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PROBLEMS IN GENERAL PHYSICS

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Problems in General Physics

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TO THE READER

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PREFACE

This collection of problems is based on the International System of Units preferred today in all the fields of science, engineering and economy.

Other units can be converted to SI units with the aid of the relevant tables given in this book.

Each section is preceded by a brief introduction describing the fundamental laws and formulas which are used to solve the problems. The solutions to the problems and the reference data are appended at the end of the book.

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ANSWERS AND SOLUTIONS

INTRODUCTION

1. International System of Units (SI)

Various physical quantities are interrelated by equations which express the relation between them. For example, the acceleration a imparted to a body with the mass m is related to the force F acting upon this body by the equation

$$F=kma \tag{1}$$

where k is a factor depending on the units in which F, m and a are measured. If the units of mass and acceleration are known, the unit of force can be so selected that the factor k in equation (1) is equal to unity, and thus

$$F=ma$$

With this aim, the unit of force should be the force which imparts a unit of acceleration to a unit of mass.

By treating any newly introduced quantity in the same manner, its unit of measurement can be found from the formula which determines this quantity; thus a system of derived units can be obtained.

Various systems differ from each other by the units taken as the basic ones.

This book is based on the International System of Units (SI) adopted by the Eleventh General Conference on Weights and Measures in 1960. The USSR State Standard GOST 9867-61 defines the SI system as the one preferable in all the fields of science, engineering and the national economy, and also in schools and colleges of the USSR.

The International System of Units (SI) is divided into several independent systems for various fields of measurement, as follows:

1. System of mechanical units (GOST 7664-61).
2. System of thermal units (GOST 8550-61).
3. System of electrical and magnetic units (GOST 8033-56).
4. System of acoustic units (GOST 8849-58).
5. System of light units (GOST 7932-56).
6. System of radioactivity and ionizing radiation units (GOST 8848-63).

The basic SI mechanical units are the metre (m), kilogramme-mass (kg) and second (s). Added to these for various fields of measurement are the following basic units: the degree Kelvin for thermal measurements, the ampere for electrical measurements and the candela for luminous intensity.

The SI system also includes two supplementary units—for a plane angle and a solid angle.

The basic and supplementary SI units are given in Table 1.

TABLE 1

Quantity	Unit	Symbol
	Basic Units	
Length	metre	m
Mass	kilogramme	kg
Time	second	s
Electric current	ampere	A
Thermodynamic temperature	degree Kelvin	°K
Luminous intensity	candela	cd
	Supplementary Units	
Plane angle	radian	rad
Solid angle	steradian	sr

Table 2 gives the prefixes used to form multiples and fractions of SI units.

TABLE 2

Prefix	Numerical value	Symbol	Prefix	Numerical value	Symbol
Atto	10^{-18}	a	Deci	10^{-1}	d
Femto	10^{-15}	f	Deca	10^{1}	da
Pico	10^{-12}	p	Hecto	10^{2}	h
Nano	10^{-9}	n	Kilo	10^{3}	k
Micro	10^{-6}	μ	Mega	10^{6}	M
Milli	10^{-3}	m	Giga	10^{9}	G
Centi	10^{-2}	c	Tera	10^{12}	T

These prefixes in Table 2 may be attached only to simple quantities (metre, gramme, etc.) and never to such as "kilogramme", which already contains the prefix "kilo". For the same reason, the unit of mass $m = 10^9$ kg $= 10^{12}$ g, for example, should be called "teragramme" (Tg).

The term "megaton" sometimes applied to this mass is wrong. The unit of length $l = 10^{-6}$ m is generally called a "micron", but the more proper name would be "micrometre" (μm).

The derived SI units are formed from the basic ones as described above. The relationship between the derived and basic units can be found from dimension formulas.

If the basic quantities are designated by l for length, m for mass, t for time, I for electric current, θ for temperature and J for luminous intensity, the dimension formula of a certain quantity x may be written in SI units as follows:

$$[x] = l^{\alpha} m^{\beta} t^{\gamma} I^{\delta} \theta^{\rho} J^{\mu}$$

To find the dimension of x, we must determine the exponents α, β, γ, δ, ρ and μ. These exponents may be positive or negative, integers or fractions.

Example 1. Find the dimension of work. Proceeding from the relation $W = Fl$, we obtain $[W] = l^2 m t^{-2}$.

Example 2. Find the dimension of specific heat. Since $c = \frac{Q}{m\Delta\theta}$ and $[Q] = [W]$, we get $[c] = l^2 t^{-2} \theta^{-1}$.

If the dimension of a physical quantity is known in the SI system, it is easy to find the dimension of its unit in this system. Thus, the unit of work obviously has the dimension m^2kgs^{-2} and the unit of specific heat—$m^2s^{-2}deg^{-1}$, etc.

Tables of derived SI units are given in the respective sections of the book: mechanical units in Chapter 1, thermal units in Chapter 2, electrical and magnetic units in Chapter 3, etc. The same chapters also contain tables which establish the relationship between the SI and other units, including non-system ones.

2. Methods of Solving Problems

When solving a problem, first of all establish the physical laws which it is based on. Then use the formulas expressing these laws to solve the problem in symbols, and finally substitute the numerical data in one system of units. Besides the International System of Units, other systems and non-system units are widely used in practice and literature. For this reason the numerical data are not always given in SI units. The relationships between the SI units, units of other systems and non-system units are given in tables at the beginning of each chapter. To solve a problem in SI units, all the initial data or data taken from reference tables should be converted into SI units. The answer, naturally, will also be in these units.

Sometimes it is not necessary to express all the data in one system. For example, if a quantity is a factor of both the numerator and the denominator, this quantity may obviously be expressed in any units provided they are the same (see Example 2 on p. 17).

When a numerical answer is obtained, pay attention to the accuracy of the final result, which should never exceed the accuracy of the initial data. Most of the problems may be solved with slide-rule accuracy. In some cases tables of four-place logarithms should be used.

As soon as the numerical data are substituted for the symbols, write the dimension of the answer.

If a graph or a diagram is required for solution, select the proper scale and origin of the coordinates, and mark the scale on the graph. The graphs in the answers to some problems are given without a scale, i. e., they show only the qualitative nature of the relationship being sought.

PROBLEMS

Chapter 1

PHYSICAL FUNDAMENTALS OF MECHANICS

MECHANICAL UNITS

The International System of Units incorporates the MKS system intended for measuring mechanical quantities (GOST 7664-61). The basic units in the MKS system are the metre (m), kilogramme (kg) and second (s).

As indicated above, the derived units of this system are formed from the basic units using the relationship between the relevant physical quantities. For example, the unit of velocity can be determined from the relation

$$v=\frac{\Delta l}{\Delta t}$$

Since the unit of length is the metre and that of time the second, the unit of velocity in the MKS system will be 1 m/s. Obviously, the unit of acceleration is 1 m/s^2.

Let us establish the unit of force. According to Newton's second law

$$F=ma$$

The unit of mass is 1 kg and the unit of acceleration 1 m/s^2. Therefore, the unit of force in the MKS system should be the force which imparts an acceleration of 1 m/s^2 to a body with a mass of 1 kg. This unit of force is known as the newton (N):

$$1\ N=1\ kg\cdot 1\ m/s^2$$

Let us now discuss the relation between the weight and mass of a body. The weight G of a body is the force with which this body is attracted by the Earth, i.e., the force which imparts an acceleration of $g=9.81$ m/s^2 to the body. Thus,

$$G=mg$$

As any other force in the MKS system, the weight of a body must be measured in newtons. Sometimes it is measured in kilogrammes

But it should always be borne in mind that the unit of weight (kilogramme) is not a unit of the MKS system. To prevent confusion, different symbols will be used for these two utterly different physical quantities: a kilogramme of mass will be denoted kg, and one of weight (force)—kgf. Let us find the relation between a kilogramme of weight and a newton. A weight of 1 kgf is defined as the weight of a body whose mass is equal to 1 kg, i.e.,

$$1 \text{ kgf} = 1 \text{ kg} \cdot 9.81 \text{ m/s}^2$$

On the other hand

$$1 \text{ N} = 1 \text{ kg} \cdot 1 \text{ m/s}^2$$

Therefore

$$1 \text{ kgf} = 9.81 \text{ N}$$

The definition of a kilogramme of weight shows that the numerical value of the weight of a body expressed in kgf is equal to the mass of this body in kg. For example, if the mass of a body is 2 kg, its weight is 2 kgf. The weight of a body in kilogrammes must be converted into newtons.

Example. The mass of a body is 4 kg. Find the weight of the body in kgf and in newtons.

Answer: $G=4$ kgf (not in the MKS system) and $G=4 \times 9.81$ N (in the MKS system).

The unit of work is determined from the relation

$$W = Fl$$

The unit of work is obviously the work performed by a force of 1 N over a distance of 1 metre. This unit of work is known as the joule (J):

$$1 \text{ J} = 1 \text{ N} \cdot 1 \text{ m}$$

Power is determined by the formula

$$P = \frac{W}{t}$$

Therefore the unit of power in the MKS system is the power of a mechanism which performs work of 1 J per second. This unit is known as the watt (W).

The same method can be used to determine the derived unit of any physical quantity in the MKS system.

Table 3 gives the basic and the most important derived units for measuring mechanical quantities in the MKS system according to GOST 7664-61.

Table 4 contains the relationships between certain mechanical SI units, and units of other systems and non-system units permitted by GOST 7664-61.

TABLE 3

Quantity and symbol	Formula	Unit	Symbol of unit	Dimension of quantity
		Basic units		
Length l	—	metre	m	l
Mass m	—	kilogramme	kg	m
Time t	—	second	s	t
		Derived units		
Area A	$A = l^2$	square metre	m^2	l^2
Volume V	$V = l^3$	cubic metre	m^3	l^3
Frequency ν	$\nu = \frac{1}{T}$	hertz	Hz	t^{-1}
Angular velocity ω	$\omega = \frac{\Delta\varphi}{\Delta t}$	radian per second	rad/s	t^{-1}
Angular acceleration α	$\alpha = \frac{\Delta\omega}{\Delta t}$	radian per second per second	rad/s^2	t^{-2}
Linear velocity v	$v = \frac{\Delta l}{\Delta t}$	metre per second	m/s	lt^{-1}
Linear acceleration a	$a = \frac{\Delta v}{\Delta t}$	metre per second per second	m/s^2	lt^{-2}
Density ρ	$\rho = \frac{m}{V}$	kilogramme per cubic metre	kg/m^3	$l^{-3}m$
Force F, weight G	$F = ma$	newton	N	lmt^{-2}
Specific weight γ	$\gamma = \frac{G}{V}$	newton per cubic metre	N/m^3	$l^{-2}mt^{-2}$
Pressure p	$p = \frac{F}{A}$	newton per square metre	N/m^2	$l^{-1}mt^{-2}$
Momentum $\overline{p}$	$\overline{p} = m\,\Delta v = F\,\Delta t$	kilogramme-metre per second	kg·m/s	lmt^{-1}
Moment of inertia I	$I = ml^2$	kilogramme-square metre	kg·m^2	l^2m
Work W and energy E	$W = Fl$	joule	J	l^2mt^{-2}
Power P	$P = \frac{\Delta W}{\Delta t}$	watt	W	l^2mt^{-3}
Dynamic viscosity η	$\eta = \frac{F}{A}\frac{\Delta l}{\Delta v}$	newton-second per square metre	N·s/m^2	$l^{-1}mt^{-1}$
Kinematic viscosity ν	$\nu = \frac{\eta}{\rho}$	square metre per second	m^2/s	l^2t^{-1}

TABLE 4

Quantity	Unit and its conversion factor to SI units
Length	1 centimetre (cm) $= 10^{-2}$ m
	1 micrometre (micron); 1 micron (μ) $= 10^{-6}$ m
	1 angström (Å) $= 10^{-10}$ m
Mass	1 gramme (g) $= 10^{-3}$ kg
	1 ton (t) $= 10^{3}$ kg
	1 centner (q) $= 10^{2}$ kg
	1 atomic unit of mass (a.u.m.) $= 1.66 \times 10^{-27}$ kg
Plane angle	1 degree (°) $= \frac{\pi}{180}$ rad
	1 minute (′) $= \frac{\pi}{108} \times 10^{-2}$ rad
	1 second (″) $= \frac{\pi}{648} \times 10^{-3}$ rad
	1 revolution (rev) $= 2\pi$ rad
Area	1 are (a) $= 100$ m^2
	1 hectare (ha) $= 10^4$ m^2
Volume	1 litre (l) $= 1.000028 \times 10^{-3}$ m^3
Force	1 dyne (dyn) $= 10^{-5}$ N
	1 kilogramme-force (kgf) $= 9.81$ N
	1 ton-force (tonf) $= 9.81 \times 10^3$ N
Pressure	1 dyn/cm^2 $= 0.1$ N/m^2
	1 kgf/m^2 $= 9.81$ N/m^2
	1 millimetre of mercury column (mm Hg) $= 133.0$ N/m^2
	1 millimetre of water column (mm H_2O) $= 9.81$ N/m^2
	1 technical atmosphere (at) $= 1$ kgf/cm^2 $= 0.981 \times 10^5$ N/m^2
	1 physical atmosphere (atm) $= 1.013 \times 10^5$ N/m^2 (this non-system unit is not listed in GOST 7664-61)
Work, energy, amount of heat	1 erg $= 10^{-7}$ J
	1 kgf-m $= 9.81$ J
	1 watt-hour (W·h) $= 3.6 \times 10^3$ J
	1 electron-volt (eV) $= 1.6 \times 10^{-19}$ J
	1 calorie (cal) $= 4.19$ J
	1 kilocalorie (1 kcal) $= 4.19 \times 10^3$ J
	1 physical litre-atmosphere (l·atm) $= 1.01 \times 10^2$ J
	1 technical litre-atmosphere (l·at) $= 98.1$ J
Power	1 erg/s $= 10^{-7}$ W
	1 kilogramme-force metre per second (kgf-m/s) $= 9.81$ W
	1 horsepower (hp) $= 75$ kgf-m/s $= 736$ W
Dynamic viscosity	1 poise (P) $= 0.1$ N·s/m^2 $= 0.1$ kg/m·s
Kinematic viscosity	1 stokes (St) $= 10^{-4}$ m^2/s

EXAMPLES OF SOLUTIONS

Example 1. A stone weighing 1.05 kgf and sliding on ice with a velocity of 2.44 m/s is stopped by friction in 10 seconds. Find the force of friction, assuming it to be constant.

Solution. From Newton's second law we have

$$F\Delta t = mv_2 - mv_1$$

where F is the force of friction under the action of which the velocity of a body with the mass m changes from v_1 to v_2 during the time Δt. In our case $v_2 = 0$, and

$$F = -\frac{mv_1}{\Delta t}$$

The minus sign shows that the force of friction F is directed opposite to the velocity v_1.

In the MKS system $m = 1.05$ kg, $v_1 = 2.44$ m/s and $\Delta t = 10$ s. Hence

$$F = -\frac{1.05\times 2.44}{10}\ \mathrm{N} = -0.256\ \mathrm{N}$$

Since the initial data are accurate to the 3rd decimal place, the answer should be given to the same accuracy with the aid of a slide rule.

By using Table 4, we can express the answer obtained in other units

$$|F| = 0.256\ \mathrm{N} = 2.56\times 10^4\ \mathrm{dyn} = 0.0261\ \mathrm{kgf}$$

Example 2. A man and a cart move towards each other. The man weighs 64 kgf and the cart 32 kgf. The velocity of the man is 5.4 km/h and of the cart 1.8 km/h. When the man approaches the cart he jumps onto it. Find the velocity of the cart carrying the man.

Solution. According to the law of conservation of momentum

$$\bar{p}_1 + \bar{p}_2 = m_1v_1 + m_2v_2 = (m_1+m_2)\,v \qquad (1)$$

where m_1 = mass of the man
v_1 = man's velocity before the jump
m_2 = mass of the cart
v_2 = velocity of the cart before the man jumps onto it
v = common velocity of the cart and man after the jump.

From formula (1)

$$v = \frac{m_1v_1 + m_2v_2}{m_1+m_2} \qquad (2)$$

Since formula (2) is homogeneous, the masses m_1 and m_2 may be written in any units provided they are the same. Formula (2) also shows that since the units of the masses will be cancelled out, the unit of the velocity v will be the same as that of the velocities v_1 and v_2. For this reason it is not necessary to convert all the data to MKS units.

The initial velocities of the cart and man were opposite in direction, and therefore their signs were different. Taking the velocity of the man to be positive, we have $v_1 = 5.4$ km/h and $v_2 = -1.8$ km/h. Besides, $m_1 = 64$ kg and $m_2 = 32$ kg. By inserting these data into formula (2), we obtain

$$v = \frac{64\times5.4-32\times1.8}{64+32} \text{ km/h} = 3.0 \text{ km/h}$$

The velocity $v > 0$. Thus, after the man jumps onto it, the cart will begin to move in the direction in which the man was walking.

Example 3. Water is pumped out of a well 20 metres deep by means of a pump with a motor rated at 5 hp. Find the efficiency of the motor if 3.8×10^5 litres of water are pumped out during 7 hours of operation.

Solution. The power of the motor P is related to the work W which the motor performs during the time t by the expression

$$P = \frac{W}{t\eta} \qquad (1)$$

where η is the efficiency of the pump. The work required to raise a mass m of water to a height h is

$$W = mgh \qquad (2)$$

The mass m of the water occupies the volume

$$V = \frac{m}{\rho}$$

whence

$$m = V\rho \qquad (3)$$

where ρ is the density of water. Substitution of $V\rho$ for m in formula (2) and the resulting expression for W in formula (1) gives

$$P = \frac{V\rho gh}{t\eta}$$

whence

$$\eta = \frac{V\rho gh}{Pt} \qquad (4)$$

Let us use Table 4 to convert the data of the example to the MKS system. It is expedient to calculate the arithmetic values in the final formula. In our case, $V=3.8\times10^5$ l$=3.8\times10^5\times10^{-3}$ m^3, $\rho = 1$ g/cm$^3 = \frac{10^{-3}}{10^{-6}}$ kg/m^3, $P=5$ hp$=5\times736$ W, $t=7\times3{,}600$ s, $g = 9.81$ m/s^2 and $h = 20$ m. Substituting these data in formula (4), we finally get

$$\eta = \frac{3.8\times10^5\times10^{-3}\times10^{-3}\times9.81\times20}{10^{-6}\times5\times736\times7\times3{,}600} = 0.8 = 80 \text{ per cent}$$

1. Kinematics

In the general case, the velocity of rectilinear motion is

$$v=\frac{ds}{dt}$$

and the acceleration

$$a=\frac{dv}{dt}=\frac{d^2s}{dt^2}$$

In uniform rectilinear motion

$$v=\frac{s}{t}=\text{const}$$

and

$$a=0$$

In uniformly variable rectilinear motion

$$s=v_0t+\frac{at^2}{2}$$

$$v=v_0+at$$

$$a=\text{const}$$

In these equations the acceleration a is positive in uniformly accelerated motion and negative in uniformly retarded motion.

In curvilinear motion the total acceleration is equal to

$$a=\sqrt{a_t^2+a_n^2}$$

where a_t is the tangential acceleration and a_n the normal (centripetal) acceleration:

$$a_t=\frac{dv}{dt} \text{ and } a_n=\frac{v^2}{r}$$

where v=velocity of motion

r=radius of curvature of the trajectory at the given point.

In the general case of rotational motion the angular velocity is

$$\omega=\frac{d\varphi}{dt}$$

and the angular acceleration

$$\alpha=\frac{d\omega}{dt}=\frac{d^2\varphi}{dt^2}$$

In uniform rotational motion the angular velocity is

$$\omega=\frac{\varphi}{t}=\frac{2\pi}{T}=2\pi\nu$$

where T=period of rotation

ν=frequency of rotation, i.e., the number of revolutions in a unit of time.

The angular velocity ω is related to the linear velocity v by the expression

$$v = \omega r$$

In rotational motion, the tangential and normal accelerations can be expressed as

$$a_t = \alpha r$$
$$a_n = \omega^2 r$$

Table 5 compares the equations for translational and rotational motion.

TABLE 5

Translational motion	Rotational motion
Uniform	
$s = vt$ $v = \text{const}$ $a = 0$	$\varphi = \omega t$ $\omega = \text{const}$ $\alpha = 0$
Uniformly variable	
$s = v_0 t + \frac{at^2}{2}$ $v = v_0 + at$ $a = \text{const}$	$\varphi = \omega_0 t + \frac{\alpha t^2}{2}$ $\omega = \omega_0 + \alpha t$ $\alpha = \text{const}$
Non-uniform	
$s = f(t)$ $v = \frac{ds}{dt}$ $a = \frac{dv}{dt} = \frac{d^2 s}{dt^2}$	$\varphi = f(t)$ $\omega = \frac{d\varphi}{dt}$ $\alpha = \frac{d\omega}{dt} = \frac{d^2\varphi}{dt^2}$

1.1. A car travels at a velocity of 80 km/h during the first half of its running time and at 40 km/h during the other half. Find the average velocity of the car.

1.2. A car covers half a distance at a velocity of 80 km/h and the other at 40 km/h. What is the average velocity of the car?

1.3. A ship goes from A to B at $v_1 = 10$ km/h and from B to A at $v_2 = 16$ km/h. Find: (1) the average velocity of the ship, and (2) the velocity of the river current.

1.4. Determine the velocity with respect to the river bank of: (1) a boat going downstream, (2) a boat going upstream, and (3) a boat traveling at an angle of $\alpha=90°$ to the current. The velocity of the current $v_1=1$ m/s, the velocity of the boat with respect to the water $v_2=$ $=2$ m/s.

1.5. An airplane is flying with the velocity of $v_1=800$ km/h relative to the air. A wind with a velocity of $v_2=15$ m/s is blowing from west to east. What is the velocity of the airplane with respect to the Earth, and what should the angle with the meridian be to fly the airplane: (1) southward, (2) northward, (3) westward and (4) eastward?

1.6. An airplane flies from A to B at a distance of 300 km eastward. Find the duration of the flight if: (1) there is no wind, (2) the wind blows from south to north, and (3) the wind blows from west to east. The velocity of the wind $v_1=20$ m/s and that of the airplane with respect to the air $v_2=600$ km/h.

1.7. A boat moves perpendicular to the bank with a velocity of 7.2 km/h. The current carries it 150 m downstream. Find: (1) the velocity of the current, (2) the time required to cross the river. The river is 0.5 km wide.

1.8. A body thrown vertically upward returns to the Earth in 3 seconds. (1) What was the initial velocity of the body? (2) What height did the body reach? Disregard the resistance of the air.

1.9. A stone is thrown upward to a height of 10 metres. (1) After what time will it fall onto the Earth? (2) What height can be reached by the stone if its initial velocity is doubled? Disregard the resistance of the air.

1.10. A stone is dropped from a balloon at an altitude of 300 metres. How much time is required for the stone to reach the Earth if: (1) the balloon is ascending with a velocity of 5 m/s, (2) the balloon is descending with a velocity of 5 m/s, (3) the balloon is stationary? Disregard the resistance of the air.

1.11. Draw a diagram showing the relationship between the height h, velocity v and time t for a body thrown vertically upward with an initial velocity of 9.8 m/s. Plot the diagram for the time interval from 0 to 2 seconds, i.e., for $0 \leqslant t \leqslant 2$ s after every 0.2 s. Disregard the resistance of the air.

1.12. A body falls vertically from the height $h=19.6$ metres with the initial velocity equal to zero. What distance will be traveled by the body: (1) during the first 0.1 second of motion, (2) during the last 0.1 second of motion? Disregard the resistance of the air.

1.13. A body falls vertically from the height $h=19.6$ metres with its initial velocity equal to zero. What time will it take the body to travel: (1) the first metre, (2) the last metre? Disregard the resistance of the air.

1.14. During the last second of its free fall a body covers half of the total distance traveled. Find: (1) the height h from which the body falls, (2) the duration of falling.

1.15. A body A is thrown vertically upward with the initial velocity v_1; a body B falls from the height h with the initial velocity $v_2=0$. Find how the distance x between the bodies A and B depends on the time t if the bodies began to move simultaneously.

1.16. The distance between two underground stations is 1.5 kilometres. The first half of the distance is covered by a train with a uniformly accelerated speed, and the second half with a uniformly retarded speed. The maximum speed of the train is 50 km/h. Find: (1) the acceleration, taking it to be numerically equal to the retardation, (2) the time the train travels between the stations.

1.17. A train is running at a speed of 36 km/h. If the supply of current to the traction motors is stopped, the train, moving with a uniformly retarded speed, will stop in 20 seconds. Find: (1) the negative acceleration of the train, (2) the distance from the station at which the current should be switched off.

1.18. Braking uniformly reduces the speed of a train from 40 km/h to 28 km/h during one minute. Find: (1) the negative acceleration of the train, (2) the distance traveled by the train during the time the brakes are applied.

1.19. A car runs at a uniformly retarded speed with a negative acceleration of -0.5 m/s^2. The initial velocity of the car is 54 km/h. In how much time and how far from the initial point will the car stop?

1.20. A body A begins to move with the initial velocity v_0' and continues to move with the constant acceleration a_1. A body B begins to move at the same time as the body A with the initial velocity v_0'' and continues to move with the constant negative acceleration a_2. What time is required for the two bodies to acquire the same velocity after motion has begun?

1.21. A body A begins to move with the initial velocity $v_0'=2$ m/s and continues to move at a constant acceleration a. In $\Delta t=10$ seconds after the body A begins to move, a body B departs from the same point with the initial velocity $v_0''=12$ m/s and moves with the same acceleration a. What is the maximum acceleration a at which the body B can overtake the body A?

1.22. The relationship between the distance s traveled by a body and the time t is given by the equation $s=At-Bt^2+Ct^3$, where $A=2$ m/s, $B=3$ m/s^2 and $C=4$ m/s^3. Find: (1) how the velocity v and acceleration a depend on the time t, (2) the distance traveled by the body, the velocity and acceleration of the body in 2 seconds after motion has begun. Plot a diagram showing the distance, velocity and acceleration for $0 \leqslant t \leqslant 3$ s after every 0.5 s.

1.23. The relationship between the distance s traveled by a body and the time t is expressed by the equation $s=A-Bt+Ct^2$, where $A=6$ m, $B=3$ m/s and $C=2$ m/s^2. Determine the average velocity and the average acceleration of the body within the time interval from 1 to 4 seconds. Plot the diagram of the distance, velocity and acceleration for $0 \leqslant t \leqslant 5$ seconds after every second.

1.24. The relationship between the distance s traveled by a body and the time t is described by the equation $s=A+Bt+Ct^2$, where $A=3$ m, $B=2$ m/s and $C=1$ m/s². Determine the average velocity and the average acceleration of the body during the first, second and third seconds of motion.

1.25. The relationship between the distance s traveled by a body and the time t is described by the equation $s=A+Bt+Ct^2+Dt^3$, where $C=0.14$ m/s² and $D=0.01$ m/s³. (1) In what time after motion begins will the acceleration of the body be equal to 1 m/s²? (2) What is the average acceleration the body acquires during this time?

1.26. A stone is thrown horizontally with the velocity $v_0=15$ m/s from a tower with a height of $H=25$ metres. Find: (1) the time during which the stone is in motion, (2) the distance s_x from the tower base to where the stone will drop onto the ground, (3) the velocity v with which it will touch the ground, (4) the angle φ formed by the trajectory of the stone with the horizontal at the point where it reaches the ground. Disregard the resistance of the air.

1.27. A stone thrown horizontally fell onto the ground after 0.5 second at a distance of 5 metres from where it was thrown. (1) From what height h was the stone thrown? (2) What was the initial velocity v_0 of the stone? (3) What velocity v did the stone touch the ground with? (4) What angle φ was formed by the trajectory of the stone with the horizontal at the point where it reached the ground? Disregard the resistance of the air.

1.28. A ball thrown horizontally strikes a wall 5 metres away. The height of the point struck by the ball is 1 metre lower than the height which it was thrown from. (1) What velocity v_0 was the ball thrown with? (2) At what angle φ did the ball reach the wall? Disregard the resistance of the air.

1.29. A stone is thrown horizontally. In 0.5 second after the stone began to move, the numerical value of its velocity was 1.5 times its initial velocity. Find the initial velocity of the stone. Disregard the resistance of the air.

1.30. A stone is thrown horizontally with the velocity $v_x=15$ m/s. Determine the normal and tangential accelerations of the stone in 1 second after it begins to move. Disregard the resistance of the air.

1.31. A stone is thrown horizontally with the velocity 10 m/s. Find the radius of curvature of its trajectory in 3 seconds after the motion began. Disregard the resistance of the air.

1.32. A ball is thrown with the velocity $v_0=10$ m/s at an angle of $\alpha=40°$ to the horizon. Find: (1) the height s_y which the ball will rise to, (2) the distance s_x from the point of throwing to where the ball will drop onto the ground, (3) the time during which the ball will be in motion. Disregard the resistance of the air.

1.33. An athlete puts a shot 16 m 20 cm in Leningrad. What distance will be covered by an identical throw in Tashkent, assuming that the initial velocity and angle to the horizon are the same? The acceleration of gravity is 981.9 cm/s^2 in Leningrad and 980.1 cm/s^2 in Tashkent.

1.34. A body is thrown with the velocity v_0 at an angle to the horizon. The duration of motion t=2.2 seconds. Find the maximum height reached by the body. Disregard the resistance of the air.

1.35. A stone thrown with the velocity v_0=12 m/s at an angle of α=45° to the horizon dropped to the ground at the distance s from the point where it was thrown. From what height h should the stone be thrown in a horizontal direction with the same initial velocity v_0 for it to fall at the same spot?

1.36. A body is thrown with the velocity v_0=14.7 m/s at an angle of α=30° to the horizon. Find the normal and tangential accelerations of the body in t=1.25 s after it began to move. Disregard the resistance of the air.

1.37. A body is thrown with the velocity v_0=10 m/s at an angle of α=45° to the horizon. Find the radius of curvature of its trajectory in t=1 s after the body began to move. Disregard the resistance of the air.

1.38. A body is thrown with the velocity v_0 at an angle of α to the horizon. Determine v_0 and α if the maximum height which the body reaches is h=3 m and the radius of curvature at the upper point of its trajectory R=3 m. Disregard the resistance of the air.

1.39. A stone is thrown from a tower with a height of H=25 m at v_0=15 m/s and an angle α=30° to the horizon. Find: (1) the time during which the stone will be in motion, (2) the distance from the tower base to where the stone will drop onto the ground, (3) the velocity with which the stone will fall to the ground, (4) the angle φ formed by the trajectory of the stone with the horizon at the point of fall. Disregard the resistance of the air.

1.40. A boy throws a ball with the velocity v_0=10 m/s at an angle of α=45° to the horizon. The ball strikes a wall at a distance of s=3 m from the boy. (1) When will the ball strike the wall (when the ball ascends or descends)? (2) Find the height y at which the ball will strike the wall (counting from the height which the ball was thrown from). (3) Determine the velocity of the ball at the moment of impact. Disregard the resistance of the air.

1.41. Find the angular velocities of: (1) daily rotation of the Earth, (2) a watch hour hand, (3) a watch minute hand, (4) an artificial satellite of the Earth rotating along a circular orbit with the period of revolution T=88 min, (5) the linear velocity of this satellite if its orbit is at a distance of 200 km from the Earth's surface.

1.42. Determine the linear velocity of revolution of points on the Earth's surface at a latitude of 60°.

1.43. What should the velocity of an airplane flying from east to west be on the equator for the passengers to see the Sun motionless in the sky?

1.44. An axle with two disks mounted at a distance of $l=0.5$ m from each other rotates with an angular velocity which corresponds to the frequency $\nu=1{,}600$ rpm. A bullet flying along the axle pierces both disks. The hole in the second disk is displaced with respect to that in the first one by the angle $\varphi=12°$. Find the velocity of the bullet.

1.45. Find the radius of a rotating wheel if the linear velocity v_1 of a point on the rim is 2.5 times greater than the linear velocity v_2 of a point 5 centimetres closer to the wheel axle.

1.46. A uniformly accelerated wheel reaches the angular velocity $\omega=20$ rad/s in $N=10$ revolutions after rotation begins. Determine the angular acceleration of the wheel.

1.47. In $t=1$ minute after it begins to rotate a flywheel acquires a velocity corresponding to $\nu=720$ rpm. Find the angular acceleration of the wheel and the number of its revolutions per minute. The motion is uniformly accelerated.

1.48. When braked, a uniformly retarded wheel reduces its velocity from 300 rpm to 180 rpm during one minute. Find the angular acceleration of the wheel and the number of revolutions it completes in this time.

1.49. A fan rotates with a velocity corresponding to a frequency of 900 rev/min. When its motor is switched off, the fan uniformly slows down and performs 75 revolutions before it comes to a stop. How much time elapsed from the moment the fan was switched off to the moment it stopped?

1.50. A shaft rotates at a constant velocity corresponding to the frequency 180 rev/min. At a certain moment the shaft is braked and begins to slow down uniformly with an angular acceleration numerically equal to 3 rad/s^2. (1) In how much time will the shaft stop? (2) What number of revolutions will it perform before stopping?

1.51. A point moves along a circle having a radius of $r=20$ cm with a constant tangential acceleration of $a_t=5$ cm/s^2. How much time is needed after motion begins for the normal acceleration a_n of the point to be: (1) equal to the tangential acceleration, (2) double the tangential acceleration?

1.52. A point moves along a circle having a radius of $r=10$ cm with a constant tangential acceleration a_t. Find the tangential acceleration a_t of the point if its velocity is $v=79.2$ cm/s at the end of the fifth revolution after motion has begun.

1.53. A point moves along a circle having a radius of $r=10$ cm with a constant tangential acceleration a_t. Find the normal acceleration a_n

of the point in $t=20$ seconds after motion begins if the linear velocity of the point is $v=10$ cm/s at the end of the fifth revolution after motion has begun.

1.54. It may be assumed to a first approximation that an electron moves in an atom of hydrogen along a circular orbit with the constant velocity v. Find the angular velocity of electron rotation around the nucleus and its normal acceleration. The radius of the orbit $r=0.5\times\times10^{-10}$ m and the velocity of the electron along this orbit $v=2.2\times\times10^{6}$ m/s.

1.55. A wheel having a radius $r=10$ cm rotates with a constant angular acceleration $\alpha=3.14$ rad/s^2. Find for points on the wheel rim at the end of the first second after motion has begun: (1) the angular velocity, (2) the linear velocity, (3) the tangential acceleration, (4) the normal acceleration, (5) the total acceleration, and (6) the angle formed by the direction of the total acceleration with the wheel radius.

1.56. A point moves along a circle with a radius of $r=2$ cm. The relationship between the distance and the time is given by the equation $x=Ct^3$, where $C=0.1$ cm/s^3. Find the normal and tangential accelerations of the point at the moment when its linear velocity $v=0.3$ m/s.

1.57. A point moves along a circle with the relationship between the distance and the time conforming to the equation $s=A+Bt+Ct^2$, where $B=-2$ m/s and $C=1$ m/s^2. Find the linear velocity of the point, and its tangential, normal and total accelerations in $t=3$ seconds after motion begins if the normal acceleration of the point when $t'=2$ seconds is $a'_n=0.5$ m/s^2.

1.58. Find the angular acceleration of a wheel if the vector of the total acceleration of a point on the rim forms an angle of 60° with the direction of the linear velocity of this point in 2 seconds after uniformly accelerated motion begins.

1.59. A wheel rotates with a constant angular acceleration $\alpha==2$ rad/s^2. In $t=0.5$ second after motion begins, the total acceleration of the wheel becomes $a=13.6$ cm/s^2. Determine the radius of the wheel.

1.60. A wheel with a radius of $r=0.1$ m so rotates that the relationship between the angle of rotation of the wheel radius and the time is described by the equation $\varphi=A+Bt+Ct^3$, where $B=2$ rad/s and $C=1$ rad/s^3. Find the following values for points lying on the wheel rim in 2 seconds after motion begins: (1) the angular velocity, (2) the linear velocity, (3) the angular acceleration, (4) the tangential acceleration, (5) the normal acceleration.

1.61. A wheel with a radius of $r=5$ cm so rotates that the relationship between the angle of rotation of the wheel radius and the time is described by the equation $\varphi=A+Bt+Ct^2+Dt^3$, where $D==1$ rad/s^3. Find the change in the tangential acceleration Δa_t during each second of motion for points on the rim.

1.62. A wheel with a radius of $r=10$ cm so rotates that the relationship between the linear velocity of points on the wheel rim and the duration of motion is described by the equation $v=At+Bt^2$, where $A=3$ cm/s^2 and $B=1$ cm/s^3. Find the angle formed by the vector of the total acceleration with the wheel radius at the moments of time $t=0$, 1, 2, 3, 4 and 5 seconds after motion begins.

1.63. A wheel so rotates that the relationship between the angle of rotation of the wheel radius and the time is described by the equation $\varphi=A+Bt+Ct^2+Dt^3$, where $B=1$ rad/s, $C=1$ rad/s^2 and $D=1$ rad/s^3. Find the radius of the wheel if the normal acceleration of points on the wheel rim is $a_n=3.46\times 10^2$ m/s^2 after two seconds of motion.

1.64. How many times does the normal acceleration of a point on the rim of a rotating wheel exceed its tangential acceleration at the moment when the vector of the total acceleration of this point forms an angle of 30° with the vector of the linear velocity?

2. Dynamics

The basic law of dynamics (Newton's second law) is expressed by the formula

$$F\,dt=d(mv)$$

If the mass is constant, then

$$F=m\frac{dv}{dt}=ma$$

where a is the acceleration acquired by a body of mass m under the action of a force F.

The work of a force F over the distance s can be expressed by the following formula

$$W=\int_s F_s\,ds$$

where F_s is the projection of the force on the direction of the distance, and ds is the distance. Integration should include the entire distance s.

In a particular case of a constant force acting at a constant angle to the motion, we have

$$W=Fs\cos\alpha$$

where α is the angle between the force F and the distance s.

Power is determined by the formula

$$P=\frac{dW}{dt}$$

When the power is constant

$$P=\frac{W}{t}$$

where W is the work performed during the time t.

The power can also be found by the formula

$$P=Fv\cos\alpha$$

i.e., by the product of the velocity of motion and the projection of the force on its direction.

The kinetic energy of a body with the mass m moving with the velocity v is equal to

$$E_k=\frac{mv^2}{2}$$

The formulas for potential energy vary depending on the nature of the acting forces.

In an isolated system the momentum of all the bodies contained in it remains the same, i.e.,

$$\bar{p}_1+\bar{p}_2+\ldots+\bar{p}_n=\text{const}$$

In the inelastic central impact of two bodies with the masses m_1 and m_2, the total velocity of motion of these bodies after the impact can be found from the formula

$$u=\frac{\bar{p}_1+\bar{p}_2}{m_1+m_2}$$

where the subscript "1" denotes the first body before the impact, and the subscript "2"—the second body before the impact.

In elastic central impact, the bodies will move at different velocities. The velocity of the first body after the impact

$$u_1=\frac{(m_1-m_2)v_1+2\bar{p}_2}{m_1+m_2}$$

and that of the second body after the impact

$$u_2=\frac{(m_2-m_1)v_2+2\bar{p}_1}{m_1+m_2}$$

In curvilinear motion, the force acting on a material particle can be resolved into two components—tangential and normal.

The normal component

$$F_n=\frac{mv^2}{r}$$

is the centripetal force. Here v is the linear velocity of motion of a body of mass m and r is the radius of curvature of the trajectory at the given point.

The force which causes an elastic deformation of x is proportional to the magnitude of the deformation, i.e.,

$$F = kx$$

where k is a coefficient numerically equal to the force causing a unit deformation (deformation coefficient).

The potential energy of elastic forces

$$E_p = \frac{kx^2}{2}$$

Two material particles (i.e., bodies whose dimensions are small relative to the distance between them) are attracted to each other by the force

$$F = \gamma \frac{m_1 m_2}{R^2}$$

where γ=gravitational constant, equal to 6.67×10^{-11} $m^3/kg \cdot s^2$
m_1 and m_2=masses of the interacting material particles
R=distance between these particles.

This law is also true for homogeneous balls, r being the distance between their centres.

The potential energy of the forces of gravitation

$$E_p = -\gamma \frac{m_1 m_2}{r}$$

The minus sign shows that when $r = \infty$, the potential energy of two interacting bodies is zero and diminishes when these bodies approach each other.

Kepler's third law has the form

$$\frac{T_1^2}{T_2^2} = \frac{r_1^3}{r_2^3}$$

where T_1 and T_2=periods of planet rotation
r_1 and r_2=major semiaxes of the planet orbits.

If a planet moves along a circular orbit, the role of the major semiaxis is played by the orbit radius.

2.1. What is the weight of ballast to be dropped from a uniformly descending balloon to enable the latter to rise with the same velocity? The balloon with the ballast weighs 1,600 kgf and its lifting capacity is 1,200 kgf. The force of the air resistance is to be taken the same during ascent and descent.

2.2. A load of G=1 kgf is suspended on a thread. Find the tension of the thread with the load if: (1) it is lifted with an acceleration of a=5 m/s^2, (2) it is lowered with the same acceleration a=5 m/s^2

2.3. A steel wire of a certain diameter withstands a weight up to 4,400 N. What is the maximum acceleration which a load of 3,900 N suspended from this wire may be lifted with so that the wire does not break?

2.4. The weight of a lift with passengers is 800 kgf. Find the acceleration and the direction of motion of the lift if the tension of the wire rope holding it is: (1) 1,200 kgf and (2) 600 kgf.

2.5. A weight is suspended on a thread. If the weight is raised with the acceleration $a_1=2$ m/s^2 the tension T of the thread will be half that at which the thread breaks. What acceleration a_2 should the weight be raised with to break the thread?

2.6. An automobile weighing 10^4 N is braked and stops after 5 seconds, covering 25 metres with uniformly retarded motion. Find: (1) the initial speed of the automobile, (2) the braking force.

2.7. A train with a mass of 500 tons is uniformly retarded by applying its brakes. Its velocity drops from 40 km/h to 28 km/h in one minute. Find the braking force.

2.8. A car weighing 1.96×10^5 N moves with an initial speed of 54 km/h. Determine the average force acting on the car if it stops in: (1) 1 minute 40 seconds, (2) 10 seconds and (3) 1 second.

2.9. What force should be applied to a car standing on rails for it to begin to move with uniform acceleration and travel the distance $s = 11$ m during $t=30$ seconds? The weight of the car $G=16$ tonf. During motion the car is acted upon by a force of friction equal to 0.05 of its weight.

2.10. When the traction motors were switched off, a train weighing 4.9×10^6 N stopped in one minute under the action of a friction force of 9.8×10^4 N. What was the speed of the train?

2.11. A railway car having a mass of 20 tons moves with a constant negative acceleration of 0.3 m/s^2. The initial speed of the car is 54 km/h. (1) What is the braking force acting on the car? (2) In how much time will the car stop? (3) What distance will be covered by the car until it stops?

2.12. A body with a mass of 0.5 kg is in rectilinear motion. The relation between the distance s traveled by the body and the time t is shown by the equation $s=A-Bt+Ct^2-Dt^3$, where $C=5$ m/s^2 and $D=1$ m/s^3. Determine the force which acts on the body at the end of the first second of motion.

2.13. Under the action of a constant force $F=1$ kgf a body so moves in a straight line that the relation between the distance s traveled by the body and the time t is described by the equation $s=A-Bt+Ct^2$. Find the mass of the body if the constant $C=1$ m/s^2.

2.14. A body with a mass $m=0.5$ kg so moves that the relationship between the distance s traveled by the body and the time t is given by the equation $s=A \sin \omega t$, where $A=5$ cm and $\omega = \pi$ rad/s. Find the force F acting on the body in $t=1/6$ second after motion begins.

2.15. A molecule with the mass $m=4.65 \times 10^{-26}$ kg flying at right angles to the wall of a vessel at a velocity of $v=600$ m/s strikes the wall and rebounds from it elastically without losing its velocity. Find the impulse of the force received by the wall during the impact.

2.16. A molecule with the mass $m=4.65 \times 10^{-26}$ kg flying at a velocity of $v=600$ m/s strikes the wall of a vessel at an angle of 60° to the normal and rebounds from it elastically at the same angle without losing its velocity. Find the impulse of the force received by the wall during the impact.

2.17. A ball weighing 0.1 kgf dropping vertically from a certain height strikes an inclined surface and rebounds from it elastically without losing its velocity. The surface is inclined at 30° to the horizon. The impulse of force received by the surface during the impact is 1.73 N·s. What time will elapse between the moment the ball strikes the surface and the moment when it is at the highest point of its trajectory?

2.18. A jet of water with a section of $A = 6$ cm^2 strikes a wall at an angle of $\alpha=60°$ to the normal and rebounds elastically from the wall without losing its velocity. Find the force acting on the wall if the velocity of the water in the jet is $v=12$ m/s.

2.19. A tramcar starts with a constant acceleration of $a=0.5$ m/s^2. Twelve seconds after it begins to move its motor is switched off, and the motion of the car uniformly retards up to the next stop. Along the entire route of the car the coefficient of friction $f=0.01$. Find: (1) the maximum speed of the car, (2) the total duration of motion, (3) the negative acceleration of the car moving with a uniform retardation, (4) the total distance covered by the car.

2.20. A motor vehicle weighs 9.8×10^3 N. During motion it is acted upon by a friction force equal to 0.1 of its weight. What should the tractive effort (force) developed by the engine be for the vehicle to move: (1) uniformly, (2) with an acceleration equal to 2 m/s^2?

2.21. What angle α to the horizon will be formed by the surface of petrol in the tank of a motor vehicle moving horizontally with a constant acceleration of $a=2.44$ m/s^2?

2.22. A ball is suspended on a thread from the ceiling of a tramcar. The brakes are applied and the speed of the car changes uniformly from $v_1=18$ km/h to $v_2=6$ km/h during the time $\Delta t=3$ seconds. By what angle α will the thread with the ball deviate from the vertical?

2.23. A railway carriage is braked and its speed changes uniformly from $v_1=47.5$ km/h to $v_2=30$ km/h during the time $\Delta t=3.3$ seconds. At what highest value of the coefficient of friction between a suit-case and a rack will the suit-case begin to slide over the rack during retardation?

2.24. A rope so lies on a table that part of it hangs over. The rope begins to slide when the length of the hanging part is 25 per cent of the entire length. What is the coefficient of friction between the rope and the table?

2.25. An automobile weighs 1 tonf and is acted upon during motion by a friction force equal to 10 per cent of its weight. Find the tractive effort developed by the engine if the automobile moves at a constant speed: (1) up a grade of 1 in 25, (2) down the same grade.

2.26. Find the tractive effort developed by the engine of an automobile moving uphill with an acceleration of 1 m/s^2. The grade is 1 in 25. The automobile weighs 9.8×10^3 N and the coefficient of friction is 0.1.

2.27. A body lies on an inclined plane forming an angle of four degrees with the horizon. (1) At what maximum value of the coefficient of friction will the body begin to slide over the plane? (2) What acceleration will the body slide with over the plane if the coefficient of friction is 0.03? (3) How much time is required for the body to travel a distance of 100 metres? (4) What will the velocity of the body be at the end of these 100 metres?

2.28. A body slides over an inclined plane forming an angle of $\alpha = 45°$ with the horizon. After a distance of $s = 36.4$ cm the body acquires a velocity of $v = 2$ m/s. What is the coefficient of friction between the body and the plane?

2.29. A body slides over an inclined plane forming an angle of 45° with the horizon. The relationship between the distance s traveled by the body and the time t is described by the equation $s = Ct^2$, where $C = 1.73$ m/s^2. Find the coefficient of friction between the body and the plane.

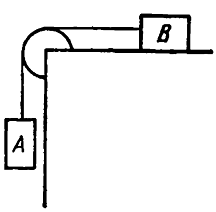

Fig. 1

2.30. Two weights $G_1 = 2$ kgf and $G_2 = 1$ kgf are linked by a thread passing over a weightless pulley. Find: (1) the acceleration with which the weights move, (2) the tension of the thread. Disregard the friction in the pulley.

2.31. A weightless pulley is attached to the edge of a table (Fig. 1). Equal weights A and B ($G_1 = G_2 = 1$ kgf) are linked by a thread passing over the pulley. The coefficient of friction between weight B and the table is $f = 0.1$. Find: (1) the acceleration which the weights move with, (2) the tension of the thread. Disregard the friction in the pulley.

2.32. A weightless pulley is attached to the top of an inclined plane (Fig. 2) forming an angle of $\alpha = 30°$ with the horizon. Equal weights A and B ($G_1 = G_2 = 1$ kgf) are linked by a thread passing over a pulley. Find: (1) the acceleration which the weights move with, (2) the tension of the thread. Disregard the friction in the pulley and the friction between weight B and the inclined plane.

2.33. Solve the previous problem if the coefficient of friction between weight B and the inclined plane $f = 0.1$. Disregard the friction in the pulley.

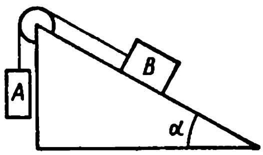

Fig. 2

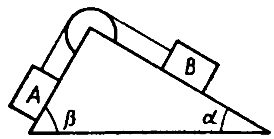

Fig. 3

2.34. A weightless pulley is attached to the apex of two inclined planes forming the angles $\alpha=30°$ and $\beta=45°$ with the horizon, respectively (Fig. 3). Equal weights A and B ($G_1=G_2=1$ kgf) are linked by a thread passing over the pulley. Find: (1) the acceleration which the weights move with, (2) the tension of the thread. Disregard the friction between weights A and B and the inclined planes and the friction in the pulley.

2.35. Solve the previous problem if the coefficients of friction between weights A and B and the inclined planes are $f_1 = f_2 = 0.1$. Disregard the friction in the pulley. Show that the solutions of problems 2.30 through 2.34 can be obtained as particular cases from the formulas used to solve the present problem.

2.36. The work $W=8$ kgf-m was performed to lift a load with the weight $G=2$ kgf vertically to a height of $h=1$ m by the constant force F. What acceleration was the load raised with?

2.37. An airplane ascends and reaches a velocity of $v=360$ km/h at an altitude of $h=5$ km. How many times is the work performed during the ascent against the force of gravity greater than that performed to increase the velocity of the airplane?

2.38. What amount of work should be performed to make a moving body with a mass of 2 kg: (1) increase its velocity from 2 m/s to 5 m/s, (2) stop from an initial velocity of 8 m/s?

2.39. A ball flying at $v_1=15$ m/s is thrown back by a racket in the opposite direction with the velocity $v_2= 20$ m/s. Find the change in the momentum of the ball if its kinetic energy changes by $\Delta E = 8.75$ J.

2.40. A stone pushed over an ice surface with the velocity $v=$ $=2$ m/s slides over a distance of $s = 20.4$ metres before it stops. Find the coefficient of friction between the stone and the ice, assuming it to be constant.

2.41. A railway car weighing 20 tonf uniformly decelerating under a force of friction of 6,000 N comes to a stop. The initial speed of the car is 54 km/h. Find: (1) the work of the friction forces, (2) the distance covered by the car before it stops.

2.42. A driver begins to brake his car 25 metres from an obstacle on the road. The force of friction in the brake shoes is constant and equal to 3,840 N. The car weighs 1 tonf. What is the maximum speed from which the car can be stopped before the obstacle? Disregard the friction of the wheels against the road.

2.43. A tramcar moves with the acceleration $a=49.0$ cm/s^2. Find the coefficient of friction if 50 per cent of the motor power is spent to overcome the friction forces and 50 per cent to increase the speed.

2.44. Find the work which must be performed to increase the velocity of a body from 2 m/s to 6 m/s over a distance of 10 metres. A constant force of friction equal to 0.2 kgf acts over the entire distance. The mass of the body is 1 kg.

2.45. A motor vehicle weighs 9.81×10^3 N. During motion it is acted upon by a constant force of friction equal to 10 per cent of its weight. What amount of petrol is spent by the engine to increase the speed of the vehicle from 10 km/h to 40 km/h over a distance of 0.5 km? The engine efficiency is 20 per cent, and the heating value of the petrol is 4.6×10^7 J/kg.

2.46. What amount of petrol is consumed by a motor vehicle engine per 100 kilometres if the average speed is 30 km/h with a mean engine power of 15 hp? The engine efficiency is 22 per cent. Take the other data necessary from the previous problem.

2.47. Find the efficiency of a motor vehicle engine if at a speed of 40 km/h the engine consumes 13.5 litres of petrol per 100 kilometres and develops a power of 16.3 hp. The density of the petrol is 0.8 g/cm^3. Take the other data necessary from Problem 2.45.

2.48. Plot a diagram showing how the kinetic, potential and total energy of a stone with a mass of 1 kg thrown vertically upward with an initial velocity of 9.8 m/s changes with time for $0 \leqslant t \leqslant 2$ seconds at intervals of 0.2 second. Use the solution of Problem 1.11.

2.49. Plot a diagram showing how the kinetic, potential and total energy of a stone changes with distance using the conditions of the previous problem.

2.50. A stone weighing 2 kgf fell from a certain height during 1.43 seconds. Find the kinetic and potential energy of the stone at half the height. Disregard the resistance of the air.

2.51. A stone is thrown horizontally with the velocity $v_0=15$ m/s from a tower with a height of $h=25$ metres. Find the kinetic and potential energy of the stone in one second after motion begins. The mass of the stone $m=0.2$ kg. Disregard the resistance of the air.

2.52. A stone is thrown at an angle of $\alpha=60°$ to the horizon with a velocity of $v_0=15$ m/s. Find the kinetic, potential and total energy of the stone: (1) in one second after motion begins, (2) at the highest point of the trajectory. The mass of the stone $m = 0.2$ kg. Disregard the resistance of the air.

2.53. The work spent to put a shot at an angle of $\alpha=30°$ to the horizon is $W = 216$ J. In how much time and how far from the point of throwing will the shot fall to the ground? The shot weighs $G=2$ kgf. Disregard the resistance of the air.

2.54. A material particle with a mass of 10 grammes moves along a circle having a radius of 6.4 cm with a constant tangential acceleration. Find this acceleration if the kinetic energy of the particle becomes equal to 8×10^{-4} J towards the end of the second revolution after motion begins.

2.55. A body with a mass of 1 kilogramme slides down an inclined plane 1 metre high and 10 metres long. Find: (1) the kinetic energy of the body at the base of the plane, (2) the velocity of the body at the base of the plane, (3) the distance traveled by the body over the horizontal part of the route until it stops. Assume the coefficient of friction to be constant over the entire route and equal to 0.05.

2.56. A body first slides along an inclined plane forming an angle of $\alpha=8°$ with the horizon and then over a horizontal plane. Find the coefficient of friction if the body covers the same distance on the horizontal plane as on the inclined one.

2.57. A body with a mass of 3 kilogrammes slides over an inclined plane 0.5 metre high and 1 metre long. The body reaches the bottom of the inclined plane with a velocity of 2.45 m/s. Find: (1) the coefficient of friction between the body and the plane, (2) the amount of heat evolved in friction. The initial velocity of the body is zero.

2.58. A motor vehicle with a mass of 2 tons runs up a grade of 1 in 25. The coefficient of friction is 8 per cent. Find: (1) the work performed by the vehicle engine over a distance of 3 kilometres, (2) the power developed by the engine if this distance was covered in 4 minutes.

2.59. Find the power developed by the engine of a vehicle with a mass of 1 ton if it moves at a constant speed of 36 km/h: (1) over a level road, (2) up a grade of 1 in 20, (3) down the same grade. The coefficient of friction is 0.07.

2.60. A vehicle weighing 1 tonf runs downhill with its engine stopped at a constant speed of 54 km/h. The grade is 1 in 25. What power must be developed by the engine for the vehicle to run up the same grade at the same speed?

2.61. A flat car with a weight of $G_1= 10$ tonf stands on rails and carries a cannon weighing $G_2=5$ tonf from which a shell is fired along the rails. The weight of the shell $G_3=100$ kgf and its initial velocity with respect to the cannon $v_0= 500$ m/s. Determine the velocity v_x of the flat car at the first moment after the shot if: (1) the flat car was standing, (2) the car was moving with a speed of $v_1= 18$ km/h and the shell was fired in the direction of motion, (3) the car was moving with a speed of $v_1 = 18$ km/h and the shell was fired in the direction opposite to its motion.

2.62. A bullet with a mass of 5×10^{-3} kg flies at a velocity of 600 m/s from a rifle with a mass of 5 kg. Find the velocity of the rifle kick.

2.63. A man weighing 60 kgf running at a speed of 8 km/h catches up with a cart weighing 80 kgf and moving at 2.9 km/h, and jumps onto it. (1) What will the velocity of the cart be after the man jumps onto it? (2) What will the velocity of the cart be if the man was running towards it?

2.64. A shell weighing 980 N flies horizontally with a velocity of 500 m/s along a railway, strikes a freight car carrying sand and weighing 10 tonf and gets stuck in it. What speed will be imparted to the car if: (1) it was standing, (2) it was moving at a speed of 36 km/h in the same direction as the shell, (3) it was moving with a speed of 36 km/h in the direction opposite to the flight of the shell.

2.65. A grenade flying at 10 m/s bursts into two fragments. The larger one having 60 per cent of the entire weight of the grenade continues to move as before but with an increased velocity equal to 25 m/s. Find the velocity of the smaller fragment.

2.66. A body with a weight of 1 kgf moves horizontally with a velocity of 1 m/s, overtakes another body weighing 0.5 kgf and collides with it inelastically. What velocity is imparted to the bodies if: (1) the second body was at rest, (2) the second body was moving at a velocity of 0.5 m/s in the same direction as the first one, (3) the second body was moving at a velocity of 0.5 m/s in a direction opposite to the motion of the first one?

2.67. A skater weighing 70 kgf standing on ice throws a stone weighing 3 kgf with a velocity of 8 m/s in a horizontal direction. Find the distance over which the skater will move back if the coefficient of friction between the skates and the ice is 0.02.

2.68. A man standing on a cart at rest throws a stone with a mass of 2 kg in a horizontal direction. The cart with the man rolls backwards. Its velocity was 0.1 m/s immediately after the stone was thrown. The cart with the man weighs 100 kgf. Find the kinetic energy of the stone in 0.5 second after it was thrown. Disregard the resistance of the air.

2.69. A body with a weight of $G_1 = 2$ kgf moves towards another body weighing $G_2 = 1.5$ kgf and collides with it inelastically. Before the impact, the velocities of the bodies were $v_1 = 1$ m/s and $v_2 = 2$ m/s, respectively. How long will these bodies move after the collision if the coefficient of friction $f = 0.05$?

2.70. An automatic gun fires 600 bullets a minute. The mass of each bullet is 4 grammes and its initial velocity is 500 m/s. Find the mean recoil.

2.71. A flat car weighing $G_1=10$ tonf stands on rails and has a cannon fastened on it weighing $G_2=5$ tonf from which a shell is fired along the rails. The shell weighs $G_3=100$ kgf, and its initial velocity with respect to the cannon is $v_0=500$ m/s. Over what distance will the car move after the shot if: (1) it is at rest, (2) it is moving with a velocity of $v_1=18$ km/h and the shot is fired in the direction of its motion, (3) it is moving with a velocity of $v_1=18$ km/h and the shot is fired in the opposite direction? The coefficient of friction between the car and the rails is 0.002.

2.72. A shell weighing 100 kgf is fired from a cannon with a mass of 5×10^3 kg. The kinetic energy of the shell at the end of the barrel is 7.5×10^6 J What kinetic energy is imparted to the cannon by the recoil?

2.73. A body weighing 2 kgf moves with a velocity of 3 m/s and overtakes another body weighing 3 kgf moving at 1 m/s. Find the velocities of the bodies after collision if: (1) the impact is inelastic, (2) the impact is elastic. The bodies move in a straight line and the impact is central.

2.74. What should the ratio between the masses of the bodies in the previous problem be for the first body to stop after an elastic impact?

2.75. A body weighing 3 kgf moves with a velocity of 4 m/sec and strikes an immobile body of the same weight. Assuming the impact to be central and inelastic, find the amount of heat evolved during the collision.

2.76. A body with a mass of 5 kg strikes an immobile body with a mass of 2.5 kg which after the impact begins to move with a kinetic energy of 5 J. Assuming the impact to be central and elastic, find the kinetic energy of the first body before and after the collision.

2.77. A body weighing 49 N strikes an immobile body weighing 2.5 kgf. The kinetic energy of the system of these two bodies becomes 5 J directly after the impact. Assuming the impact to be central and inelastic, find the kinetic energy of the first body before the collision.

2.78. Two bodies move towards each other and collide inelastically. The velocity of the first body before the impact $v_1=2$ m/s and of the second $v_2=4$ m/s. The total velocity of the bodies after collision coincides in direction with the velocity v_1 and is equal to $v=1$ m/s. How many times did the kinetic energy of the first body exceed that of the second one?

2.79. Two balls are suspended on parallel threads of the same length so that they contact each other. The mass of the first ball is 0.2 kg and that of the second 100 g. The first ball is deflected so that its centre of gravity rises to a height of 4.5 cm and is then released. What height will the balls rise to after the collision if: (1) the impact is elastic, (2) the impact is inelastic?

2.80. A bullet flying horizontally strikes a ball suspended from a very light rigid rod and gets stuck in it. The mass of the ball is

1,000 times greater than that of the bullet. The distance from the point of suspension to the centre of the ball is 1 metre. Find the velocity of the bullet if the rod with the ball deviates by an angle of 10° after the impact.

2.81. A bullet flying horizontally strikes a ball suspended from a light rigid rod and gets stuck in the ball. The mass of the bullet $m_1=$ $=5$ g and that of the ball $m_2=0.5$ kg. The velocity of the bullet $v_1=500$ m/s. At what maximum length of the rod (distance from the point of suspension to the centre of the ball) will the ball rise to the top of the circle as a result of the impact?

2.82. A mallet weighing 0.5 kgf strikes an immobile wall. At the moment of impact the velocity of the mallet is 1 m/s. Assuming the coefficient of restitution to equal 0.5 in impact, find the amount of heat evolved during the impact. The coefficient of restitution is the ratio between the velocity of the body after the impact and that before the impact.

2.83. Find the impulse of the force acting on the wall during the impact using the conditions of the previous problem.

2.84. A wooden ball is dropped vertically from a height of 2 m with an initial velocity of zero. When the ball strikes the floor, the coefficient of restitution is 0.5. Find: (1) the height which the ball rises to after striking the floor, (2) the amount of heat evolved during the impact. The mass of the ball is 100 grammes.

2.85. A plastic ball falls from a height of 1 metre and rebounds several times from the floor. What is the coefficient of restitution during the impact with the floor if 1.3 seconds pass from the first impact to the second one?

2.86. A steel ball falls from a height of 1.5 metres onto a steel plate and rebounds from it with the velocity $v_2=0.75\ v_1$, where v_1 is the velocity which it approaches the plate with. (1) What height does the ball rise to? (2) How much time passes from the moment the ball begins to move to its second impact with the plate?

2.87. A metal ball falls from a height of $h_1=1$ m onto a steel plate and jumps up to a height of $h_2=81$ cm. Find the coefficient of restitution of the ball material.

2.88. A steel ball with a mass of $m=20$ g falls from a height of $h_1=1$ m onto a steel plate and rebounds to a height of $h_2=81$ cm. Find: (1) the impulse of the force received by the plate during the impact, (2) the amount of heat evolved during the impact.

2.89. A moving body with a mass of m_1 strikes an immobile body with a mass of m_2. Assuming the impact to be inelastic and central, find the part of the initial kinetic energy transformed into heat during the impact. First solve the problem in its general form, and then consider the cases when: (1) $m_1=m_2$, (2) $m_1=9\ m_2$.

2.90. A moving body with a mass of m_1 strikes an immobile body with a mass of m_2. Assuming the impact to be elastic and central, find the part of the initial kinetic energy transmitted by the first body to the second during the impact. First solve the problem in its general form, and then consider the cases when: (1) $m_1=m_2$, (2) $m_1=9\ m_2$.

2.91. A moving body with a mass of m_1 strikes an immobile body with a mass of m_2. (1) What should the ratio of the masses m_1/m_2 be to decrease the velocity of the first body 1.5 times with a central elastic impact? (2) What kinetic energy E_2 will the second body begin to move with in this case if the initial kinetic energy E_1 of the first body is 1 kJ?

2.92. A neutron (mass m_0) strikes an immobile nucleus of a carbon atom ($m=12\ m_0$). Assuming the impact to be central and elastic, find the reduction in the kinetic energy of the neutron after the impact.

2.93. A neutron (mass m_0) strikes an immobile nucleus of: (1) a carbon atom ($m=12\ m_0$), (2) an uranium atom ($m=235\ m_0$). Assuming the impact to be central and elastic, find the part of the velocity lost by the neutron during the impact.

2.94. How much of its weight does a body lose on the equator due to the Earth's rotation about its axis?

2.95. In what time should the Earth make a full revolution for any body on the equator to be weightless?

2.96. A tramcar with a mass of 5 tons runs over a curve having a radius of 128 metres. Find the force which the wheels press laterally against the rails with if the speed is 9 km/h.

2.97. A bucket with water attached to a rope 60 cm long uniformly rotates in a vertical plane. Find: (1) the minimum velocity of rotation of the bucket when the water will not spill out at the highest point, (2) the tension of the rope at this velocity at the highest and lowest points of the circumference. The mass of the bucket with the water is 2 kg.

2.98. A stone tied to a string with a length of $l=50$ cm uniformly rotates in a vertical plane. At what number of revolutions per second will the string break if it is known to do so under a load equal to the ten-fold weight of the stone?

2.99. A stone tied to a string uniformly rotates in a vertical plane. Find the mass of the stone if the difference between the maximum and minimum tension of the rope is 1 kgf.

2.100. A weight tied to a thread 30 cm long describes a circle with a radius of 15 cm in a horizontal plane. What number of revolutions per minute does the rotational velocity of the weight correspond to?

2.101. A weight with a mass of 50 grammes tied to a thread 25 centimetres long describes a circle in a horizontal plane. The weight rotates at 2 rev/s. Find the tension of the thread.

2.102. A disk rotates at 30 rev/min around a vertical axis. A body lies on the disk at a distance of 20 cm from the axis of rotation. What should the coefficient of friction between the body and the disk be so that the body will not slide off the disk?

2.103. An airplane flying at a velocity of 900 km/h loops a loop. What should the loop radius be so that the maximum force pressing the pilot against the seat is: (1) five times his weight, (2) ten times his weight?

2.104. A motorcyclist rides over a level road with a speed of 72 km/h and makes a turn with a radius of 100 m. To what angle should he tilt his body to remain seated?

2.105. A ball is suspended on a thread from the ceiling of a tramcar. The car negotiates a curve with a radius of 36.4 m at a speed of 9 km/h. Through what angle will the thread with the ball deviate?

2.106. The rods of a centrifugal governor (Fig. 4) are 12.5 cm long. What number of revolutions per second does the centrifugal governor make if the weights deviated from the vertical during rotation through the angle: (1) 60°, (2) 30°?

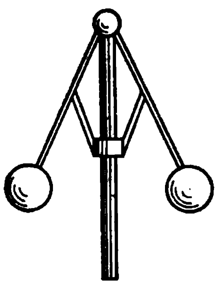

Fig. 4

2.107. A highway has a curve with a bank of 10°, the radius of the curve being 100 m. What speed is the curve intended for?

2.108. A weight of 1 kgf suspended on a thread deviates through an angle of 30°. Find the tension of the thread at the moment the weight passes through the position of equilibrium.

2.109. A boy swings around a "giant stride" making 16 rev/min. The length of the ropes is 5 m. (1) What angle will the ropes of the "giant stride" form with the vertical? (2) What is the tension of the ropes if the boy weighs 45 kgf? (3) What is the linear velocity of the swinging boy?

2.110. A load with a mass of $m=1$ kg hanging on a weightless rod with a length of $l=0.5$ m oscillates in a vertical plane. (1) At what angle α of deviation of the rod from the vertical will the kinetic energy of the load in its bottom position be equal to $E_k=2.45$ J? (2) How many times is the tension of the rod greater in the middle position than in the extreme one with this angle of deviation.

2.111. A weight G is suspended on a weightless rod. The weight is moved through an angle of 90° and released. Find the tension of the rod when it passes through the position of equilibrium.

2.112. A steel wire resists a load up to 300 kgf. A load with a mass of 150 kg is suspended from the wire. What is the maximum angle through which the wire with the load can be deviated so that it does not fail when the load passes through the position of equilibrium?

2.113. A stone weighing 0.5 kgf tied to a rope with a length of $l=50$ cm rotates uniformly in a vertical plane. The tension of the rope at the bottom point of the circumference $T=44$ N. What height will the stone rise to if the rope breaks the moment the velocity is directed vertically upward?

2.114. Water flows along a horizontal pipe with a curve having a radius of $r=20.0$ m. Find the lateral pressure of the water caused by the centrifugal force. The pipe diameter $d=20$ cm, and $M=300$ tons of water pass through a cross section of the pipe per hour.

2.115. Water flows along a horizontal channel 0.5 m wide and having a curve with a radius of 10 m. The velocity of the water is 5 m/s. Find the lateral pressure of the water caused by the centrifugal force.

2.116. Find the work which must be done to compress a spring by 20 cm if the force is proportional to the deformation and a force of 29.4 N compresses the spring by one centimetre.

2.117. Find the maximum deflection of a leaf spring caused by a load placed on its middle if the static deflection of the spring due to the same load is $x_0=2$ cm. What will the maximum initial deflection be if the same load is dropped onto the middle of the spring from a height of $h=1$ m with an initial velocity equal to zero?

2.118. An acrobat jumps onto a net from a height of $H_1=8$ m. At what minimum height h_1 should the net be stretched above the floor so that the acrobat will not hit it when he jumps? If the acrobat jumps down from a height of $H_2=1$ m, the net deflects by $h_2=0.5$ m.

2.119. A weight is placed on the pan of a balance. How many graduations will be indicated by the balance pointer upon its first deviation if it shows 5 graduations after the oscillations stop?

2.120. A load weighing 1 kgf falls onto the pan of a balance from a height of 10 cm. What will the balance show at the moment of impact? After the pan comes to a stop it will be lower by 0.5 cm under the weight of the load.

2.121. What speed did a railway car with a mass of 20 tons move with if each buffer was compressed by 10 cm when the car struck a wall? It is known that the spring of each buffer is compressed by 1 cm under the action of a force of 1 tonf.

2.122. A boy stretches the rubber cord of his catapult so that it becomes 10 cm longer. What velocity will a stone with a mass of 20 g be projected with? Stretching of the cord by 1 cm requires a force of 1 kgf. Disregard the resistance of the air.

2.123. A spring carrying a load is attached to the bottom end of another spring suspended vertically. The deformation coefficient of the upper spring is k_1 and of the lower spring k_2. Find the ratio between the potential energies of these springs, neglecting their weight with respect to that of the load.

2.124. A rod whose weight may be neglected is suspended from two parallel springs of the same length. The deformation coefficients of the springs are $k_1=2$ kgf/cm and $k_2=3$ kgf/cm, respectively. The length of the rod is equal to the distance between the springs $L = 10$ cm. What point on the rod should a load be attached to for the rod to remain horizontal?

2.125. A rubber ball with a mass of $m=0.1$ kg flies horizontally with a certain velocity and strikes a fixed vertical wall. During the time $\Delta t = 0.01$ second the ball is compressed by $\Delta x = 1.37$ cm. The same time Δt is spent to restore the initial shape of the ball. Find the mean force acting on the wall during the impact.

2.126. A weight of $G=4.9$ N attached to a rubber cord with a length of l_0 describes a circle in a horizontal plane. The weight rotates with a frequency of $\nu=2$ rev/s. The angle through which the rubber cord is deflected from the vertical is $\alpha=30°$. Find the length l_0 of the unstretched rubber cord. A force of $F_1=6.0$ N is required to stretch the cord by $x_1=1$ cm.

2.127. A load of $G=0.5$ kgf attached to a rubber cord with a length of $l_0 = 9.5$ cm is deflected through an angle of $\alpha=90°$ and released. Find the length l of the rubber cord when the load passes through the position of equilibrium. The deformation coefficient of the rubber cord $k=1$ kgf/cm.

2.128. A ball with a radius of $r=10$ cm floats in water so that its centre is at a height of $h=9$ cm above the surface of the water. What work should be performed to submerge the ball up to the diametral plane?

2.129. A ball with a radius of $r = 6$ cm is held by an external force under water so that its top touches the surface. The density of the ball material $\rho=500$ kg/m^3. What work will be performed by the force of expulsion if the ball is released and allowed to float freely?

2.130. A ball with a diameter of $D=30$ cm floats in water. What work should be performed to submerge the ball deeper by $h=5$ cm? The density of the ball material $\rho=500$ kg/m^3.

2.131. A block of ice with an area of $A=1$ m^2 and a height of $h=0.4$ m floats in water. What work should be performed to submerge the ice block completely into the water?

2.132. Find the force of attraction between two protons which are at a distance of $r=10^{-10}$ m from each other. The mass of a proton $m=1.67 \times 10^{-27}$ kg. Regard the protons as point masses.

2.133. Two copper balls with diameters of $d_1=4$ cm and $d_2=6$ cm are in contact with each other. Find the gravitational potential energy of this system.

2.134. Calculate the gravity constant knowing the Earth's radius R, its mean density ρ and the acceleration of gravity g near the Earth (see the tables in the Appendix).

2.135. Taking the acceleration of gravity at the Earth's surface equal to $g = 9.80$ m/s^2 and using the data in Table III of the Appendix, compile a table for the average densities of the solar planets.

2.136. A cosmic rocket flies to the Moon. At what point of the straight line connecting the centres of the Moon and Earth will the rocket be attracted by the Earth and the Moon with the same force?

2.137. Compare the acceleration of gravity on the Moon with that on the Earth.

2.138. How will the period of oscillations of a mathematical pendulum transferred from the Earth to the Moon change?

Note. The formula for the period of oscillations of a mathematical pendulum is given in Section 12.

2.139. Find the numerical value of the first cosmic velocity, i.e., the velocity to be imparted to a body in a horizontal direction at the Earth's surface for the body to begin moving around the Earth as a satellite in a circular orbit.

2.140. Find the numerical value of the second cosmic velocity, i.e., the velocity to be imparted to a body at the Earth's surface for the body to overcome the Earth's force of gravity and depart from the Earth forever.

2.141. Taking the acceleration of gravity at the Earth's surface equal to $g=980$ cm/s^2 and using the data in Table III of the Appendix, compile a table for the first and second cosmic velocities (in km/s) at the surfaces of the solar planets.

2.142. Find the linear velocity with which the Earth moves along its orbit. Assume the Earth's orbit to be circular.

2.143. With what linear velocity v will an artificial satellite of the Earth move in a circular orbit: (1) at the Earth's surface (disregard the resistance of the air), (2) at an altitude of $h_1=200$ km and $h_2=7{,}000$ km? Find the period of revolution T of an artificial satellite around the Earth in these conditions.

2.144. (1) Find the relationship between the period of revolution of an artificial satellite moving in a circular orbit at the surface of a central body and the density of this body. (2) Use the data obtained in solving Problem 2.135 to compile a table for the periods of revolution of artificial satellites moving in a circular orbit at the surfaces of the solar planets.

2.145. Find the centripetal acceleration with which an artificial satellite of the Earth moves in a circular orbit at an altitude of 200 km from the Earth's surface.

2.146. The planet Mars has two satellites—Phobus and Deimos. The former is at a distance of $R_1=9{,}500$ km from the centre of Mars and the latter at a distance of $R_2=24{,}000$ km. Find the periods of revolution of these satellites around Mars.

2.147. An artificial satellite of the Earth moves in a circular orbit in the plane of the equator from west to east. At what distance from the Earth's surface should the satellite be for it to remain immobile with respect to an observer on the Earth?

2.148. An artificial satellite of the Moon moves in a circular orbit at a distance of 20 km from the Moon's surface. Find the linear velocity of the satellite and its period of revolution around the Moon.

2.149. Find the numerical values of the first and second cosmic velocities for the Moon (see the conditions of Problems 2.139 and 2.140).

2.150. Find the relationship between the acceleration of gravity and the altitude above the Earth's surface. At what altitude will the acceleration of gravity be 25 per cent of that at the Earth's surface?

2.151. At what distance from the Earh's surface is the acceleration of gravity equal to 1 m/s^2?

2.152. How many times is the kinetic energy of an artificial Earth's satellite moving in a circular orbit smaller than its gravitational potential energy?

2.153. Find the change in the acceleration of gravity when a body is lowered to a depth of h. At what depth will the acceleration of gravity be 25 per cent of that at the Earth's surface? Consider the Earth's density to be constant.

Note. Remember that a body lying at a depth of h under the Earth's surface is not attracted by the overlying spherical layer with a thickness of h since the attraction by separate layers is mutually compensated.

2.154. What is the ratio between the height H of a mountain and the depth h of a mine if a pendulum swings with the same period at the top of the mountain and at the bottom of the mine?

Note. The formula for the period of oscillations of a mathematical pendulum is given in Section 12.

2.155. Find the period of revolution of an artificial planet around the Sun if the major semiaxis of the planet's elliptical orbit is greater than that of the Earth's orbit by 24 million kilometres.

2.156. An artificial planet moves in almost a circular orbit. Assuming the planet's orbit to be circular, find the linear velocity of its motion and the period of its revolution about the Sun, knowing the Sun's diameter and mean density. The mean distance from the planet to the Sun $R=1.71\times10^8$ km.

2.157. The major axis of the orbit of the world's first artificial satellite of the Earth was smaller than that of the orbit of the second satellite by 800 km. The period of revolution of the first satellite around the Earth at the beginning of motion was 96.2 min. Find: (1) the major axis of the orbit of the second artificial satellite, (2) the period of its revolution about the Earth.

2.158. The minimum distance of the spaceship "Vostok-2" from the Earth's surface was 183 km and the maximum distance 244 km. Find the period of revolution of the spaceship about the Earth.

2.159. A ring is made of thin wire with a radius of r. Find the force with which this ring will attract a material particle with a mass of m

on the axis of the ring at a distance of L from its centre. The radius of the ring is R and the density of the wire material is ρ.

2.160. A ring is made of thin copper wire having a radius of 1 mm. The radius of the ring is 20 cm. (1) Find the force F with which the ring attracts a material particle with a mass of 2 g on the axis of the ring at a distance of $L=0$, 5, 10, 15, 20 and 50 cm from its centre. Compile a table of the values of F and draw a diagram showing the relationship $F=f(L)$. (2) At what distance L_{max} from the centre of the ring will the force of interaction between the ring and the material particle be maximum? (3) Find the numerical value of the maximum force of interaction between the ring and the material particle.

2.161. The force of interaction F between a wire ring and a material particle on the axis of the ring is maximum when the particle is at a distance of L_{max} from the centre of the ring. How many times is the force of interaction between the ring and the material point located at a distance of $L=0.5L_{max}$ from the centre of the ring smaller than the maximum force?

3. Rotational Motion of Solids

The moment M of the force F with respect to any axis of revolution is determined by the formula

$$M=Fl$$

where l is the distance from the axis of revolution to the straight line along which this force acts.

The moment of inertia of a material particle relative to any axis of revolution is

$$I=mr^2$$

where m = mass of the material particle
r = distance from the particle to the axis.

The moment of inertia of a solid with respect to its axis of revolution is

$$I=\int r^2\,dm$$

where integration should be extended over the entire volume of the body. The following formulas can be obtained by integration:

1. The moment of inertia of a solid homogeneous cylinder (disk) with respect to the cylinder axis

$$I=\frac{1}{2}mr^2$$

where r = radius of the cylinder
m = mass of the cylinder.

2. The moment of inertia of a hollow cylinder (hoop) with an internal radius r_1 and an external radius r_2 relative to the cylinder axis

$$I = m\frac{r_1^2 + r_2^2}{2}$$

For a thin-walled hollow cylinder $r_1 \cong r_2 = r$ and

$$I \cong mr^2$$

3. The moment of inertia of a homogeneous ball with a radius r relative to the axis passing through its centre

$$I = \frac{2}{5}mr^2$$

4. The moment of inertia of a homogeneous rod relative to an axis passing through its middle perpendicular to its length l

$$I = \frac{1}{12}ml^2$$

If the moment of inertia I_0 of any body relative to an axis passing through its centre of gravity is known, the moment of inertia with respect to any axis parallel to the first one can be found from Steiner's theorem (also called the parallel-axis theorem)

$$I = I_0 + md^2$$

where m = mass of the body
d = distance from the centre of gravity of the body to its axis of revolution.

The fundamental law of dynamics of rotational motion is expressed by the equation

$$Mdt = d\,(I\omega)$$

where M = moment of the forces applied to a body whose moment of inertia is equal to I
ω = angular velocity of rotation of the body.

If I = const, then

$$M = I\frac{d\omega}{dt} = I\alpha$$

where α is the angular acceleration acquired by the body under the action of the torque M.

The kinetic energy of a rotating body

$$E_k = \frac{I\omega^2}{2}$$

where I = moment of inertia of the body
ω = its angular velocity.

The equations of the dynamics of rotational motion are compared with those of translational motion in Table 6.

TABLE 6

Translational motion	Rotational motion
Newton's second law	
$F\Delta t = mv_2 - mv_1 = \bar{p}_2 - \bar{p}_1$ or $F = ma$	$M\Delta t = I\omega_2 - I\omega_1$ or $M = I\alpha$
Law of conservation of momentum $\Sigma \bar{p} = \Sigma mv = \text{const}$	Law of conservation of angular momentum $\Sigma I\omega = \text{const}$
Work and kinetic energy	
$W = Fs = \frac{mv_2^2}{2} - \frac{mv_1^2}{2}$	$W = M\varphi = \frac{I\omega_2^2}{2} - \frac{I\omega_1^2}{2}$

The period of small oscillations of a physical pendulum

$$T = 2\pi \sqrt{\frac{I}{mdg}}$$

where I=moment of inertia of the pendulum relative to its axis of revolution
m=mass of the pendulum
d=distance from the axis of revolution to the centre of gravity
g=acceleration of gravity.

3.1. Find the moment of inertia and the angular momentum of the Earth relative to its axis of revolution.

3.2. Two balls with the radii $r_1 = r_2 = 5$ cm are attached to the ends of a thin rod with a weight much smaller than that of the balls. The distance between the centres of the balls $R = 0.5$ m. The mass of each ball $m = 1$ kg. Find: (1) the moment of inertia I_1 of this system with respect to an axis passing through the middle of the rod perpendicular to its length, (2) the moment of inertia I_2 of this system relative to the same axis assuming the balls to be material particles whose masses are concentrated at their centres, (3) the relative error $\delta = \frac{I_1 - I_2}{I_2}$ made in calculating the moment of inertia of this system when we use I_2 instead of I_1.

3.3. A constant tangential force $F = 98.1$ N is applied to the rim of a homogeneous disk with a radius of $r = 0.2$ m. When the disk rotates, it is acted upon by the moment of friction forces $M_{fr} = 0.5$ kgf·m.

Find the weight G of the disk when it rotates with a constant angular acceleration of $\alpha=100$ rad/s^2.

3.4. A homogeneous rod with a length of 1 m and a weight of 0.5 kgf rotates in a vertical plane about a horizontal axis passing through the middle of the rod. What angular acceleration will the rod rotate with if the rotational moment is 9.81×10^{-2} N·m?

3.5. A homogeneous disk with a radius of $r=0.2$ m and a weight of $G=5$ kgf rotates around an axis passing through its centre. The relation between the angular velocity of disk rotation and the time is described by the equation $\omega=A+Bt$, where $B=8$ rad/s^2. Find the tangential force applied to the rim of the disk. Disregard friction.

3.6. A flywheel with the moment of inertia $I=63.6$ kg·m^2 rotates with a constant angular velocity $\omega=31.4$ rad/s. Find the braking moment M which stops the flywheel in $t=20$ seconds.

3.7. A tangential force of 10 kgf is applied to the rim of a wheel having the form of a disk with a radius of 0.5 m and a mass $m=50$ kg. Find: (1) the angular acceleration of the wheel, (2) in what time after the force is applied will the wheel rotate at 100 rev/s.

3.8. A flywheel with a radius of $r=0.2$ m and a mass of $m=10$ kg is connected to a motor by means of a drive belt. The tension of the belt which runs without slipping is constant and equals $T=14.7$ N. What number of revolutions will be developed by the flywheel per second in $\Delta t=10$ seconds after motion begins? Consider the flywheel as a homogeneous disk. Disregard friction.

3.9. A flywheel with a moment of inertia of 245 kg·m^2 rotates at 20 rev/s. The wheel stops in one minute after the torque stops acting on it. Find: (1) the moment of the forces of friction, (2) the number of revolutions completed by the wheel from the moment the forces stop acting on it until it stops.

3.10. Two weights $G_1=2$ kgf and $G_2=1$ kgf are linked by a thread and thrown over a pulley weighing $G=1$ kgf. Find: (1) the acceleration a with which the weights move, (2) the tensions T_1 and T_2 of the threads which the weights are attached to. Consider the pulley as a homogeneous disk. Disregard friction.

3.11. A load with a mass of $m=2$ kg is attached to the end of a cord wrapped around a drum having a mass of $M=9$ kg. Find the acceleration of the load. Consider the drum to be a homogeneous cylinder. Disregard friction.

3.12. A load of $G_1=10$ kgf is attached to the end of a cord wrapped around a drum with a radius of $r=0.5$ m. Find the moment of inertia of the drum if the load is lowered with an acceleration of $a=2.04$ m/s^2.

3.13. A load of $G_1=0.5$ kgf is attached to the end of a cord wrapped around a drum with a radius of $r=20$ cm. The moment of inertia of the drum $I=0.1$ kg·m^2. Before the drum begins to rotate, the height of the load G_1 above the floor is $h_1=1$ m. Find: (1) the time needed by the load to reach the floor, (2) the kinetic energy of the load at the moment of impact against the floor, (3) the tension of the cord. Disregard friction.

3.14. Two different weights are connected by a thread passing over a pulley whose moment of inertia $I=50$ kg·m² and radius $r=20$ cm. The pulley rotates with friction and the moment of the friction forces $M_{fr}=98.1$ N·m. Find the difference in the tensions of the thread T_1-T_2 on both sides of the pulley if it rotates with a constant angular acceleration $\alpha=2.36$ rad/s².

3.15. A pulley weighing $G=1$ kgf is secured to the edge of a table (see Fig. 1 and Problem 2.31). The equal weights A and B ($G_1=G_2=1$ kgf) are linked by a thread thrown over the pulley. The coefficient of friction of the weight B against the table $f=0.1$. Consider the pulley to be a homogeneous disk and disregard the friction in the pulley. Find: (1) the acceleration which the weights move with, (2) the tensions T_1 and T_2 of the threads.

3.16. A disk weighing 2 kgf rolls without slipping over a horizontal plane with a velocity of 4 m/s. Find the kinetic energy of the disk.

3.17. A ball 6 cm in diameter rolls without slipping over a horizontal plane with a velocity of 4 rev/s. The mass of the ball is 0.25 kg. Find the kinetic energy of the rolling ball.

3.18. A hoop and a disk have the same weight G and roll without slipping with the same linear velocity v. The kinetic energy of the hoop $E_1=4$ kgf-m. Find the kinetic energy of the disk E_2.

3.19. A ball with a mass of $m=1$ kg rolls without slipping, strikes a wall and rolls back. Before the impact the velocity of the ball $v_1=10$ cm/s and after the impact $v_2=8$ cm/s. Find the amount of heat Q evolved during the impact.

3.20. Determine the relative error obtained in calculating the kinetic energy of a rolling ball if rotation of the ball is neglected.

3.21. A disk with a weight of 1 kgf and a diameter of 60 cm rotates about an axis passing through its centre perpendicular to its plane at 20 rev/s. What work should be performed to stop the disk?

3.22. The kinetic energy of a shaft rotating at a constant velocity of 5 rev/s is 60 J. Find the angular momentum of the shaft.

3.23. Find the kinetic energy of a cyclist riding at a speed of $v=9$ km/h. The cyclist with his bicycle weighs $G=78$ kgf, and the wheels $G_1=3$ kgf. Consider the bicycle wheels as hoops.

3.24. A boy drives a hoop over a horizontal path with a speed of 7.2 km/h. Over what distance can the hoop run uphill at the expense of its kinetic energy? The slope of the hill is 1 in 10.

3.25. What is the minimum height h from which a cyclist can start to travel by inertia (without friction) over a path in the form of a loop with a radius of $R=3$ m so as not to leave the path at the top of the loop? The mass of the cyclist together with the bicycle $m=75$ kg, the mass of the wheels being $m_1=3$ kg. Consider the bicycle wheels as hoops.

3.26. A copper ball with a radius of $r=10$ cm rotates with a velocity corresponding to $\nu = 2$ rev/s about an axis passing through its centre. What work should be performed to increase the angular velocity of rotation of the ball twofold?

3.27. Find the linear accelerations of the centres of gravity of: (1) a ball, (2) a disk, and (3) a hoop, which roll without slipping down an inclined plane. The angle of inclination is 30°, and the initial velocity of all the bodies is zero. (4) Compare these accelerations with that of a body which slides off the inclined plane without friction.

3.28. Find the linear velocities of the centres of gravity of: (1) a ball, (2) a disk, and (3) a hoop, which roll without slipping down an inclined plane. The height of the inclined plane $h=0.5$ m, and the initial velocity of all the bodies is zero. (4) Compare these velocities with that of a body which slides off the inclined plane without friction.

3.29. The surfaces of two cylinders—aluminium (solid) and lead (hollow)—having the same radius $r=6$ cm and the same weight $G=$ $=0.5$ kgf are painted the same colour. (1) How can the cylinders be distinguished by observing their translational velocities at the base of the inclined plane? (2) Find the moments of inertia of these cylinders. (3) How much time does it take each cylinder to roll down the inclined plane without slipping? The height of the inclined plane $h=0.5$ m and its angle of inclination $\alpha = 30°$. The initial velocity of each cylinder is zero.

3.30. A wheel is uniformly retarded by braking and its velocity of rotation drops from 300 to 180 rev/min in one minute. The moment of inertia of the wheel is 2 kg·m². Find: (1) the angular acceleration of the wheel, (2) the braking moment, (3) the work of braking, (4) the number of revolutions completed by the wheel during this minute.

3.31. A fan rotates with a velocity of 900 rev/min. When its motor is switched off, the fan has uniformly retarded rotation and makes 75 revolutions before it stops. The work of the braking forces is 44.4 J. Find: (1) the moment of inertia of the fan, (2) the moment of the friction force.

3.32. A flywheel with a moment of inertia of $I=245$ kg·m² rotates at 20 rev/s. After the action of the torque is discontinued, the wheel stops upon completing 1,000 revolutions. Find: (1) the moment of the friction forces, (2) the time which elapses from the moment the action of the torque discontinues to the moment when the wheel stops.

3.33. A load of 1 kgf is fixed to the end of a thread passing around the rim of a pulley fitted on the same axle as a flywheel. Over what distance should the load lower for the wheel and the pulley to acquire a velocity of 60 rev/min? The moment of inertia of the wheel and the pulley is 0.42 kg·m², and the pulley radius is 10 cm.

3.34. A flywheel begins to rotate with a constant angular acceleration of $\alpha=0.5$ rad/s^2 and acquires an angular momentum $I\omega=$ $=73.5$ kg·m^2/s in $t_1=15$ seconds after motion begins. Find the kinetic energy of the wheel in $t_2=20$ seconds after rotation begins.

3.35. A flywheel rotates with a constant velocity corresponding to $\nu=10$ rev/s and its kinetic energy $E_k=800$ kgf-m. In what time will the torque $M=50$ N·m applied to the flywheel double its angular velocity?

3.36. A constant tangential force $F=2$ kgf is applied to the rim of a disk with a mass of $m=5$ kg. What kinetic energy will be imparted to the disk in $\Delta t=5$ seconds after the force begins to act?

3.37. Through what angle should a homogeneous rod suspended from a horizontal axis passing through the upper end of the rod deviate for the lower end of the rod to move at 5 m/s when it passes through the position of equilibrium? The rod is 1 m long.

3.38. A homogeneous rod 85 cm long is suspended from a horizontal axis passing through its upper end. What minimum velocity should be imparted to the lower end of the rod to make it complete one full revolution about the axis?

3.39. A pencil placed vertically on a table falls down. What will the angular and linear velocities be at the end of the fall of: (1) the middle of the pencil, (2) its upper end? The pencil is 15 cm long.

3.40. A horizontal platform with a mass of 100 kg rotates at 10 rev/min around a vertical axis passing through its centre. A man weighing 60 kgf is standing on its edge. What velocity will the platform begin to rotate with if the man moves from the edge of the platform to its centre? Regard the platform as a circular homogeneous disk and the man as a point mass.

3.41. What work will be performed by a man moving from the edge of the platform to its centre in the conditions of the previous problem? The radius of the platform is 1.5 m.

3.42. A horizontal platform with a weight of 80 kgf and a radius of 1 m rotates at an angular velocity corresponding to 20 rev/min. A man stands in the centre of the platform and holds weights in his outstretched hands. How many revolutions will the platform make per minute if the man lowers his hands, thus reducing his moment of inertia from 2.94 kg·m^2 to 0.98 kg·m^2? Consider the platform as a circular homogeneous disk.

3.43. How many times will the kinetic energy of the platform with the man increase in the previous problem?

3.44. A man weighing 60 kgf stands on an immobile platform with a mass of 100 kg. What number of revolutions will be made by the platform a minute if the man moves along a circle with a radius of 5 m around the axis of rotation? The man moves relative to the platform with a velocity of 4 km/h. The radius of the platform is 10 m. Consider the platform as a homogeneous disk and the man as a point mass.

3.45. A homogeneous rod oscillates in a vertical plane about a horizontal axis passing through its top. The length of the rod $l=0.5$ m. Find the period of oscillations of the rod.

3.46. Find the period of oscillations of the rod in the previous problem if the axis of rotation passes through a point 10 centimetres from its top end.

3.47. Two weights are attached to the ends of a vertical rod. The centre of gravity of these weights is below the middle of the rod by $d=5$ cm. Find the length of the rod if the period of small oscillations of the rod with the weights around a horizontal axis passing through its centre $T=2$ sec. Neglect the weight of the rod with respect to that of the weights.

3.48. A hoop 56.5 cm in diameter hangs on a nail hammered into a wall and performs small oscillations in a plane parallel to the wall. Find the period of the oscillations.

3.49. What should be the minimum length l of a thread on which a homogeneous ball with a diameter of $D=4$ cm is suspended to regard this ball as a mathematical pendulum in determining the period of small oscillations? The error made when assuming this should not exceed 1 per cent.

3.50. A homogeneous ball is suspended from a thread with a length equal to the radius of the ball. How many times is the period of small oscillations of this pendulum greater than that of a mathematical pendulum suspended at the same distance from the centre of gravity?

4. Mechanics of Fluids

Liquids and gases are also known under the common name of fluids. The steady motion of an ideal incompressible fluid is described by the Bernoulli equation

$$p+\frac{\rho v^2}{2}+\rho gh=\text{const}$$

where ρ=density of the fluid
v=velocity of the fluid in the given cross section of the pipe
h=height of this cross section above a certain level
p=pressure.

It follows from the Bernoulli equation that a fluid flows out from a small orifice with the velocity $v=\sqrt{2gh}$, where h is the height of the surface of the fluid above the orifice. Since the same quantities of fluid pass through any cross section of a pipe, then $A_1v_1=A_2v_2$, where v_1 and v_2 are the velocities of the fluid in two sections of the pipe with the areas A_1 and A_2.

The force of resistance acting on a ball falling in a viscous fluid is determined by the Stokes formula

$$F=6\,\pi\eta rv$$

where η=coefficient of internal friction of the fluid (dynamic viscosity)
r=radius of the ball
v=velocity of the ball.

The Stokes law is true only for laminar motion, when the volume of a fluid passing during the time t through a capillary tube with the radius r and the length l can be determined from the Poiseuille formula

$$V=\frac{\pi r^4 t\Delta p}{8l\eta}$$

where η=dynamic viscosity of the fluid
Δp=difference of pressures at the tube ends.

The nature of motion of a fluid is determined by the dimensionless Reynolds number

$$\mathrm{Re}=\frac{Dv\rho}{\eta}=\frac{Dv}{\nu}$$

where D=quantity characterizing the linear dimensions of the body around which the fluid flows
v=velocity of flow
ρ=density
η=dynamic viscosity.

The ratio $\nu=\eta/\rho$ is known as the kinematic viscosity. The critical value of the Reynolds number which determines the transition from laminar to turbulent motion is different for bodies of different shape.

4.1. Find the flow velocity of carbon dioxide gas along a pipe if 0.51 kg of gas flows in half an hour through the cross section of the pipe. The density of the gas is 7.5 kg/m^3 and the pipe diameter is 2 cm (in Problems 4.1 through 4.9 the fluids are considered as ideal and incompressible).

4.2. The bottom of a cylindrical vessel nas a circular hole d=1 cm in diameter. The diameter of the vessel D=0.5 m. Find the relationship between the velocity v with which the water level in the vessel drops and the height h of this level. Also determine the numerical value of this velocity for the height h=0.2 m.

4.3. A vessel filled with water stands on a table. In its side the vessel has a small orifice arranged at the distance h_1 from the bottom of the vessel and at the distance h_2 from the level of the water, which is constant. At what distance from the orifice (in a horizontal direction) will the jet of water fall onto the table? Solve the problem for the following cases: (1) $h_1=25$ cm and $h_2=16$ cm, (2) $h_1=16$ cm and $h_2=25$ cm.

4.4. A vessel A filled with water (Mariotte vessel) communicates with the atmosphere through a glass tube a passing through the throat of the vessel (Fig. 5) A faucet F is $h_2=2$ cm from the bottom of the vessel. Find the velocity with which the water flows out of the faucet F when the distance between the end of the tube and the bottom of the vessel is: (1) $h_1=2$ cm, (2) $h_1=7.5$ cm, and (3) $h_1=10$ cm.

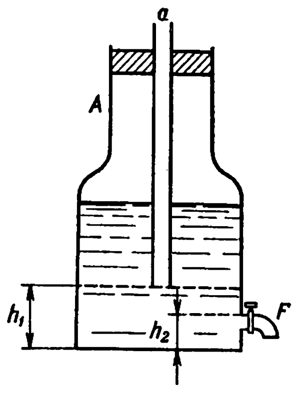

Fig. 5

4.5. A cylindrical tank with a height of $h=1$ m is filled with water up to its brim. (1) What time is required to empty the tank through an orifice in its bottom? The cross-sectional area of the orifice is 1/400 of that of the tank. (2) Compare this time with that required for the same amount of water to flow out of the tank if the water level in the tank is maintained constant at a height of $h=1$ m from the orifice.

4.6. Water is poured into a vessel at a rate of 0.2 litre a second. What should the diameter d of an orifice in the bottom of the vessel be for the water to remain at a constant level of $h=8.3$ cm?

4.7. What pressure will be built up by a compressor in a paint gun if a stream of liquid paint flows out of it with a velocity of 25 m/s? The density of the paint is 0.8 g/cm³.

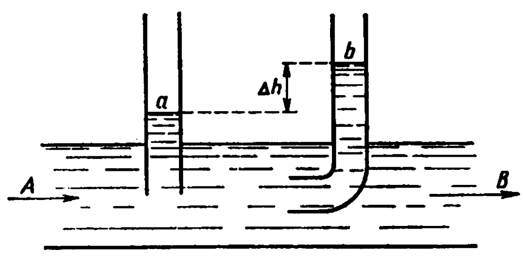

Fig. 6

4.8. A liquid flows along a horizontal pipe AB (Fig. 6). The difference between the levels of the liquid in tubes a and b is 10 cm. The diameters of tubes a and b are the same. Determine the velocity of the liquid flowing along pipe AB.

4.9. Air is blown through a pipe AB (Fig. 7) at a rate of 15 litres per minute. The cross-sectional area of the broad portion of pipe AB

is 2 cm^2, and of the narrow portion and of tube *abc* is 0.5 cm^2. Find the difference Δh between the levels of the water poured into tube *abc*. Assume the density of air to be equal to 1.32 kg/m^3.

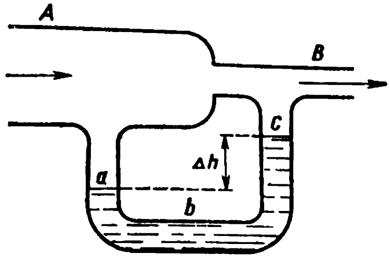

Fig. 7

4.10. A ball rises to the surface at a constant velocity in a liquid whose density is four times greater than that of the material of the ball. How many times is the force of friction acting on the rising ball greater than its weight?

4.11. What maximum velocity can be attained by a rain drop $d = 0.3$ mm in diameter if the dynamic viscosity of air is 1.2×10^{-4} g/cm·s?

4.12. A steel ball 1 mm in diameter falls at a constant velocity of 0.185 cm/s in a large vessel filled with castor oil. Find the dynamic viscosity of the oil.

4.13. A mixture of lead pellets 3 mm and 1 mm in diameter is lowered into a tank with glycerine 1 m deep. How much later will the smaller pellets drop to the bottom than the greater ones? At the temperature of the experiment the dynamic viscosity is 14.7 g/cm·s.

4.14. A cork ball with a radius of 5 mm rises to the surface in a vessel filled with castor oil. What are the dynamic and kinematic viscosities of the castor oil in the conditions of the experiment if the ball rises with a constant velocity of 3.5 cm/s?

4.15. A horizontal capillary tube with an internal radius of $r = 1$ mm and a length of $l = 2$ cm is inserted into the side surface of a cylindrical vessel having a radius of $R = 2$ cm. The vessel is filled with castor oil whose dynamic viscosity $\eta = 12$ g/cm·s. Find the relationship between the velocity v with which the level of the castor oil sinks in the cylindrical vessel and the height h of this level above the capillary tube. Determine the numerical value of this velocity for $h = 26$ cm.

4.16. A horizontal capillary tube with an internal radius of $r = 1$ mm and a length of $l = 1.5$ cm is inserted into the side surface of a vessel. The latter is filled with glycerine whose dynamic viscosity is $\eta = 1.0$ N·s/m^2 in the conditions of the experiment. The level of the glycerine in the vessel is kept constant at a height of $h = 0.18$ m above the capillary tube. How much time is required for 5 cm^3 of glycerine to flow out of the capillary tube?

4.17. A horizontal capillary tube is inserted into the side surface of a vessel standing on a table at a height of $h_1 = 5$ cm from the bottom. The internal radius of the capillary tube $r = 1$ mm and its length $l = 1$ cm. The vessel is filled with machine oil with a density of $\rho =$

$=900$ kg/m^3 and a dynamic viscosity of $\eta=0.5$ N·s/m^2. The level of the oil in the vessel is kept at a height of $h_2=50$ cm above the capillary tube. Find the distance from the end of the capillary tube (along a horizontal line) to the place where the stream of oil drops onto the table.

4.18. A steel ball falls in a broad vessel filled with transformer oil having a density of $\rho=900$ kg/m^3 and a dynamic viscosity of $\eta=0.8$ N·s/m^2. Assuming that the Stokes law is true when $Re\leqslant 0.5$ (if in calculating Re the ball diameter is taken to be the quantity D), find the maximum diameter of the ball.

4.19. Assuming that laminar motion of a fluid is retained in a cylindrical pipe when $Re\leqslant 3{,}000$ (if when calculating Re the pipe diameter is taken to be the quantity D), show that the conditions of Problem 4.1 correspond to laminar motion. The kinematic viscosity of the fluid is to be taken equal to $\nu=1.33\times10^{-6}$ m^2/s.

4.20. Water flows along a pipe at a rate of 200 cm^3 per second. The dynamic viscosity of the water in the conditions of the experiment is 0.001 N·s/m^2. At what maximum pipe diameter will the water flow remain laminar? (See the conditions of the previous problem.)

Chapter 2

MOLECULAR PHYSICS AND THERMODYNAMICS

THERMAL UNITS

The International System of Units (SI) incorporates the MKSD system designed for measuring thermal units (GOST 8550-61). Table 7 gives the basic and the most important derived units used to measure thermal quantities in this system.

TABLE 7

Quantity and symbol	Formula	Unit	Symbol of unit	Dimension of quantity
		Basic Units		
Length l	—	metre	m	l
Mass m	—	kilogramme	kg	m
Time t	—	second	s	t
Temperature T	—	degree	deg	θ
		Derived Units		
Amount of heat	$Q = W = E$	joule	J	l^2mt^{-2}
Heat capacity of a system	$C = \frac{Q}{\Delta T}$	joule per degree	J/deg	$l^2mt^{-2}\theta^{-1}$
Entropy of a system	$S = \frac{\Delta Q}{T}$	joule per degree	J/deg	$l^2mt^{-2}\theta^{-1}$
Specific heat	$c = \frac{Q}{m\Delta T}$	joule per kilogramme-degree	J/kg·deg	$l^2t^{-2}\theta^{-1}$
Specific entropy	$s = \frac{S}{m}$	joule per kilogramme-degree	J/kg·deg	$l^2t^{-2}\theta^{-1}$
Specific heat of phase transition	$q = \frac{Q}{m}$	joule per kilogramme	J/kg	l^2t^{-2}
Temperature gradient	$\text{grad}\ T = \frac{\Delta T}{\Delta l}$	degree per metre	deg/m	$l^{-1}\theta$
Thermal flux	$\Phi = \frac{\Delta Q}{\Delta t}$	watt	W	l^2mt^{-3}
Surface radiation density, density of thermal flux	$q = \frac{\Phi}{A}$	watt per square metre	W/m^2	mt^{-3}

Table 7, concluded

Quantity and symbol	Formula	Unit	Symbol of unit	Dimension of quantity
Thermal conductivity coefficient	$\lambda = \frac{Q}{\Delta t A \frac{\Delta T}{\Delta l}}$	watt per metre-degree	W/m·deg	$lmt^{-3}\theta^{-1}$
Thermal diffusivity	$a = \frac{\lambda}{c\rho}$	square metre per second	m^2/s	l^2t^{-1}
Coefficient of heat transfer	$\alpha = \frac{\Phi}{A\Delta T}$	watt per square metre-degree	W/m^2·deg	$mt^{-3}\theta^{-1}$

GOST 8550-61 also allows the use of non-system units based on the calorie to measure thermal quantities (Table 8).

TABLE 8

Quantity	Unit and its relation to SI units
Amount of heat	1 calorie (cal) = 4.19 J 1 kilocalorie (kcal) = 4.19×10^3 J
Heat capacity and entropy of a system	1 cal/deg = 4.19 J/deg 1 kcal/deg = 4.19×10^3 J/deg
Specific heat and specific entropy	1 cal/g·deg = 4.19×10^3 J/kg·deg 1 kcal/kg·deg = 4.19×10^3 J/kg·deg
Specific heat of phase transition	1 cal/g = 4.19×10^3 J/kg 1 kcal/kg = 4.19×10^3 J/kg
Thermal flux	1 cal/s = 4.19 W 1 kcal/h = 1.163 W
Density of thermal flux	1 cal/cm^2·s = 4.19×10^4 W/m^2 1 $kcal/m^2$·h = 1.163 W/m^2
Thermal conductivity coefficient	1 cal/cm·s·deg = 4.19×10^2 W/m·deg 1 kcal/m·h·deg = 1.163 W/m·deg

Units of molar quantities are obtained from the units given in Tables 7 and 8 by replacing the gramme by the gramme-mole (mole) and the kilogramme by the kilogramme-mole (kmole), where a kilomole is the quantity of a substance whose mass in kilogrammes is equal to the molecular weight.

EXAMPLES OF SOLUTIONS

Example 1. A vessel with a volume of 20 litres contains 4 g of hydrogen at a temperature of 27° C. Find the pressure of the hydrogen.

Solution. Ideal gases obey the Mendeleyev-Clapeyron equation

$$pV = \frac{M}{\mu} RT \qquad (1)$$

which relates the volume of a gas V, its pressure p, absolute temperature T and mass M. In equation (1), R is the gas constant equal in SI units to 8.31×10^3 J/kmole·deg, μ is the mass of one kilomole and M/μ is the number of kilomoles.

From equation (1)

$$p=\frac{MRT}{\mu V} \tag{2}$$

In our case $M=4\times10^{-3}$ kg, $\mu=2$ kg/kmole, $T=27°$ C$=300°$ K, $V=20$ l$=2\times10^{-2}$ m^3. Upon inserting these data in equation (2), we obtain

$$p=\frac{4\times10^{-3}\times8.31\times10^3\times300}{2\times2\times10^{-2}}\ \frac{\text{N}}{\text{m}^2}=2.5\times10^5\ \text{N/m}^2$$

By using Table 4, the answer can be expressed in other units

$$p=2.5\times10^5\ \text{N/m}^2=1{,}880\ \text{mm Hg}=2.55\ \text{kgf/cm}^2=2.46\ \text{atm}$$

Example 2. Find the specific heat of a polyatomic gas at a constant volume if the density of this gas in standard conditions is $7.95\times\times10^{-4}$ g/cm^3.

Solution. The specific heat at a constant volume is determined by the formula

$$c_v=\frac{Ri}{2\mu} \tag{1}$$

where R = gas constant
i = number of degrees of freedom of a polyatomic gas
μ = mass of one kilomole of the gas.

A formula for the density of a gas can be easily obtained from the Mendeleyev-Clapeyron equation

$$\rho=\frac{M}{V}=\frac{p\mu}{RT} \tag{2}$$

From equations (1) and (2)

$$c_v=\frac{Ri}{2}\ \frac{p}{\rho RT}=\frac{pi}{2\rho T} \tag{3}$$

Since the gas is in standard conditions, $p=1$ atm$=1.013\times\times10^5$ N/m^2, $T=0°$ C$=273°$ K. For polyatomic gases $i=6$. Besides, in accordance with the initial conditions, $\rho=7.95\times\times10^{-4}$ g/cm^3$=0.795$ kg/m^3. By inserting these data in equation (3), we get $c_v=1{,}400$ J/kg·deg.

With the aid of Table 8, the result obtained can be expressed in cal/g·deg

$$c_v=1{,}400\ \text{J/kg}\cdot\text{deg}=\frac{1{,}400}{4.19\times10^3}\ \text{cal/g}\cdot\text{deg}=0.334\ \text{cal/g}\cdot\text{deg}$$

5. Physical Fundamentals of the Molecular-Kinetic Theory and Thermodynamics

Ideal gases conform to the Mendeleyev-Clapeyron equation of state

$$pV = \frac{M}{\mu} RT$$

where p = pressure of a gas
V = volume of the gas
T = absolute temperature
M = mass of the gas
μ = mass of one kilomole of the gas
R = gas constant.

The ratio $\frac{M}{\mu}$ shows the number of kilomoles.

In SI units the gas constant is numerically equal to $R=8.31\times\times10^3$ J/kmole·deg.

According to Dalton's law, the pressure of a mixture of gases is equal to the sum of their partial pressures. By the partial pressure is meant the pressure which a gas of the mixture would have if it alone filled the entire volume at the given temperature.

The basic equation of the kinetic theory of gases has the form

$$p = \frac{2}{3} n\overline{E}_0 = \frac{2}{3} n \frac{m\overline{v^2}}{2}$$

where n = number of molecules in a unit of volume
$\overline{E}_0$ = mean kinetic energy of the translational motion of one molecule
m = mass of a molecule
$\sqrt{\overline{v^2}}$ = mean square velocity of the molecules.

These quantities are determined by the following formulas.

The number of molecules in a unit of volume

$$n = \frac{p}{kT}$$

where $k=\frac{R}{N_A}$ is Boltzmann's constant (N_A is Avogadro's number). Since $R=8.31\times10^3$ J/kmole·deg and $N_A=6.02\times10^{26}$ kmole^{-1}, then $k=1.38\times10^{-23}$ J/deg$=1.38\times10^{-16}$ erg/deg.

The mean kinetic energy of the translational motion of molecules

$$\overline{E}_0 = \frac{3}{2} kT$$

The mean square velocity of molecules

$$\sqrt{\overline{v^2}} = \sqrt{\frac{3RT}{\mu}} = \sqrt{\frac{3kT}{m}}$$

where $m = \frac{\mu}{N_A}$.

The energy of the thermal motion of molecules (internal energy of a gas)

$$U = \frac{M}{\mu} \frac{i}{2} RT$$

where i is the number of degrees of freedom of the molecules.

The relation between the molecular C and specific c heats can be determined from their definition:

$$C = \mu c$$

The molecular heat of a gas at a constant volume

$$C_v = \frac{i}{2} R$$

and at a constant pressure

$$C_p = C_v + R$$

The molecular heat is therefore determined by the number of degrees of freedom of the gas molecules. For a monoatomic gas $i=3$, and

$$C_v = 12.5 \times 10^3 \text{ J/kmole·deg} \cong 3 \text{ cal/mole·deg}$$
$$C_p = 20.8 \times 10^3 \text{ J/kmole·deg} \cong 5 \text{ cal/mole·deg}$$

For a biatomic gas $i = 5$, and

$$C_v = 20.8 \times 10^3 \text{ J/kmole·deg} \cong 5 \text{ cal/mole·deg}$$
$$C_p = 29.1 \times 10^3 \text{ J/kmole·deg} \cong 7 \text{ cal/mole·deg}$$

For a polyatomic gas $i = 6$, and

$$C_v = 24.9 \times 10^3 \text{ J/kmole·deg} \cong 6 \text{ cal/mole·deg}$$
$$C_p = 33.2 \times 10^3 \text{ J/kmole·deg} \cong 8 \text{ cal/mole·deg}$$

The law of distribution of molecules by velocities (Maxwell's law) can be used to find the number of molecules ΔN whose relative velocities are within the interval from u to $u + \Delta u$

$$\Delta N = \frac{4}{\sqrt{\pi}} N e^{-u^2} u^2 \Delta u$$

Here $u = \frac{v}{v_{pr}}$ = relative velocity
v = given velocity

v_{pr} = the maximum probable velocity of molecules equal to $\sqrt{\frac{2RT}{\mu}}$

Δu = interval of the relative velocities, which is small in comparison with u

e = base of natural logarithms.

In solving problems relating to the law of distribution of molecules by velocities, it is convenient to use Table 9, which gives the values of $\frac{\Delta N}{N \Delta u}$ for various u.

TABLE 9

u	$\frac{\Delta N}{N\Delta u}$	u	$\frac{\Delta N}{N\Delta u}$	u	$\frac{\Delta N}{N\Delta u}$
0	0	0.9	0.81	1.8	0.29
0.1	0.02	1.0	0.83	1.9	0.22
0.2	0.09	1.1	0.82	2.0	0.16
0.3	0.18	1.2	0.78	2.1	0.12
0.4	0.31	1.3	0.71	2.2	0.09
0.5	0.44	1.4	0.63	2.3	0.06
0.6	0.57	1.5	0.54	2.4	0.04
0.7	0.68	1.6	0.46	2.5	0.03
0.8	0.76	1.7	0.36		

The arithmetic mean velocity of molecules

$$\bar{v} = \sqrt{\frac{8RT}{\pi\mu}}$$

It is often important to know the number of molecules N_x whose velocities exceed the given velocity u. Table 10 gives the values $\frac{N_x}{N} = f(u)$, where N is the total number of molecules.

TABLE 10

u	$\frac{N_x}{N}$	u	$\frac{N_x}{N}$
0	1.000	0.8	0.734
0.2	0.994	1.0	0.572
0.4	0.957	1.25	0.374
0.5	0.918	1.5	0.213
0.6	0.868	2.0	0.046
0.7	0.806	2.5	0.0057

The barometric formula describes the law of reduction of the pressure of a gas with altitude in the gravity field

$$p_h = p_0 e^{-\frac{\mu g h}{RT}}$$

Here p_h = gas pressure at the altitude h
p_0 = pressure at the altitude $h=0$
g = gravity acceleration.
The formula is approximate, since the temperature T cannot be assumed identical for great differences in altitude.

The mean free path of gas molecules

$$\bar{l} = \frac{\bar{v}}{\bar{z}} = \frac{1}{\sqrt{2}\pi\sigma^2 n}$$

where $\bar{v}$ = arithmetic mean velocity
$\bar{z}$ = mean number of collisions of each molecule with the others in a unit of time
σ = effective diameter of a molecule
n = number of molecules in a unit of volume.
The total number of collisions of all the molecules in a unit of volume per unit of time is

$$Z = \frac{1}{2}\bar{z}n$$

The mass M transferred during the time Δt in diffusion is described by the equation

$$M = -D\frac{\Delta\rho}{\Delta x}\Delta A\Delta t$$

where $\frac{\Delta\rho}{\Delta x}$ = density gradient in a direction perpendicular to the area ΔA
D = diffusion coefficient equal to

$$D = \frac{1}{3}\overline{vl}$$

Here $\bar{v}$ = mean velocity
$\bar{l}$ = mean free path of the molecules.

The momentum transferred by the gas during the time Δt determines the force of internal friction F in the gas

$$F = -\eta\frac{\Delta v}{\Delta x}\Delta A$$

where $\frac{\Delta v}{\Delta x}$ = velocity gradient of gas flow in a direction perpendicular to the area ΔA

η = coefficient of internal friction (dynamic viscosity).

$$\eta = \frac{1}{3}\bar{v}\bar{l}\rho$$

The amount of heat Q transferred during the time Δt by conduction is

$$Q = -\lambda \frac{\Delta T}{\Delta x} \Delta A \Delta t$$

where $\frac{\Delta T}{\Delta x}$ = temperature gradient in a direction perpendicular to the area ΔA

λ = thermal conductivity coefficient equal to

$$\lambda = \frac{1}{3}\bar{v}\bar{l}c_v\rho$$

The first law of thermodynamics can be written as

$$dQ = dU + dW$$

where dQ = amount of heat received by a gas

dU = change in the internal energy of the gas

$dW = pdV$ = the work performed by the gas upon a change in its volume.

The change in the internal energy of the gas is

$$dU = \frac{M}{\mu}\frac{i}{2}RdT$$

where dT is the change in the temperature. The total work upon a change in the volume of the gas is

$$W = \int_{V_1}^{V_2} pdV$$

The work performed upon an isothermal change in the volume of a gas

$$W = RT\frac{M}{\mu}\log_e \frac{V_2}{V_1}$$

The pressure and volume of a gas are related in an adiabatic process by Poisson's equation

$$pV^{\varkappa} = \text{const}$$

i. e.,

$$\frac{p_1}{p_2} = \left(\frac{V_2}{V_1}\right)^{\varkappa}$$

where

$$\varkappa = \frac{C_p}{C_v}$$

Poisson's equation can also be written in the form

$$TV^{\varkappa-1} = \text{const}$$

i.e.,

$$\frac{T_1}{T_2} = \left(\frac{V_2}{V_1}\right)^{\varkappa-1}$$

or

$$Tp^{\frac{1-\varkappa}{\varkappa}} = \text{const}$$

i.e.,

$$\frac{T_1}{T_2} = \left(\frac{p_1}{p_2}\right)^{\frac{\varkappa-1}{\varkappa}} = \left(\frac{p_2}{p_1}\right)^{\frac{1-\varkappa}{\varkappa}}$$

The work performed upon an adiabatic change in the volume of a gas can be found from the formula

$$W = \frac{RT_1}{\varkappa-1}\frac{M}{\mu}\left[1-\left(\frac{V_1}{V_2}\right)^{\varkappa-1}\right] = \frac{RT_1}{\varkappa-1}\frac{M}{\mu}\left(1-\frac{T_2}{T_1}\right) = \frac{p_1V_1(T_1-T_2)}{(\varkappa-1)T_1}$$

where p_1 and V_1 are the pressure and volume of the gas at the temperature T_1.

The equation of a polytropic process has the form

$$pV^n = \text{const}$$

or

$$p_1V_1^n = p_2V_2^n$$

where n is the polytropic exponent $(1 < n < \varkappa)$.

The efficiency of a heat machine

$$\eta = \frac{Q_1-Q_2}{Q_1}$$

where Q_1= heat transmitted to the working body
Q_2= heat rejected to the low-temperature sink.

For the ideal Carnot cycle

$$\eta = \frac{T_1-T_2}{T_1}$$

where T_1= temperature of the high-temperature source
T_2= temperature of the low-temperature sink.

The difference between the entropies $S_B - S_A$ of two states B and A is described by the formula

$$S_B - S_A = \int_A^B \frac{dQ}{T}$$

5.1. What is the temperature of 2 grammes of nitrogen occupying a volume of 820 cm^3 at a pressure of 2 atm?

5.2. What volume is occupied by 10 grammes of oxygen at a pressure of 750 mm Hg and a temperature of 20° C?

5.3. A cylinder with a capacity of 12 litres is filled with nitrogen at a pressure of 8.1×10^6 N/m^2 and a temperature of 17° C. How much nitrogen does the cylinder contain?

5.4. The pressure of air inside a tightly sealed bottle was 1 atm at a temperature of 7° C. The cork flew out of the bottle when it was heated. Find the temperature which the bottle was heated to if the cork flew out when the pressure of the air in the bottle was 1.3 atm.

5.5. What can be the smallest volume of a cylinder containing 6.4 kg of oxygen if the walls of the cylinder withstand a pressure of 160 kgf/cm^2 at a temperature of 20° C?

5.6. A cylinder contained 10 kg of gas at a pressure of 10^7 N/m^2. Find the quantity of gas taken out of the cylinder if the final pressure became 2.5×10^6 N/m^2. Assume that the temperature of the gas is constant.

5.7. Find the mass of sulphur dioxide gas (SO_2) occupying a volume of 25 litres at a temperature of 27° C and a pressure of 760 mm Hg.

5.8. Find the mass of the air in a hall 5 metres high with a floor area of 200 m^2. The pressure of the air is 750 mm Hg and the temperature in the room 17° C. (Assume that the mass of one kilomole of air is equal to 29 kg/kmole.)

5.9. How many times is the weight of air filling a room in winter (7° C) greater than its weight in summer (37° C)? The pressure is the same.

5.10. Plot isothermal lines for 0.5 gramme of hydrogen at the temperatures: (1) 0° C, and (2) 100° C.

5.11. Plot isothermal lines for 15.5 grammes of oxygen at the temperatures: (1) 29° C, and (2) 180° C.

5.12. How many kilomoles of gas are contained in a cylinder 10 m^3 in volume at a pressure of 720 mm Hg and a temperature of 17° C?

5.13. Five grammes of nitrogen contained in a closed vessel 4 litres in volume at a temperature of 20° C are heated to 40° C. Find the pressure of the gas before and after heating.

5.14. A column of mercury with a length of $l = 20$ cm is in the middle of a horizontal capillary tube evacuated and soldered at both ends. If the capillary tube is placed vertically, the mercury column will shift through a distance of $\Delta l = 10$ cm. Determine the pressure to which the capillary tube was evacuated. The length of the tube $L = 1$ m.

5.15. There is a well known riddle: "What is heavier, a ton of lead or a ton of cork?" Determine how much the actual weight of the cork, which is 1 tonf in air, exceeds the actual weight of the lead, which is also 1 tonf in air. The temperature of the air is 17° C and the pressure 760 mm Hg.

5.16. What should the weight of the rubber skin of a balloon 25 cm in diameter filled with hydrogen be for the resulting lifting force of the balloon to equal zero, i.e., for the balloon to be suspended? The air and hydrogen are in standard conditions. The pressure inside the balloon is equal to the external pressure.

5.17. The elasticity of saturated water vapours is 92.5 mm Hg at a temperature of 50° C. What is the density of the water vapours in these conditions.

5.18. Find the density of hydrogen at a temperature of 15° C and a pressure of 730 mm Hg.

5.19. The density of a gas is 0.34 kg/m^3 at a temperature of 10° C and a pressure of 2×10^5 N/m^2. What is the mass of one kilomole of this gas?

5.20. What is the density of air in a vessel evacuated to the maximum rarefaction possible with up-to-date laboratory methods ($p = 10^{-11}$ mm Hg)? The temperature of the air is 15° C.

5.21. 12 grammes of gas occupy a volume of 4×10^{-3} m^3 at a temperature of 7° C. After the gas is heated at a constant pressure, its density becomes equal to 6×10^{-4} g/cm^3. What is the temperature which the gas was heated to?

5.22. 10 grammes of oxygen are subjected to a pressure of 3 atm at a temperature of 10°C. Heating at a constant pressure expanded the oxygen to 10 litres. Find: (1) the volume of the gas before expansion, (2) the temperature of the gas after expansion, (3) the density of the gas before expansion, (4) the density of the gas after expansion.

5.23. Water contained in a soldered vessel occupies a volume equal to half that of the vessel. Determine the pressure and density of the water vapours (steam) at a temperature of 400° C if all the water is converted into steam at this temperature.

5.24. Plot a diagram showing how the density of oxygen depends on: (1) the pressure at a temperature of $T = \text{const} = 390°$ K ($0 \leqslant p \leqslant 4$ at at intervals of 0.5 at); (2) the temperature at $p = \text{const} = 4$ at (200° K $\leqslant T \leqslant$ 300° K at intervals of 20°).

5.25. A closed vessel with a capacity of 1 m^3 contains 0.9 kg of water and 1.6 kg of oxygen. Find the pressure in the vessel at a temperature of 500° C, at which all the water will be converted into steam.

5.26. A vessel A with a capacity of $V_1 = 3$ litres contains gas at a pressure of $p_0' = 2$ at, and a vessel B with a capacity of $V_2 = 4$ litres contains the same gas at a pressure of $p_0''=1$ at. The temperature is the same in both vessels. What pressure will the gas be under if vessels A and B are connected by a tube?

5.27. A vessel with a volume of 2×10^{-3} m^3 is filled with 6 grammes of carbon dioxide gas (CO_2) and 5 grammes of nitrogen monoxide (N_2O). What is the total pressure in the vessel at a temperature of 127° C?

5.28. A vessel contains 14 grammes of nitrogen and 9 grammes of hydrogen at a temperature of 10°C and a pressure of 10^6 N/m^2. Find: (1) the mass of one kilomole of the mixture, (2) the volume of the vessel.

5.29. Diethyl ether ($C_2H_5OC_2H_5$) is introduced into a closed vessel with a volume of $V=2$ litres filled with air. The air is in standard conditions. After all the ether evaporates, the pressure in the vessel becomes equal to 1,050 mm Hg. How much ether was introduced into the vessel?

5.30. A vessel with a capacity of 0.5 litre contains 1 gramme of vaporous iodine. The pressure in the vessel is 700 mm Hg at a temperature of 1,000° C. Determine the degree of dissociation of the iodine molecules I_2 into atoms I in these conditions. The mass of one kilomole of I_2 is equal to 254 kg/kmole.

5.31. A vessel is filled with carbon dioxide gas. At a certain temperature the degree of dissociation of the carbon dioxide molecules into oxygen and carbon monoxide is 25 per cent. How many times will the pressure in the vessel in these conditions be greater than the pressure if the carbon dioxide molecules are not dissociated?

5.32. Assuming that air contains 23.6 parts of oxygen and 76.4 parts of nitrogen, find the density of the air at a pressure of 750 mm Hg and a temperature of 13° C. Also determine the partial pressures of the oxygen and nitrogen in these conditions.

5.33. A vessel contains a mixture of 10 grammes of carbon dioxide gas and 15 grammes of nitrogen. Find the density of this mixture at a temperature of 27° C and a pressure of 1.5×10^5 N/m^2.

5.34. Determine the mass of an atom of: (1) hydrogen, (2) helium.

5.35. A hydrogen molecule flying with a velocity of 600 m/s strikes the wall of a vessel perpendicularly and rebounds elastically without losing any of its velocity. Find the impulse of the force imparted to the wall during the impact.

5.36. An argon molecule flies with a velocity of 500 m/s and elastically strikes the wall of a vessel. The direction of the molecule velocity and a perpendicular to the wall form an angle of 60°. Find the impulse of the force imparted to the wall during the impact.

5.37. A nitrogen molecule flies with a velocity of 430 m/s. Determine the momentum of this molecule.

5.38. What number of molecules is contained in 1 gramme of water vapour?

5.39. A vessel with a capacity of 4 litres contains 1 gramme of hydrogen. What number of molecules is present in 1 cm^3 of the vessel?

5.40. What number of molecules is contained in a room 80 m^3 in volume at a temperature of 17°C and a pressure of 750 mm Hg?

5.41. How many molecules are present in 1 cm^3 of a vessel at 10° C if it is evacuated to the highest rarefaction possible using up-to-date laboratory methods ($p=10^{-11}$ mm Hg)?

5.42. To obtain a good vacuum in a glass vessel, its walls should be heated during evacuation to remove the adsorbed gas. Calculate the possible increase of pressure in a spherical vessel with a radius of $r=10$ cm if the adsorbed molecules pass into the vessel from the walls. The cross-sectional area of a molecule is 10^{-15} cm^2 and the layer is monomolecular. The temperature $t=300$°C.

5.43. What number of particles is present in 1 gramme of vaporous iodine if its degree of dissociation is 50 per cent? The mass of one kilomole of iodine I_2 is 254 kg/kmole.

5.44. How many particles are contained in 16 grammes of half dissociated oxygen?

5.45. A vessel contains 10^{-10} kmole of oxygen and 10^{-6} grammes of nitrogen. The temperature of the mixture is 100° C. The pressure in the vessel is 10^{-3} mm Hg. Find: (1) the volume of the vessel, (2) the partial pressures of the oxygen and nitrogen, (3) the number of molecules in 1 cm^3 of the vessel.

5.46. Determine the mean square velocity of air molecules at a temperature of 17°C assuming the air to be a homogeneous gas with the mass of one kilomole equal to $\mu=29$ kg/kmole.

5.47. Find the relationship between the mean square velocities of the molecules of helium and nitrogen at the same temperatures.

5.48. An explosion of an atomic bomb develops a temperature of about 10^6 degrees. Assuming that all the molecules are completely dissociated into atoms, and that the atoms are ionized, find the mean square velocity of a hydrogen ion.

5.49. Find the number of hydrogen molecules in 1 cm^3 if the pressure is 200 mm Hg and the mean square velocity of the hydrogen molecules is 2,400 m/s in these conditions.

5.50. The density of a gas is 6×10^{-2} kg/m^3 and the mean square velocity of the gas molecules is 500 m/s. Find the pressure exerted by the gas on the walls of the vessel.

5.51. How many times is the mean square velocity of a dust particle suspended in air smaller than that of the air molecules? The mass of the particle is 10^{-8} g. Assume the air to be a homogeneous gas with the mass of one kilomole equal to 29 kg/kmole.

5.52. Determine the momentum of a hydrogen molecule at a temperature of 20° C. The velocity of the molecule is equal to the mean square velocity.

5.53. A vessel with a capacity of 2 litres contains 10 grammes of oxygen under a pressure of 680 mm Hg. Find: (1) the mean square velocity of the gas molecules, (2) the number of molecules in the vessel, (3) the density of the gas.

5.54. Gamboge particles with a diameter of $D=1\ \mu$ participate in Brownian motion. The density of the gamboge $\rho=1$ g/cm^3. Find the mean square velocity of the gamboge particles at $t=0^\circ$ C.

5.55. The mean square velocity of the molecules of a certain gas is 450 m/s. The gas pressure is 5×10^4 N/m^2. Determine the density of the gas in these conditions.

5.56. (1) Find the mean square velocity of the molecules of a gas whose density is 8.2×10^{-5} g/cm^3 at a pressure of 750 mm Hg. (2) What is the mass of one kilomole of this gas if its density is given for a temperature of 17° C?

5.57. The mean square velocity of the molecules of a certain gas is 461 m/s in standard conditions. How many molecules are contained in 1 gramme of this gas?

5.58. What is the energy of the thermal motion of 20 grammes of oxygen at a temperature of 10° C? What part of this energy falls to the share of translational motion and of rotational motion?

5.59. Determine the kinetic energy of the thermal motion of the molecules present in 1 gramme of air at a temperature of 15° C. Assume the air to be a homogeneous gas, the mass of one kilomole of which is equal to 29 kg/kmole.

5.60. What is the energy of the rotational motion of the molecules contained in 1 kg of nitrogen at a temperature of 7° C?

5.61. What is the energy of the thermal motion of the molecules of a biatomic gas enclosed in a vessel 2 litres in volume under a pressure of 1.5×10^5 N/m^2?

5.62. The kinetic energy of translational motion of the nitrogen molecules present in a cylinder having a volume of 0.02 m^3 is equal to 5×10^3 J, and the mean square velocity of the nitrogen molecules is 2×10^3 m/s. Find: (1) the quantity of nitrogen in the cylinder, (2) the pressure acting on the nitrogen.

5.63. At what temperature will the mean kinetic energy of the thermal motion of helium atoms be sufficient for the helium atoms to overcome the gravitational force of the Earth and leave the atmosphere forever? Also solve this problem for the Moon.

5.64. One kilogramme of a biatomic gas is at a pressure of $p=8\times10^4$ N/m^2 and has a density of $\rho=4$ kg/m^3. Find the energy of thermal motion of the gas molecules in these conditions.

5.65. What number of molecules of a biatomic gas occupy a volume of $V=10$ cm^3 at a pressure of $p=40$ mm Hg and a temperature of $t=27^\circ$ C? What is the energy of the thermal motion of these molecules?

5.66. Find the specific heat of oxygen: (1) at V=const, and (2) at p=const.

5.67. Find the specific heat at constant pressure of the following gases: (1) hydrogen chloride, (2) neon, (3) nitrogen oxide, (4) carbon monoxide, and (5) mercury vapours.

5.68. Determine for oxygen the ratio of the specific heat at constant pressure to the specific heat at constant volume.

5.69. The specific heat at constant pressure for a biatomic gas is 3.5 cal/g·deg. What is the mass of one kilomole of this gas?

5.70. What are the specific heats c_v and c_p of a biatomic gas if its density is 1.43 kg/m^3 in standard conditions?

5.71. Find the specific heats c_v and c_p of a gas if the mass of one kilomole of this gas is μ=30 kg/kmole and the ratio c_p/c_v=1.4.

5.72. How many times is the heat capacity of oxyhydrogen gas greater than that of the water vapours produced by the combustion of the gas? Solve the problem for the cases when (1) V=const, and (2) p=const.

5.73. What is the degree of dissociation of oxygen if its specific heat is 1,050 J/kg·deg at constant pressure?

5.74. Determine the specific heats c_v and c_p of vaporous iodine if its degree of dissociation is 50 per cent. The mass of one kilomole of iodine I_2 is equal to 254 kg/kmole.

5.75. Determine the degree of dissociation of nitrogen if its ratio c_p/c_v is equal to 1.47.

5.76. Find the specific heat at constant pressure of a gas mixture consisting of 3 kmoles of argon and 2 kmoles of nitrogen.

5.77. Find the ratio c_p/c_v for a gas mixture consisting of 8 grammes of helium and 16 grammes of oxygen.

5.78. The specific heat of a gas mixture consisting of one kilomole of oxygen and several kilomoles of argon at constant volume is equal to 430 J/kg·deg. How much argon is there in the gas mixture?

5.79. Ten grammes of oxygen are under a pressure of 3×10^5 N/m^2 at a temperature of 10° C. After heating at constant pressure, the gas occupies a volume of 10 litres. Find: (1) the amount of heat received by the gas, (2) the energy of the thermal motion of the gas molecules before and after heating.

5.80. Twelve grammes of nitrogen are contained in a closed vessel 2 litres in volume at a temperature of 10° C. After heating, the pressure in the vessel becomes 10^4 mm Hg. What amount of heat has been received by the gas during heating?

5.81. Two litres of nitrogen are under a pressure of 10^5 N/m^2. What amount of heat should be imparted to the nitrogen: (1) to double its volume at p=const, (2) to double its pressure at V=const?

5.82. A closed vessel contains 14 grammes of nitrogen under a pressure of 10^5 N/m^2 and a temperature of 27° C. After heating, the pressure in the vessel increases five times. Find: (1) the temperature which the gas was heated to, (2) the volume of the vessel, (3) the amount of heat received by the gas.

5.83. What amount of heat should be imparted to 12 grammes of oxygen to heat it by 50° C at a constant pressure?

5.84. To heat 40 grammes of oxygen from 16° C to 40° C, 150 calories are required. What conditions was the gas heated in (at a constant volume or a constant pressure)?

5.85. A closed vessel 10 litres in volume contains air under a pressure of 10^5 N/m^2. What amount of heat should be imparted to the air to increase the pressure in the vessel five times?

5.86. (1) What amount of carbon dioxide gas can be heated from 20° C to 100° C by an amount of heat of 0.053 kcal? (2) By how much will the kinetic energy of one molecule change? During heating the gas expands at p=const.

5.87. A closed vessel with a volume of $V=2$ litres contains nitrogen with a density of $\rho=1.4$ kg/m^3. What amount of heat Q should be imparted to the nitrogen to heat it by $\Delta t=100$°C in these conditions?

5.88. A closed vessel 3 litres in volume contains nitrogen at a temperature of 27° C and a pressure of 3 at. After heating, the pressure in the vessel rose to 25 at. Find: (1) the temperature of the nitrogen after heating, (2) the quantity of heat received by the nitrogen.

5.89. To heat a certain amount of gas by 50° C at a constant pressure 160 cal are required. If the same amount of gas is cooled by 100° C at a constant volume, 240 cal will be evolved. How many degrees of freedom do the molecules of this gas have?

5.90. A closed vessel contains 10 grammes of nitrogen at a temperature of 7° C. (1) What amount of heat should be imparted to the nitrogen to double the mean square velocity of its molecules? (2) How many times will the temperature of the gas change? (3) How many times will the pressure of the gas on the walls of the vessel change?

5.91. Helium is contained in a closed vessel with a volume of 2 litres at a temperature of 20° C and a pressure of 10^5 N/m^2. (1) What amount of heat should be imparted to the helium to increase its temperature by 100° C? (2) What will the mean square velocity of its molecules be at the new temperature? (3) What will the pressure be? (4) What will the density of the helium be? (5) What will the energy of the thermal motion of its molecules be?

5.92. A closed vessel 2 litres in volume contains m grammes of nitrogen and m grammes of argon in standard conditions. What quantity of heat should be imparted to this gas mixture to raise its temperature by 100° C?

5.93. Find the mean arithmetic, the mean square and the most probable velocity of the molecules of a gas with a density of 0.3 gramme per litre at a pressure of 300 mm Hg.

5.94. At what temperature is the mean square velocity of nitrogen molecules higher than their maximum probable velocity by 50 m/s?

5.95. What part of oxygen molecules have a velocity from 100 m/s to 110 m/s at 0° C?

5.96. What part of nitrogen molecules have a velocity from 300 m/s to 325 m/s at 150° C?

5.97. What part of hydrogen molecules have a velocity from 2,000 m/s to 2,100 m/s at 0° C?

5.98. How many times is the number of molecules ΔN_1 whose velocities range from $\sqrt{\overline{v^2}}$ to $\sqrt{\overline{v^2}}+\Delta v$ smaller than the number of molecules ΔN_2 whose velocities range from v_{pr} to $v_{pr}+\Delta v$?

5.99. What part of nitrogen molecules at a temperature of T have velocities ranging from v_{pr} to $v_{pr}+\Delta v$, where $\Delta v=20$ m/s? Solve the problem for: (1) $T=400°$ K, and (2) $T=900°$ K.

5.100. What part of nitrogen molecules at a temperature of $T=$ $=150°$ C has velocities ranging from $v_1=300$ m/s to $v_2=800$ m/s?

5.101. What part of the total number N of molecules has a velocity: (1) higher than the maximum probable velocity, and (2) lower than the maximum probable velocity?

5.102. A cylinder contains 2.5 grammes of oxygen. Find the number of oxygen molecules whose velocities exceed the mean square velocity.

5.103. A vessel contains 8 grammes of oxygen at a temperature of 1,600° K. How many oxygen molecules have a kinetic energy of translational motion which exceeds $E_0= 6.65\times 10^{-20}$ J?

5.104. The energy of charged particles is frequently measured in electron-volts. One electron-volt (1 eV) is the energy acquired by an electron when it passes through a potential difference of 1 V in an electric field. 1 eV$=1.6 \times 10^{-19}$ J (see Table 4 on p. 16) Find: (1) the temperature at which the mean kinetic energy of translational motion of the molecules is equal to 1 eV, (2) the temperature at which 50 per cent of all molecules have a mean kinetic energy of translational motion which exceeds 1 eV.

5.105. The work of ionization of potassium atoms is 10^5 kcal/kg-atom. Determine the temperature of a gas at which 10 per cent of all the molecules have a kinetic energy of translational motion which exceeds the energy required to ionize one atom of potassium.

5.106. There is a Soviet high-altitude cosmic station on the mountain Alaghez in Armenia at an altitude of 3,250 m above sea level. Find the air pressure at this altitude. Consider the temperature of the air constant and equal to 5° C. The mass of one kilomole of air is 29 kg/kmole and the pressure of air at sea level is 760 mm Hg.

5.107. At what altitude is the pressure of air equal to 75 per cent of that at sea level? Consider the temperature to be constant and equal to 0° C.

5.108. A passenger plane flies at an altitude of 8,300 m. To dispense with oxygen masks, a constant pressure, which corresponds to an altitude of 2,700 m, is maintained in the cabins with the aid of a compressor. Determine the difference between the pressures inside and outside the cabin. Take the mean temperature of the outside air as equal to 0° C.

5.109. Find in the previous problem how many times the density of the air in the cabin is higher than that outside if the ambient temperature is —20° C and the temperature inside the cabin is +20° C.

5.110. What is the weight of 1 m^3 of air: (1) at the Earth's surface, (2) at an altitude of 4 km from the Earth's surface? Consider the air temperature constant and equal to 0° C. The air pressure at the Earth's surface is 10^5 N/m^2.

5.111. At what altitude is the density of a gas equal to 50 per cent of its density at sea level? Consider the temperature constant and equal to 0° C. Solve the problem for: (1) air, and (2) hydrogen.

5.112. When Perrin observed through a microscope the change in the concentration of suspended particles of gamboge with altitude, he used the barometric formula to determine Avogadro's number experimentally. One of his experiments showed that when the distance between any two layers is 100 μ, the number of suspended gamboge particles in one layer is double that in the other. The temperature of the gamboge was 20° C. The particles of gamboge 0.3×10^{-4} cm in diameter were suspended in a liquid whose density was 0.2 g/cm^3 less than that of the particles. Use these data to find Avogadro's number.

5.113. Determine the mean free path of carbon dioxide gas molecules at a temperature of 100° C and a pressure of 0.1 mm Hg. Take the diameter of a carbon dioxide gas molecule equal to 3.2×10^{-8} cm.

5.114. An ionization gauge installed in the third Soviet artificial satellite showed that 1 cm^3 of the atmosphere contained about a thousand million particles of gas at a height of 300 km from the Earth's surface. Find the mean free path of the gas particles at this height. Take the diameter of the particles equal to 2×10^{-10} m.

5.115. Find the mean free path of air molecules in standard conditions. Take the diameter of an air molecule equal to 3×10^{-8} cm.

5.116. Find the mean number of collisions per second of carbon dioxide gas molecules at a temperature of 100° C if the mean free path is 8.7×10^{-2} cm in these conditions.

5.117. Find the mean number of collisions per second of nitrogen molecules at a temperature of $t=27°$ C and a pressure of $p=$ $=400$ mm Hg.

5.118. A vessel with a volume of 0.5 litre contains oxygen in standard conditions. Find the total number of collisions between the oxygen molecules in this volume during 1 second.

5.119. How many times will the number of collisions per second of the molecules of a biatomic gas decrease if the volume of the gas increases adiabatically twofold?

5.120. Find the mean free path of nitrogen molecules at a temperature of 17° C and a pressure of 10^4 N/m^2.

5.121. Find the mean free path of helium atoms for conditions when the density of the helium $\rho=2.1\times10^{-2}$ kg/m^3.

5.122. What is the mean free path of hydrogen molecules at a pressure of $p=10^{-3}$ mm Hg and a temperature of $t=50°$ C?

5.123. The mean free path of oxygen molecules is 9.5×10^{-8} m at 0° C and a certain pressure. What is the mean number of collisions of the oxygen molecules per second if the vessel is evacuated to 0.01 of the initial pressure? The temperature is constant.

5.124. Under certain conditions the mean free path of molecules of a gas is 1.6×10^{-7} m and the arithmetic mean velocity of its molecules is 1.95 km/s. What is the mean number of collisions of the gas molecules per second if the gas pressure is reduced 1.27 times at the same temperature?

5.125. A flask with a volume of 100 cm^3 contains 0.5 gramme of nitrogen. Find the mean free path of nitrogen molecules in these conditions.

5.126. A vessel is filled with carbon dioxide gas whose density $\rho=$ $=1.7$ kg/m^3. In these conditions the mean free path of its molecules $\bar{l}=7.9\times10^{-6}$ cm. Determine the diameter σ of the carbon dioxide molecules.

5.127. Determine the mean time between two consecutive collisions of nitrogen molecules at a temperature of 10° C and a pressure of 1 mm Hg.

5.128. A vessel with air is evacuated to a pressure of 10^{-6} mm Hg. What is the density of the air in the vessel, the number of molecules in 1 cm^3 of the vessel and the mean free path of the molecules? Assume the diameter of the air molecules to be 3×10^{-8} cm and the mass of one kilomole $\mu=29$ kg/kmole. The temperature of the air is 17° C.

5.129. What maximum number of molecules of a gas should be contained in 1 cm^3 of a spherical vessel with a diameter of 15 cm so that the molecules do not collide with each other? The diameter of a gas molecule is 3×10^{-8} cm.

5.130. What pressure should be built up inside a spherical vessel with a diameter of: (1) 1 cm, (2) 10 cm and (3) 100 cm so that the molecules do not collide with each other? The diameter of a gas molecule is 3×10^{-8} cm and the temperature of the gas is 0° C.

5.131. The distance between the cathode and anode in a discharge tube is 15 cm. What pressure should be built up in the tube so that the electrons do not collide with the air molecules on their path from the cathode to the anode? The temperature is 27° C. The diameter of an air molecule is 3×10^{-8} cm. The mean free path of an electron in the gas is approximately 5.7 times greater than that of the gas molecules.

5.132. A spherical flask with a volume of 1 litre is filled with nitrogen. At what density of the nitrogen will the mean free path of its molecules be greater than the dimensions of the vessel?

5.133. Find the mean number of collisions of the molecules of a certain gas per second if the mean free path is 5×10^{-4} cm in these conditions and the mean square velocity of its molecules is 500 m/s.

5.134. Find the diffusion coefficient of hydrogen in standard conditions if the mean free path of the molecules is 1.6×10^{-7} m.

5.135. Find the diffusion coefficient of helium in standard conditions.

5.136. Plot a diagram showing the diffusion coefficient of hydrogen versus the temperature within the range of $100°\ K\leqslant T\leqslant 600°\ K$ for intervals of 100° at a constant pressure of $p=\text{const}=1$ at.

5.137. Find the amount of nitrogen that passes owing to diffusion through an area of 100 cm^2 in 10 seconds if the density gradient in a direction perpendicular to the area is 1.26 kg/m^4. The temperature of the nitrogen is 27° C and the mean free path of the nitrogen molecules is 10^{-5} cm.

5.138. Two vessels A and B are connected by a tube whose diameter $d=1$ cm and length $l=1.5$ cm. The tube is provided with a cock. When the cock is closed, the pressure of the air in vessel A is p_1. Vessel B is evacuated to a pressure of $p_2\ll p_1$. Determine the amount of air that diffuses from vessel A into vessel B during the first second after the cock is opened. The temperature of the air in both vessels is 17° C and the diameter of the air molecules $\sigma=3$ Å.

5.139. Find the mean free path of helium molecules at a temperature of 0° C and a pressure of 760 mm Hg if the coefficient of internal friction (dynamic viscosity) is equal to 1.3×10^{-4} g/cm·s.

5.140. Determine the coefficient of internal friction of nitrogen in standard conditions if the diffusion coefficient is 0.142 cm^2/s.

5.141. Find the diameter of an oxygen molecule if the coefficient of internal friction for oxygen at 0°C is equal to $\eta=18.8\times10^{-6}$ N·s/m^2.

5.142. Plot a diagram showing the coefficient of internal friction of nitrogen versus the temperature within the range of $100°\ K\leqslant T\leqslant 600°\ K$ for intervals of 100°.

5.143. Determine the coefficients of diffusion and of internal friction of air at a pressure of 760 mm Hg and a temperature of 10° C. The diameter of an air molecule is 3×10^{-10} m.

5.144. How many times is the coefficient of internal friction of oxygen greater than that of nitrogen? The temperature of the gases is the same.

5.145. The coefficients of diffusion and internal friction of hydrogen are under certain conditions equal to $D=1.42$ cm^2/s and $\eta=8.5\times$

$\times 10^{-6}$ N·s/m², respectively. Find the number of hydrogen molecules in 1 m³ in these conditions.

5.146. The coefficients of diffusion and internal friction of oxygen are equal to 1.22×10^{5} m²/s and $\eta=1.95\times 10^{-5}$ kg/m·s, respectively. Find in these conditions: (1) the density of the oxygen, (2) the mean free path of its molecules, and (3) the arithmetic mean velocity of its molecules.

5.147. What maximum velocity can a rain drop 0.3 mm in diameter reach? The diameter of an air molecule is 3×10^{-10} m and the air temperature is 0° C. Assume that the Stokes law is true for the rain drop.

5.148. An airplane flies at a velocity of 360 km/h. Assuming that the layer of air at the airplane wing carried along owing to viscosity is 4 cm, find the tangential force acting on each square metre of the wing surface. The diameter of an air molecule is 3×10^{-8} cm. The temperature of the air is 0° C.

5.149. The space between two coaxial cylinders is filled with gas. The radii of the cylinders are equal to $r=5$ cm and $R=5.2$ cm, respectively. The height of the internal cylinder $h=25$ cm. The external cylinder rotates with a velocity corresponding to $\nu=360$ rev/min. For the internal cylinder to remain immobile, a tangential force of $F=1.38\times 10^{-3}$ N should be applied to it. Considering this case to the first approximation as a plane problem, use the data of this experiment to determine the viscosity coefficient of the gas between the cylinders.

5.150. Find the thermal conductivity coefficient of hydrogen if its coefficient of internal friction is 8.6×10^{-6} N·s/m² in these conditions.

5.151. Find the thermal conductivity coefficient of air at a temperature of 10° C and a pressure of 10^{5} N/cm². The diameter of an air molecule is 3×10^{-8} cm.

5.152. Plot a diagram showing the thermal conductivity coefficient of hydrogen versus the temperature within the range of $100°\text{ K}\leqslant T\leqslant 600°\text{ K}$ for intervals of 100°.

5.153. A vessel with a volume of $V=2$ litres contains $N=4\times 10^{22}$ molecules of a biatomic gas. The thermal conductivity coefficient of the gas $\lambda=0.014$ W/m·deg. Find the diffusion coefficient of the gas in these conditions.

5.154. Carbon dioxide gas and nitrogen are under the same temperature and pressure. Find for these gases the ratio of: (1) the diffusion coefficients, (2) the coefficients of internal friction, and (3) the thermal conductivity coefficients. Assume the diameters of the molecules of these gases to be identical.

5.155. The distance between the walls of a Dewar flask is 8 mm. At what pressure will the thermal conductivity of the air between the walls of the flask begin to diminish during its evacuation? The temperature of the air is 17° C and the diameter of an air molecule is $3\times$ $\times 10^{-7}$ mm.

5.156. A cylindrical vacuum bottle with an external radius of $r_2=$ $=10$ cm, an internal radius of $r_1=9$ cm and a height of $h=20$ cm is filled with ice. The temperature of the ice is 0° C and the ambient temperature of the air is 20° C. (1) Find the maximum air pressure between the walls of the vacuum bottle at which the thermal conductivity coefficient will still depend on the pressure. The diameter of the air molecules is 3×10^{-8} cm and the temperature of the air between the bottle walls is equal to the arithmetical mean of the temperatures of the ice and the ambient medium. (2) Find the thermal conductivity coefficient of the air between the bottle walls at pressures of: (a) 760 mm Hg, (b) 10^{-4} mm Hg ($\mu=29$ kg/mole). (3) What amount of heat passes in one minute through the side surface of the vacuum bottle with a mean radius of 9.5 cm due to thermal conductivity? Solve the problem for pressures of: (a) 760 mm Hg, and (b) 10^{-4} mm Hg.

5.157. What amount of heat is lost every hour through a double window owing to the thermal conductivity of the air enclosed between the panes? The area of each pane is 4 m^2 and the distance between them is 30 cm. The temperature in the room is 18° C, and outside it is —20° C. The diameter of the air molecules is 3×10^{-8} cm, and the temperature of the air between the panes is equal to the arithmetic mean between the temperatures in the room and outdoors. The pressure is 760 mm Hg.

5.158. There is air between two plates arranged at a distance of 1 mm from each other and a temperature difference of $\Delta T=1°$ is maintained between them. The area of each plate $A=100$ cm^2. What amount of heat is transferred by conductivity from one plate to the other during 10 minutes? The air is in standard conditions and the diameter of an air molecule is equal to 3×10^{-10} m.

5.159. Ten grammes of oxygen are under a pressure of 3×10^5 N/m^2 at a temperature of 10° C. The gas heated at a constant pressure occupies a volume of 10 litres. Find: (1) the amount of heat received by the gas, (2) the change in the internal energy of the gas, (3) the work performed by the gas during expansion.

5.160. Hydrogen at a temperature of 27° C and in an amount of 6.5 grammes expands twofold at $p=$const owing to the influx of heat from outside. Find: (1) the work of expansion, (2) the change in the internal energy of the gas, (3) the amount of heat received by the gas.

5.161. A closed vessel contains 20 grammes of nitrogen and 32 grammes of oxygen. Find the change in the internal energy of the gas mixture when it cools by 28° C.

5.162. Two kilomoles of carbon dioxide gas are heated by 50° C at constant pressure. Find: (1) the change in the internal energy of the gas, (2) the work of expansion, (3) the amount of heat received by the gas.

5.163. Five hundred calories of heat are imparted to a biatomic gas. The gas expands at constant pressure. Determine the work of expansion of the gas.

5.164. Work equal to 16 kgf-m was performed during isobaric expansion of a biatomic gas. What amount of heat did the gas receive?

5.165. A gas occupying a volume of 5 litres under a pressure of 2×10^5 N/m^2 and a temperature of 17° C is heated and expands isobarically. The work of expansion is 20 kgf-m. What temperature was the gas heated to?

5.166. Seven grammes of carbon dioxide gas are heated by 10° C in conditions of free expansion. Find the work of expansion of the gas and the change in its internal energy.

5.167. One kilomole of a multiatomic gas is heated by 100° C in conditions of free expansion: Find: (1) the amount of heat received by the gas, (2) the change in its internal energy, (3) the work of expansion.

5.168. One gramme of nitrogen is present in a vessel under a piston. (1) What amount of heat should be spent to heat the nitrogen by 10° C? (2) How much will the piston rise? The piston weighs 1 kgf and its cross-sectional area is 10 cm^2. The pressure above the piston is 1 at.

5.169. Oxyhydrogen gas is in a vessel under a piston. Determine the amount of heat evolved upon explosion of the gas if its internal energy changes by 80.2 calories and the piston rises 20 cm. The piston weighs 2 kgf and its cross-sectional area is 10 cm^2. The air above the piston is in standard conditions.

5.170. Nitrogen amounting to 10.5 grammes expands isothermally at a temperature of —23° C from a pressure of $p_1=2.5$ at to $p_2=1$ at. Find the work performed by the gas during expansion.

5.171. Upon the isothermal expansion of 10 grammes of nitrogen at a temperature of 17° C, work was performed equal to 860 J. How many times did the pressure of the nitrogen change upon expansion?

5.172. The work of isothermal expansion of 10 grammes of a gas from the volume V_1 to $V_2=2V_1$ is 575 J. Determine the mean square velocity of the gas molecules at the same temperature as above.

5.173. One litre of helium in standard conditions expands isothermally to a volume of two litres at the expense of heat received from a hot source. Find: (1) the work performed by the gas during expansion, (2) the amount of heat received by the gas.

5.174. Upon the isothermal expansion of 2 m^3 of a gas its pressure changes from $p_1=5$ at to $p_2=4$ at. Determine the work performed.

5.175. What temperature will air at 0° C be cooled to if it expands adiabatically from a volume of V_1 to $V_2=2V_1$?

5.176. Oxygen amounting to 7.5 litres is compressed adiabatically to a volume of 1 litre, and the pressure at the end of compression is 1.6×10^6 N/m^2. Under what pressure was the gas before compression?

5.177. Air is compressed adiabatically in the cylinders of an internal-combustion engine and its pressure changes from $p_1=1$ at to $p_2=$ $=35$ at. The initial temperature of the air is 40° C. Find its temperature at the end of compression.

5.178. A gas expands adiabatically, and its volume doubles, while its absolute temperature drops 1.32 times. What number of degrees of freedom do the gas molecules have?

5.179. A biatomic gas at a temperature of 27° C and under a pressure of 2×10^6 N/m^2 is compressed adiabatically from the volume V_1 to $V_2= 0.5V_1$. Find the temperature and pressure of the gas after compression.

5.180. Oxyhydrogen gas in a vessel under a piston occupies a volume of 10^{-4} m^3 in standard conditions. When compressed rapidly the gas ignites. Find the temperature of ignition of the gas if the work of compression is equal to 4.73 kgf-m.

5.181. A gas is in a vessel under a piston in standard conditions. The distance between the bottom of the vessel and the crown of the piston is 25 cm. A load of 20 kgf is placed on the piston and it lowers by 13.4 cm. Assuming the compression to be adiabatic, find the ratio c_p/c_v for this gas. The cross-sectional area of the piston is 10 cm^2. Disregard the weight of the piston.

5.182. A biatomic gas occupies a volume of $V_1=0.5$ litre under a pressure of $p_1=0.5$ at. The gas is compressed adiabatically to a certain volume V_2 and a pressure p_2 and is then cooled to the initial temperature at a constant volume of V_2. Here its pressure becomes equal to $p_0=1$ at. (1) Plot the diagram of this process. (2) Determine the volume V_2 and the pressure p_2.

5.183. A gas so expands adiabatically that its pressure drops from 2 at to 1 at. The gas is then heated at a constant volume to the initial temperature and its pressure increases to 1.22 at. (1) Find the ratio c_p/c_v for this gas. (2) Plot the diagram of this process.

5.184. One kilomole of nitrogen in standard conditions expands adiabatically from the volume V_1 to $V_2=5V_1$. Find: (1) the change in the internal energy of the gas, (2) the work performed during expansion.

5.185. It is necessary to compress 1×10^{-2} m^3 of air to a volume of 2×10^{-3} m^3. What is the best compression process—adiabatic or isothermal?

5.186. The work of 146 kJ is spent to compress adiabatically one kilomole of a biatomic gas. How much will the temperature of the gas increase during compression?

5.187. How many times will the mean square velocity of the molecules of a biatomic gas decrease upon a twofold adiabatic increase in the volume of the gas?

5.188. Ten grammes of oxygen in standard conditions are compressed to a volume of 1.4×10^{-3} m^3. Find the pressure and tempera ture of the oxygen after compression if: (1) the oxygen is compressed isothermally, (2) the oxygen is compressed adiabatically. Find the work of compression in each case.

5.189. Nitrogen in an amount of 28 grammes at a temperature of 40° C and a pressure of 750 mm Hg is compressed to a volume of 13 litres. Find the temperature and pressure of the nitrogen after compression if: (1) the nitrogen is compressed isothermally, (2) the nitrogen is compressed adiabatically. Find the work of compression in each case.

5.190. How many times will the free path of the molecules of a biatomic gas increase if its pressure is halved? Consider the cases when: (1) the gas expands isothermally, (2) the gas expands adiabatically.

5.191. Two different gases—one monoatomic and the other biatomic—are at the same temperature and occupy the same volume. The gases are so compressed adiabatically that their volume is halved. Which of the gases will be heated more and how many times?

5.192. One kilogramme of air at a temperature of 30° C and a pressure of 1.5 atm expands adiabatically and the pressure drops to 1 atm. Find: (1) the expansion ratio, (2) the final temperature, (3) the work performed by the gas during expansion.

5.193. The volume of 1 kmole of oxygen in standard conditions increases to $V=5V_0$. Plot a diagram showing the relationship $p=f(V)$ if: (1) expansion occurs isothermally, and (2) expansion occurs adiabatically. Find the value of p for the volumes: V_0, $2V_0$, $3V_0$, $4V_0$ and $5V_0$.

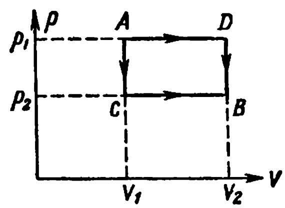

Fig. 8

5.194. A certain quantity of oxygen occupies a volume of $V_1=3$ litres at a temperature of $t_1=27°$ C and a pressure of $p_1=8.2\times10^5$ N/m^2 (Fig. 8). In the second state the gas parameters are: $V_2=4.5$ litres and $p_2=6\times10^5$ N/m^2. Find the amount of heat received by the gas, the work performed by the gas during expansion, and the change in the internal energy of the gas. Solve the problem on condition that the transition of the gas from the first to the second state is via: (1) *ACB*, and (2) *ADB*.

5.195. An ideal heat engine operates according to the Carnot cycle and receives 600 cal from a hot source each cycle. The temperature of the hot source is 400° K and that of the cold sink 300° K. Find the work performed by the engine per cycle and the amount of heat rejected to the cold sink per cycle.

5.196. An ideal heat engine operates according to the Carnot cycle. Determine the efficiency of the cycle if work equal to 300 kgf-m was performed during one cycle and the cooler received 3.2 kcal.

5.197. An ideal heat engine operates according to the Carnot cycle and performs work equal to 7.35×10^4 J during one cycle. The temperature of the hot source is 100° C and that of the cold sink 0°C. Find: (1) the engine efficiency, (2) the amount of heat received by the engine per cycle from the hot source, (3) the amount of heat rejected to the cold sink during one cycle.

5.198. An ideal heat engine operates according to the Carnot cycle. Eighty per cent of the heat received from the hot source is rejected to the cold sink. The amount of heat received from the hot source is 1.5 kcal. Find: (1) the cycle efficiency, (2) the work performed during a complete cycle.

5.199. An ideal heat engine operates according to the Carnot cycle using heated air taken at an initial pressure of 7 atm and a temperature of 127° C. The initial volume of the air is 2×10^{-3} m^3. After the first isothermal expansion the air occupies a volume of 5 litres, and after adiabatic expansion 8 litres. Find: (1) the coordinates of the intersection of the isothermal and adiabatic lines, (2) the work on each section of the cycle, (3) the total work performed during the entire cycle, (4) the cycle efficiency, (5) the amount of heat received from the hot source per cycle, (6) the amount of heat rejected to the cold sink during one cycle.

5.200. One kilomole of an ideal gas completes a cycle consisting of two isochoric and two isobaric lines. The volume of the gas changes from $V_1=25$ m^3 to $V_2=50$ m^3 and the pressure from $p_1=1$ atm to $p_2=2$ atm. How many times is the work performed in this cycle less than the work in a Carnot cycle whose isothermal lines correspond to the maximum and minimum temperatures of the cycle under consideration if the volume doubles in isothermal expansion?

5.201. An ideal refrigerator operates according to the reverse Carnot cycle and performs work equal to 3.7×10^4 J per cycle. It receives heat from a body with a temperature of —10° C and transfers this heat to a body with a temperature of +17° C. Find: (1) the cycle efficiency, (2) the amount of heat rejected from the cold body per cycle, (3) the amount of heat imparted to the hot body per cycle.

5.202. An ideal refrigerator operates as a heat pump according to the reverse Carnot cycle. It receives heat from water with a temperature of 2° C and transfers it to air with a temperature of 27° C. Find: (1) the quantity η_1—the ratio between the amount of heat imparted to the air during a certain time and the amount rejected during the same time from the water, (2) the quantity η_2—the ratio between the amount of heat rejected from the water during a certain time and the energy spent during the same time to operate the refrigerator (η_2 is known as the refrigeration coefficient), (3) the quantity η_3—the ratio between the energy spent to operate the refrigerator during a certain time and the amount of heat transferred during the same time to the air (η_3 is known as the cycle efficiency). Find the relationship between the quantities η_1, η_2 and η_3.

5.203. An ideal refrigerator operates according to the reverse Carnot cycle and transmits heat from a cold source with water at a temperature of 0° C to a boiler with water at a temperature of 100° C. What amount of water must be frozen in the cooler to convert 1 kg of water into vapour in the boiler?

5.204. A room is heated by a refrigerator operating according to the reverse Carnot cycle. How many times is the amount of heat Q_0 received by the room by burning firewood in a furnace lower than the heat Q_1 imparted to the room by a refrigerator actuated by a heat engine consuming the same amount of firewood. The engine operates within the temperature range $T_1=100°$ C and $T_2=0°$ C. A temperature of $T_1'=16°$ C should be maintained in the room. The ambient temperature $T_2'=-10°$ C.

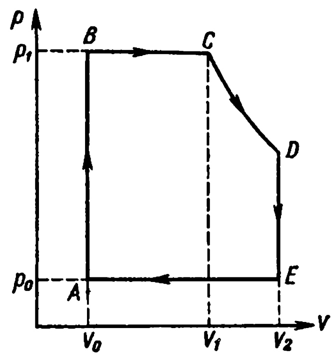

Fig. 9

5.205. The working cycle of an ideal steam engine is shown in Fig. 9: (a) when admission of the steam from the boiler into the cylinder is begun, the pressure in the latter increases at a constant volume V_0 from p_0 to p_1 (line *AB*); (b) upon further admission of the steam, the piston moves from left to right (line *BC*) at a constant pressure p_1; (c) when the piston continues to move to the right, the supply of steam from the boiler into the cylinder is shut off and the steam expands adiabatically (line *CD*); (d) when the piston is in its extreme right-hand position the steam emerges from the cylinder into a cold sink, and the pressure drops at a constant volume V_2 to p_0 (line *DE*); (e) during the reverse stroke the piston expels the remaining steam at a constant pressure p_0 and the volume drops from V_2 to V_0 (line *EA*). Determine the work done by the engine per cycle if $V_0=0.5$ litre, $V_1=1.5$ litres, $V_2=3.0$ litres, $p_0=1$ at, $p_1=12$ at and the adiabatic exponent is 1.33.

5.206. A steam engine rated at 14.7 kW consumes 8.1 kg of coal with a calorific value of 3.3×10^7 J/kg during one hour of operation. The temperature of the boiler is 200° C and that of the cold sink 58° C. Determine the actual efficiency of the engine η_1 and compare it with the efficiency η_2 of an ideal heat engine operating according to the Carnot cycle within the same temperature range.

5.207. The piston area in a steam engine rated at 20 hp is 200 cm^2 and the piston stroke $l=45$ cm. The isobaric process *BC* (Fig. 9) takes place when the piston travels one-third of its stroke. The volume V_0 as compared to the volumes V_1 and V_2 may be neglected. The steam pressure in the boiler is 16 at and that in the cold sink 1 at. Determine the number of cycles per minute performed by the engine if the adiabatic exponent is 1.3.

5.208. The cycle of an internal-combustion carburettor and a gas four-stroke engine is shown in Fig. 10: (a) during the first piston stroke, fuel is sucked into the cylinder (in carburettor engines the combustible mixture consists of petrol vapours and air mixed in carburettors, while in gas engines the working mixture—gas and air—is supplied from a gas generator plant). Here p_0=const and the volume increases from V_2 to V_1 (line AB—suction); (b) during the second piston stroke (line BC—compression) the fuel is compressed adiabatically from V_1 to V_2, the temperature increases from T_0 to T_1 and the pressure from p_0 to p_1; (c) the fuel is then ignited by a spark, the pressure increases from p_1 to p_2 at a constant volume (line CD) and the temperature rises to T_2; (d) the third piston stroke is adiabatic expansion of the fuel from V_2 to V_1 (the working stroke—line DE) when the temperature drops to T_3; (e) when the piston is in its extreme position (point E) the exhaust valve opens, and the pressure drops at a constant volume to p_0 (line EB); (f) the fourth piston stroke is isobaric compression (line BA—exhaust, i.e., expulsion of the used gas). Find the cycle efficiency if the compression ratio $\frac{V_1}{V_2}=5$ and the adiabatic exponent is 1.33.

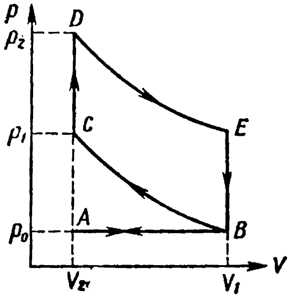

Fig. 10

5.209. Gas is compressed polytropically to $V_2=\frac{1}{6}V_1$ (the compression ratio is 6) in the cylinders of an internal-combustion carburettor engine. Assuming the initial pressure to be equal to 9×10^4 N/m² and the initial temperature to 127° C, find the pressure and temperature of the gas in the cylinders after compression. The polytropic exponent is 1.3.

5.210. Gas is so compressed polytropically in the cylinders of an internal-combustion carburettor engine that the temperature of the gas is 427° C after compression. The initial temperature of the gas is 140° C. The compression ratio is 5.8. What is the polytropic exponent?

5.211. The diameter of a cylinder of an internal-combustion carburettor engine is 10 cm and the piston stroke 11 cm. (1) What is the volume of the compression chamber if the initial pressure of the gas is 1 at, its initial temperature 127° C and the final pressure in the chamber after compression 10 at? (2) What will the gas temperature in the chamber be after compression? (3) Find the work performed during compression. The polytropic exponent is 1.3.

5.212. Determine the efficiency of an internal-combustion carburettor engine if the polytropic exponent is 1.33 and the compression ratio is: (1) $\frac{V_1}{V_2}=4$, (2) $\frac{V_1}{V_2}=6$, (3) $\frac{V_1}{V_2}=8$.

5.213. The carburettor engine of a "Volga" car consumes a minimum of 265 grammes of petrol per hp/h. Determine the losses due to friction, thermal conductivity, etc. The compression ratio is 6.2 and the calorific value of the petrol 4.6×10^7 J/kg. The polytropic exponent is equal to 1.2.

5.214. The cycle of a four-stroke Diesel engine is depicted in Fig. 11: (a) line *AB*—air is sucked into a cylinder ($p_0=1$ at); (b) line *BC*—the air is compressed adiabatically to the pressure p_1; (c) at the end of the compression stroke, fuel is injected into the cylinder, it ignites in the hot air and burns; the piston moves to the right first isobarically (line *CD*) and then adiabatically (line *DE*); (d) at the end of adiabatic expansion, the exhaust valve opens and the pressure drops to p_0 (line *EB*); (e) when the piston moves to the left, the mixture is expelled from the cylinder (line *BA*). Find the efficiency of the engine.

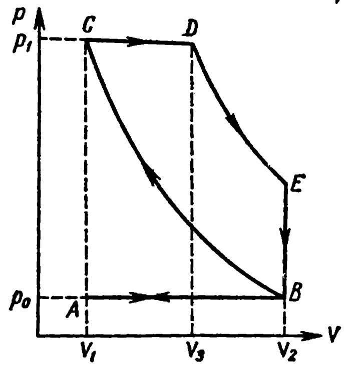

Fig. 11

5.215. The adiabatic compression ratio of an internal-combustion Diesel engine is 16 and the adiabatic expansion ratio 6.4. What is the minimum amount of fuel oil consumed by the engine per hour if its power is 50 hp, the polytropic exponent 1.3 and the calorific value of the fuel 4.6×10^7 J/kg?

5.216. Find the change in entropy upon the conversion of 10 grammes of ice at —20° C into steam at 100° C.

5.217. Find the increment in entropy upon the conversion of 1 gramme of water at 0° C into steam at 100° C.

5.218. Find the change in entropy upon the melting of 1 kg of ice at 0° C.

5.219. Liquid lead at the melting point is poured onto ice at 0° C. Find the change in entropy during this process if the amount of lead is 640 grammes.

5.220. Find the change in entropy during the transition of 8 grammes of oxygen from a volume of 10 litres at a temperature of 80° C to a volume of 40 litres at a temperature of 300° C.

5.221. Find the change in entropy during the transition of 6 grammes of hydrogen from a volume of 20 litres at a pressure of 1.5×10^5 N/m² to a volume of 60 litres at a pressure of 1×10^6 N/m².

5.222. Hydrogen amounting to 6.6 grammes expands isobarically until its volume doubles. Find the change in entropy during expansion.

5.223. Find the change in entropy during the isobaric expansion of 8 grammes of helium from the volume $V_1=10$ litres to $V_2=25$ litres.

5.224. Find the change in entropy upon the isothermal expansion of 6 grammes of hydrogen from 10^5 to 0.5×10^5 N/m².

5.225. Nitrogen amounting to 10.5 grammes expands isothermally from a volume of $V_1=2$ litres to $V_2=5$ litres. Find the entropy increment during this process.

5.226. Oxygen (10 grammes) is heated from $t_1=50°$ C to $t_2=150°$ C. Find the change in entropy if the oxygen is heated: (1) isochorically, (2) isobarically.

5.227. The heating of 1 kmole of a biatomic gas increases its absolute temperature 1.5 times. Find the change in entropy if the gas is heated: (1) isochorically, (2) isobarically.

5.228. The heating of 22 grammes of nitrogen increases its absolute temperature 1.2 times and its entropy by 4.19 J/deg. Under what conditions was the nitrogen heated (at a constant volume or a constant pressure)?

5.229. Find the change in entropy upon the transition of a gas from state *A* to state *B* for the conditions of Problem 5.194 if the transition occurs: (1) along *ACB*, (2) along *ADB* (see Fig. 8).

5.230. One cubic metre of air at a temperature of 0° C and a pressure of 2 kgf/cm² expands isothermally from the volume V_1 to $V_2=2V_1$. Find the change in entropy during this process.

5.231. The change in entropy between the two adiabatic lines of a Carnot cycle is 1 kcal/deg. The temperature difference between the two isothermal lines is equal to 100° C. What amount of heat will be converted into work during this cycle?

6. Real Gases

The equation of state for real gases (Van der Waals equation) for one kilomole has the form:

$$\left(p+\frac{a}{V_0^2}\right)(V_0-b)=RT$$

where V_0 = volume of one kilomole of a gas
a and b = constants different for different gases
p = pressure
T = absolute temperature
R = gas constant.

The Van der Waals equation referred to any mass m of a gas may be written as

$$\left(p+\frac{m^2}{\mu^2}\frac{a}{V^2}\right)\left(V-\frac{m}{\mu}b\right)=\frac{m}{\mu}RT$$

where V = volume of all the gas
μ = mass of one kilomole.

In this equation $\frac{m^2a}{\mu^2V^2}=p_i$ is the pressure due to the forces of interaction of the molecules, and $\frac{m}{\mu}b=V_i$ is the volume connected with the own volume of the molecules.

The constants a and b of a given gas are related to its critical temperature T_{cr}, critical pressure p_{cr} and critical volume V_{0cr} as follows:

$$V_{0cr}=3b; \qquad p_{cr}=\frac{a}{27b^2}; \qquad T_{cr}=\frac{8a}{27bR}$$

These equations can be solved with respect to the constants a and b:

$$a=\frac{27T_{cr}^2R^2}{64p_{cr}} \quad \text{and} \quad b=\frac{T_{cr}R}{8p_{cr}}$$

If we introduce the following notation:

$$\tau=\frac{T}{T_{cr}}, \qquad \pi=\frac{p}{p_{cr}}, \qquad \omega=\frac{V_0}{V_{0cr}}$$

the Van der Waals equation will take the transformed form (for one kilomole)

$$\left(\pi+\frac{3}{\omega^2}\right)(3\omega-1)=8\tau$$

6.1. Express in SI units the constants a and b contained in the Van der Waals equation.

6.2. By using data on the critical values of T_{cr} and p_{cr} for certain gases (see Table V), find for them the constants a and b in the Van der Waals equation.

6.3. What is the temperature of 2 grammes of nitrogen occupying a volume of 820 cm^3 at a pressure of 2 atm? Consider the gas to be: (1) ideal, (2) real.

6.4. What is the temperature of 3.5 grammes of oxygen occupying a volume of 90 cm^3 at a pressure of 28 atm? Consider the gas to be: (1) ideal, (2) real.

6.5. 10 grammes of helium occupy a volume of 100 cm^3 at a pressure of 10^8 N/m^2. Find the temperature of the gas, considering it as: (1) ideal, (2) real.

6.6. One kilomole of carbon dioxide gas has a temperature of 100° C. Find the pressure of the gas, considering it as: (1) real, (2) ideal. Solve the problem for the volumes: (a) $V_1=1$ m^3, and (b) $V_2=0.05$ m^3.

6.7. A closed vessel with a volume of $V=0.5$ m^3 contains 0.6 kmole of carbon dioxide gas at a pressure of 3×10^6 N/m^2. By using the Van der Waals equation, find the number of times the temperature of this gas is to be raised to double the pressure.

6.8. One kilomole of oxygen has a temperature of $t=27°$ C and a pressure of $p=10^7$ N/m^2. Find the volume of the gas, assuming that it behaves in these conditions as a real gas.

6.9. One kilomole of nitrogen has a temperature of $t=27°$ C and a pressure of 5×10^6 N/m^2. Find the volume of the gas assuming that it behaves in these conditions as a real gas.

6.10. Find the effective diameter of an oxygen molecule, assuming that the critical values T_{cr} and p_{cr} for the oxygen are known.

6.11. Find the effective diameter of a nitrogen molecule: (1) from the given value of the mean free path of the molecules in standard conditions $\bar{l}=9.5\times10^{-6}$ cm, (2) from the known value of the constant b in the Van der Waals equation.

6.12. Find the mean free path of a carbon dioxide molecule in standard conditions. Calculate the effective diameter of the molecule, assuming the critical temperature T_{cr} and pressure p_{cr} to be known for carbon dioxide gas.

6.13. Find the diffusion coefficient of helium at a temperature of $t=17°$ C and a pressure of $p=1.5\times10^5$ N/m^2. Calculate the effective diameter of a helium atom, assuming T_{cr} and p_{cr} to be known for helium.

6.14. Plot the isothermal lines $p=f(V)$ for one kilomole of carbon dioxide gas at a temperature of 0° C. Consider the gas as: (1) ideal, and (2) real. Take the following values of V in m^3/kmole for the real gas: 0.07, 0.08, 0.10, 0.12, 0.14, 0.16, 0.18, 0.20, 0.25, 0.30, 0.35 and 0.40, and for the ideal gas within $0.2\leqslant V\leqslant0.4$ m^3/kmole.

6.15. Find the pressure due to the forces of interaction of the molecules contained in one kilomole of a gas in standard conditions. The critical temperature and pressure of this gas are $T_{cr}=417°$ K and $p_{cr}=$ $=76$ atm, respectively.

6.16. The forces of interaction between hydrogen molecules are negligible, the most important factor being the size of the molecules. (1) Write the equation of state for such a semi-ideal gas. (2) Find the error if the dimensions of the molecules are disregarded in calculating the number of kilomoles of hydrogen in a certain volume at a temperature of $t=0°$ C and a pressure of $p=2.8\times10^7$ N/m^2.

6.17. A vessel with a volume of 10 litres contains 0.25 kg of nitrogen at a temperature of 27° C. (1) What part of the gas pressure will the pressure due to the forces of molecular interaction be? (2) What part of the volume of the vessel will be occupied by the own volume of the molecules?

6.18. A certain gas amounting to 0.5 kmole occupies a volume of $V_1=1$ m^3. When the gas expands to $V_2=1.2$m^3, work is done against the forces of interaction of the molecules equal to $W=580$ kgf-m. Find the constant a in the Van der Waals equation for this gas.

6.19. Twenty kilogrammes of nitrogen expand adiabatically into a vacuum from $V_1=1$ m^3 to $V_2=2$ m^3. Find the drop of the temperature during expansion if the constant a in the Van der Waals equation is known for nitrogen. (See the answer to Problem 6.2.)

6.20. Half a kilomole of a triatomic gas expands adiabatically into a vacuum from $V_1=0.5$ m^3 to $V_2=3$ m^3. The temperature of the gas drops by 12.2°. By using these data, find the constant a in the Van der Waals equation.

6.21. (1) What pressure is required to convert carbon dioxide gas into liquid carbon dioxide at a temperature of: (a) 31° C and (b) 50° C? (2) What maximum volume can be occupied by 1 kg of liquid carbon dioxide? (3) What is the highest pressure of the saturated vapours of liquid carbon dioxide?

6.22. Find the density of water vapours in the critical state if their constant b in the Van der Waals equation is known. (See the answer to Problem 6.2.)

6.23. Find the density of helium in the critical state, assuming the critical values of T_{cr} and p_{cr} to be known for it.

6.24. One kilomole of oxygen occupies a volume of 0.056 m^3 at a pressure of 920 at. Find the temperature of the gas using the transformed Van der Waals equation.

6.25. One kilomole of helium occupies a volume of $V=0.237$ m^3 at a temperature of $t=-200°$ C. Find the pressure of the gas using the transformed Van der Waals equation.

6.26. Determine how many times the pressure of a gas is greater than its critical pressure if the volume and temperature of the gas are double their critical values.

7. Saturated Vapours and Liquids

The absolute humidity is the partial pressure of the water vapours present in the air. The relative humidity w is the ratio of the absolute humidity to the partial pressure of water vapours saturating the space at the given temperature.

The specific heat of vaporization (evaporation) r is the amount of heat required to convert a unit mass of a liquid into vapour at a constant temperature.

The molecular heat of vaporization r_0 is equal to

$$r_0=\mu r$$

where μ is the mass of one kilomole.

The relationship between the pressure of a saturated vapour p_s and the temperature is described by the Clausius-Clapeyron equation

$$\frac{dp_s}{dT}=\frac{r_0}{T\,(V_v-V_l)}$$

where V_v= volume of one kilomole of vapour
V_l= volume of one kilomole of liquid.

The relative change in the volume of a liquid when heated is determined by the formula

$$\frac{\Delta V}{V}=\gamma\Delta t$$

where γ is the coefficient of volume expansion.

The relative change in the volume of a liquid when the pressure changes is

$$\frac{\Delta V}{V}=-k\Delta p$$

where k is the coefficient of compression.

The coefficient of surface tension α is numerically equal to the force applied to a unit length of the edge of the surface film of a liquid, i.e.,

$$\alpha=\frac{F}{l}$$

When the film area changes by ΔA the following work is performed

$$\Delta W=\alpha\Delta A$$

The additional pressure caused by curvature of the surface of the liquid is determined by the Laplace formula

$$\Delta p=\alpha\left(\frac{1}{R_1}+\frac{1}{R_2}\right)$$

where R_1 and R_2 are the radii of curvature of two mutually perpendicular cross sections of the surface of the liquid. The radius R is positive if the centre of curvature is inside the liquid (a convex meniscus) and negative if it is outside the liquid (a concave meniscus).

The height of the liquid in a capillary tube

$$h=\frac{2\alpha\cos\theta}{r\rho g}$$

where r=radius of the tube
ρ=density of the liquid
θ=wetting angle.

With complete wetting $\theta=0$ and with no wetting $\theta=\pi$.

The pressure of saturated vapours p_1 above a concave surface of a liquid is less, and above a convex surface more than the pressure p_0

above a flat surface. The additional pressure is

$$\Delta p = p_1 - p_0 = \pm \frac{2\alpha\rho_0}{\rho R}$$

where ρ=density of the liquid
ρ_0=density of the saturated vapours of the liquid
R=radius of curvature of the surface of the liquid.

The osmotic pressure p of a solution is related to its absolute temperature T by the Van't Hoff formula

$$p = CRT$$

where R is the gas constant and $C=\frac{m}{\mu V}$ is the number of kilomoles of the substance being dissolved in a unit volume of the solution (molar concentration of the solution).

For solutions of undissociated molecules of a substance

$$C = \frac{m}{\mu V} = \frac{N}{N_A}$$

where N_A=Avogadro's number
N=number of molecules of the dissolved substance in a unit volume.

When dissociation occurs, the number of particles in a unit volume will be greater, and the osmotic pressure will increase.

The pressure of the saturated vapours above the solution is smaller than above the pure solvent. With a sufficiently low concentration of the solution, the relative drop in the pressure of the saturated vapour above the solution can be determined from the Raoult law

$$\frac{p_0 - p}{p_0} = \frac{z'}{z + z'}$$

where p_0=pressure of the saturated vapour above the pure solvent
p=pressure of the saturated vapour above the solution
z'=number of kilomoles of the dissolved substance
z=number of kilomoles of the liquid.

Problems relating to the viscosity of liquids are given in Section 4 of Chapter 1.

7.1. Table VI gives the pressure of water vapours saturating a space at various temperatures. How can these data be used to compile a table showing the amount of water vapours in 1 m³ of air saturated with the vapours at various temperatures? By way of example, calculate the amount of saturated water vapours in 1 m³ of air at a temperature of 50° C.

7.2. Find the density of saturated water vapours at a temperature of 50° C.

7.3. How many times is the density of saturated water vapours at a temperature of 16°C lower than that of water?

7.4. How many times is the density of saturated water vapours at a temperature of 200° C greater than at a temperature of 100° C?

7.5. What is the weight of the water vapours in 1 m^3 of air on a warm day at a temperature of 30° C and a relative humidity of 75 per cent?

7.6. The relative humidity of the air in a closed space with a volume of $V=1$ m^3 is equal to 60 per cent at a temperature of 20° C. How much more water should be evaporated into this space for the vapours to be saturated?

7.7. The temperature in a room is 18° C and the relative humidity is 50 per cent. A metal tea-kettle is filled with cold water. What is the temperature of the water at which the kettle stops being covered with mist?

7.8. Find the number of molecules of saturated water vapour contained in 1 cm^3 at a temperature of 30° C.

7.9. Half a gramme of water vapour occupies a volume of 10 litres at a temperature of 50° C. (1) Determine the relative humidity. (2) What amount of vapour will be condensed if the volume is halved isothermally?

7.10. A Wilson cloud chamber with a volume of 1 litre contains air saturated with water vapours. The initial temperature of the chamber is 20° C. As the piston moves, the volume of the chamber increases 1.25 times. The expansion is adiabatic, $\varkappa=C_p/C_v$ being equal to 1.4. Find: (1) the pressure of the water vapours before expansion, (2) the amount of water vapours in the chamber before expansion, (3) the density of the water vapours before expansion, (4) the temperature of the vapour after expansion (disregard the change in the temperature due to heat evolution during vapour condensation), (5) the amount of water vapours condensed into water, (6) the density of the water vapours after condensation, (7) the degree of supersaturation, i.e., the ratio of the density of the water vapour after expansion (but before condensation) to the density of the water vapour saturating the space at the temperature which set in after condensation.

7.11. Find the specific volume of water in the liquid and vaporous states in standard conditions.

7.12. Using the first law of thermodynamics and the data in Tables V and VI, find the specific heat of water vaporization at 200° C. The critical temperature for water $T_{cr}=647°$ K and the critical pressure $p_{cr}=217$ atm. Check the result obtained with the aid of the data in Table VII.

7.13. What part of the specific heat of water vaporization at a temperature of 100° C is spent to increase the internal energy of the system?

7.14. The specific heat of vaporization of benzene (C_6H_6) at a temperature of 77° C is equal to 95 cal/g. What is the change in the internal energy upon the evaporation of 20 grammes of benzene at this temperature?

7.15. Using the Clausius-Clapeyron equation and the data in Table VI, find the specific heat of water vaporization at a temperature of 5° C. Check the result obtained with the aid of the data in Table VII.

7.16. The pressure of saturated mercury vapours at temperatures of $t_1=100°$ C and $t_2=120°$ C is equal to $p_1=0.28$ mm Hg and $p_2=0.76$ mm Hg, respectively. Determine the mean specific heat of mercury vaporization within this temperature range.

7.17. The boiling point of benzene (C_6H_6) at $p=1$ atm is equal to 80.2° C. Determine the pressure of its saturated vapours at a temperature of 75.6° C if the mean specific heat of vaporization is equal to 4×10^5 J/kg within the given temperature range.

7.18. The pressure of the saturated vapours of ethyl alcohol (C_2H_5OH) is equal to 133 mm Hg at a temperature of 40° C, and to 509 mm Hg at a temperature of 68° C. Find the change in entropy upon the vaporization of 1 gramme of ethyl alcohol at a temperature of 50° C.

7.19. The change in entropy upon the vaporization of 1 kmole of a certain liquid at a temperature of 50° C is 133 J/deg. The pressure of the saturated vapours of this liquid at a temperature of 50° C is equal to 92.5 mm Hg. By how much will the pressure of the saturated vapours of this liquid change when the temperature changes from 50 to 51° C?

7.20. Find the limit pressure to which a vessel can be evacuated by means of a mercury-diffusion pump operating without a mercury trap if the temperature of the pump water jacket is 15° C. The pressure of the saturated mercury vapours at a temperature of 0° C is equal to 1.6×10^{-4} mm Hg. The specific heat of vaporization of mercury should be taken equal to 75.6 cal/g within the temperature range of 0-15° C.

7.21. Knowing that the density of mercury at a temperature of 0° C is 13.6 g/cm^3, find its density at 300° C. The coefficient of volume expansion of mercury is to be considered constant and its mean value equal to 1.85×10^{-4} deg^{-1} within the given temperature range.

7.22. The density of mercury is equal to 13.4 g/cm^3 at a temperature of 100° C. At what temperature is the mercury density equal to 13.1 g/cm^3? The coefficient of volume expansion of mercury is equal to 1.8×10^{-4} deg^{-1}.

7.23. Taking the mean value of the coefficient of water compression equal to 4.8×10^{-5} atm, find the density of sea water at a depth of 5 km if its density at the surface is 1,030 kg/m^3. In calculating the hydrostatic pressure of the sea water, assume its density to be approximately equal to the density of the water at the surface.

7.24. At 0° C and atmospheric pressure, the coefficient of compression of benzene is equal to 9×10^{-5} atm^{-1} and its coefficient of volume expansion to 1.24×10^{-3} deg^{-1}. What external pressure must be applied for the volume of the benzene to remain unchanged when heated by 1° C.

7.25. The coefficient of volume expansion of mercury is equal to $\gamma=1.82\times10^{-4}$ deg^{-1}. Find the coefficient of compression if the external pressure must be increased by 47 atm for the volume of the mercury to remain unchanged when it is heated by 1° C.

7.26. Find the difference in the levels of mercury in two identical communicating glass tubes if the left-hand tube is maintained at a temperature of 0° C and the right-hand one is heated to 100° C. The height of the left-hand tube is 90 cm. The coefficient of volume expansion of mercury is 1.82×10^{-4} deg^{-1}. Disregard the expansion of the glass.

7.27. Mercury is poured into a glass vessel with a height of $H=10$ cm. At a temperature of $t=20°$ C the level of the mercury is $h=1$ mm below the upper edge of the vessel. By how much can the mercury be heated so that it does not flow out of the vessel? The coefficient of volume expansion of mercury is $\gamma=1.82\times10^{-4}$ deg^{-1}. Disregard the expansion of the glass.

7.28. A glass vessel filled with mercury at a temperature of 0° C up to its edges weighs 1 kgf. The empty vessel weighs 0.1 kgf. Neglecting the expansion of the glass, find the amount of mercury which can be contained in the vessel at a temperature of 100° C. The coefficient of volume expansion of mercury is 1.8×10^{-4} deg^{-1}.

7.29. Solve the previous problem taking into account the expansion of the glass. Assume that the coefficient of volume expansion of the glass is equal to 3×10^{-5} deg^{-1}.

7.30. A glass vessel is filled up to its edges with liquid oil at a temperature of 0° C. When the vessel with the oil was heated to 100° C, six per cent of the oil flowed out. Find the coefficient of volume expansion of the oil γ_{oil}, assuming the coefficient of volume expansion of the glass to be equal to $\gamma=3\times10^{-5}$ deg^{-1}.

7.31. What will the relative error in determining the coefficient of volume expansion of the oil in the conditions of the previous problem be if the expansion of the glass is neglected?

7.32. The temperature in a room is 37° C and the atmospheric pressure 760 mm Hg. What pressure (in mm Hg) will be shown by a mercury barometer hanging in the room? Consider the expansion of the glass to be small as compared with that of the mercury. The coefficient of volume expansion of mercury is 1.82×10^{-4} deg^{-1}.

7.33. (1) What force must be applied to a horizontal aluminium ring with a height of $h=10$ mm, an internal diameter of $d_1=50$ mm and an external diameter of $d_2 = 52$ mm to tear the ring away from the surface of water? (2) What part of the force determined is due to the force of surface tension?

7.34. A ring with an internal diameter of 25 mm and an external diameter of 26 mm is suspended on a spring with a deformation coefficient of 10^{-4} kgf/mm and touches the surface of a liquid. When the surface of the liquid lowered, the ring broke away from it upon expansion of the spring by 5.3 mm. Find the coefficient of surface tension of the liquid.

7.35. Frame *ABCD* (Fig. 12) with a movable bar *KL* is covered with a soap film. (1) What should the diameter of the copper bar *KL* be for it to remain in equilibrium? (2) What is the length l of the bar if isothermal work equal to 4.5×10^{-5} J is performed when the bar moves over a distance of 1 cm. For soapy water $\alpha = 0.045$ N/m.

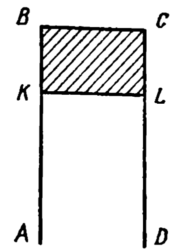

Fig. 12

7.36. Alcohol flows out drop by drop from a vessel through a vertical tube with an internal diameter of 2 mm. Find the time during which 10 grammes of the alcohol will flow out if the interval between drops is 1 second. Assume that the diameter of the neck of a drop at the moment it breaks away is equal to the internal diameter of the tube.

7.37. Water flows out drop by drop from a vessel through a vertical tube with an internal diameter of $d = 3$ mm. When the water cools from $t_1 = 100°$ C to $t_2 = 20°$ C, the weight of each drop changes by $\Delta G = 13.5 \times 10^{-6}$ kgf. Knowing the coefficient of surface tension of water at 20° C, find this coefficient at 100° C. Assume that the diameter of the neck of a drop at the moment it breaks away is equal to the internal diameter of the tube.

7.38. Twenty drops of lead were formed when the lower end of a vertically suspended lead wire 1 mm in diameter was melted. By how much did the wire become shorter? The coefficient of surface tension of liquid lead is 0.47 N/m. Assume that the diameter of the neck of a drop at the moment it breaks away is equal to the diameter of the wire.

7.39. Drops of water fall from a vertical tube with an internal radius of $r = 1$ mm. Find the radius of a drop at the moment when it breaks away, considering it to be spherical. Assume that the diameter of the neck of a drop at the moment it breaks away is equal to the internal diameter of the tube.

7.40. By how much will a mercury drop obtained from the merging of two drops each with a radius of 1 mm be heated?

7.41. What work should be performed against the forces of surface tension to split a spherical mercury drop with a radius of 3 mm into two identical drops?

7.42. What work should be performed against the forces of surface tension to double the volume of a soap bubble with a radius of 1 cm. The coefficient of surface tension of a soap solution is 43×10^{-3} N/m.

7.43. What work should be performed against the forces of surface tension to blow a soap bubble ($\alpha=0.043$ N/m) 4 cm in diameter?

7.44. Determine the pressure of the air (in mm Hg) in an air bubble with a diameter of $d=0.01$ mm at a depth of $h=20$ cm below the surface of water. The external pressure $p_1=765$ mm Hg.

7.45. The pressure of the air inside a soap bubble is 1 mm Hg greater than the atmospheric pressure. What is the diameter of the bubble? The coefficient of surface tension of the soap solution is 0.043 N/m.

7.46. Find the depth of an air bubble under water if the density of the air in the bubble is 2 kg/m^3. The diameter of the bubble is 0.015 mm, the temperature 20° C and the atmospheric pressure 760 mm Hg.

7.47. How many times is the density of air in a bubble in water at a depth of 5 m below the surface greater than the density of the air at atmospheric pressure (at the same temperature)? The radius of the bubble is 5×10^{-4} mm.

7.48. An open capillary tube with an internal diameter of $d=$ $=3$ mm is lowered into a vessel with mercury. The difference between the levels of the mercury in the vessel and in the capillary tube $\Delta h=$ $=3.7$ mm. What is the radius of curvature of the mercury meniscus in the capillary tube?

7.49. An open capillary tube with an internal diameter of $d=$ $=1$ mm is lowered into a vessel with water. The difference between the levels of the water in the vessel and in the capillary tube is $\Delta h=$ $=2.8$ cm. (1) What is the radius of curvature of the meniscus in the capillary tube? (2) What would the difference between the levels in the vessel and the capillary tube be if wetting were complete?

7.50. To what height will benzene rise in a capillary tube whose internal diameter is $d=1$ mm? Consider wetting to be complete.

7.51. What should the internal diameter of a capillary tube be for the water to rise in it by 2 cm with complete wetting? Solve the problem for the cases when the capillary tube is: (1) on the Earth, (2) on the Moon.

7.52. Find the difference in the levels of mercury in two communicating capillary tubes with the diameters $d_1=1$ mm and $d_2=2$ mm, respectively. Consider that there is absolutely no wetting.

7.53. What should the maximum diameter of the pores in the wick of an oil stove be for the oil to rise from the bottom of the stove to the burner (height $h=10$ cm)? Consider the pores as cylindrical tubes and wetting to be complete.

7.54. A capillary tube with an internal radius of 2 mm is lowered into a liquid. Find the coefficient of surface tension of the liquid if the weight of the liquid that has risen in the capillary tube is 9×10^{-6} kgf.

7.55. A capillary tube with an internal radius of $r=0.16$ mm is lowered vertically into a vessel with water. What should the air pressure above the liquid in the capillary tube be for the water level in the capillary tube and in a broad vessel to be the same? The external pressure $p_0=760$ mm Hg, and wetting is complete.

7.56. A capillary tube is lowered vertically into a vessel with water. The upper end of the tube is soldered. For the level of the water in the tube and in a broad vessel to be the same, the tube has to be submerged into the water by 1.5 per cent of its length. What is the internal radius of the tube? The external pressure is 750 mm Hg and wetting is complete.

7.57. The internal diameter d of barometric tube A filled with mercury (Fig. 13) is: (a) 5 mm, (b) 1.5 cm. Can the atmospheric pressure be determined directly from the height of the mercury column? Find the height of the mercury column in each case if the atmospheric pressure $p_0=758$ mm Hg. Consider that there is absolutely no wetting.

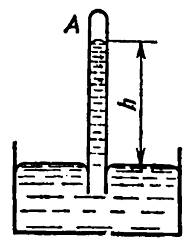

Fig. 13

7.58. The internal diameter of a barometric tube is 0.75 cm. What correction should be made when measuring the atmospheric pressure according to the height of the mercury column? Consider that there is no wetting.

7.59. What will the relative error be in calculating the atmospheric pressure equal to 760 mm Hg according to the height of a mercury column if the internal diameter of the barometric tube is: (1) 5 mm and (2) 10 mm? Consider that there is no wetting.

7.60. A greased steel needle which is unwettable by water is placed onto the surface of water. What will the maximum diameter of the needle be at which it will still remain on the surface?

7.61. Will a greased (unwettable by water) platinum wire 1 mm in diameter float on the surface of water?

7.62. The bottom of a vessel with mercury has a hole. What can the maximum diameter of the hole be at which no mercury will flow out from the vessel when the mercury column is 3 cm high?

7.63. The bottom of a glass vessel with an area of $A=30$ cm^2 has a round hole with a diameter of $d=0.5$ mm. The vessel is filled with mercury. How much mercury will remain in the vessel?

7.64. Find the weight of a water skater running over water if under each of the six legs of the insect a hemisphere with a radius of 0.1 mm is formed.

7.65. What force must be applied to detach two wetted photographic plates 9×12 cm in size from each other without shifting them? The thickness of the water layer between the plates is 0.05 mm and the wetting is complete.

7.66. A liquid is poured between two vertical flat and parallel glass plates at a distance of 0.25 mm from each other. Find the density of the liquid if the height which it rises to between the plates is 3.1 cm (α=30 dyne/cm). Consider wetting to be complete.

7.67. There are five grammes of mercury between two horizontal flat and parallel glass plates. A load of 5 kgf is placed on the upper plate and the distance between the plates becomes equal to 0.087 mm. Neglecting the weight of the plate as compared with that of the load, find the coefficient of surface tension of the mercury. Consider that there is no wetting.

7.68. An open capillary tube contains a drop of water. When the tube is in its vertical position the drop forms a column with a length of: (1) 2 cm, (2) 4 cm, (3) 2.98 cm. The internal diameter of the capillary tube is 1 mm. Determine the radii of curvature of the upper and lower meniscuses in each case. Consider wetting to be complete.

7.69. Water is pumped into a horizontal capillary tube with an internal diameter of $d=2$ mm so that a column $h=10$ cm long is formed. How many grammes of the water will flow out of the tube if it is placed vertically? Consider wetting to be complete.

Note. Bear in mind that the maximum length of the water column left in the capillary tube should correspond to the radius of curvature of the lower meniscus, equal to the radius of the tube (see the solution of the previous problem).

7.70. A column of alcohol is contained in an open vertical capillary tube with an internal radius of $r=0.6$ mm. The lower meniscus of the column hangs from the bottom end of the tube. Find the height h of the alcohol column at which the radius of curvature R of the lower meniscus is equal to: (1) 3 r, (2) 2 r, and (3) r. Consider wetting to be complete.

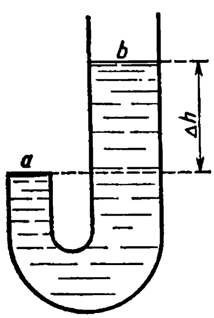

Fig. 14

7.71. The tube shown in Fig. 14 is open at both ends and filled with kerosene. The internal radii a and b are equal to $r_1=0.5$ mm and $r_2=0.9$ mm, respectively. At what difference Δh between the levels will the meniscus at the end of tube a be: (1) concave with a radius of curvature of $R_x=r_1$, (2) flat, (3) convex with a radius of curvature of $R_x=r_2$, (4) convex and equal to r_1? Consider wetting to be complete.

7.72. A capillary tube is so submerged into a broad vessel with water that the upper end of the tube is above the level of the water in the vessel by $h=2$ cm. The internal radius of the capillary tube r =

= 0.5 mm. Find the radius of curvature R of the meniscus in the tube. Consider wetting to be complete.

7.73. An aerometer floats in water which wets its walls completely. The diameter of the vertical cylindrical tube of the aerometer $d=9$ mm. How much will the depth of submergence of the aerometer change if several drops of alcohol are poured onto the surface of the water?

7.74. An aerometer floats in a liquid having a density of $\rho = 800$ kg/m^3 and a coefficient of surface tension of $\alpha=30$ dyne/cm. The liquid completely wets the walls of the aerometer. The diameter of the vertical cylindrical tube of the aerometer $d=9$ mm. How much will the depth of submergence of the aerometer change if greasing makes it completely unwettable?

7.75. When 10 grammes of sugar ($C_{12}H_{22}O_{11}$) are dissolved in 0.5 litre of water the osmotic pressure of the solution is equal to 1.52×10^5 N/m^2. What is the temperature of the solution? The sugar molecules are not dissociated.

7.76. The osmotic pressure of a solution at a temperature of 87° C is equal to 1.65×10^6 N/m^2. What number of water molecules is there per molecule of the substance dissolved in the solution? There is no dissociation.

7.77. Two grammes of table salt are dissolved in 0.5 litre of water. The degree of dissociation of the salt molecules is 75 per cent. Determine the osmotic pressure of the solution at a temperature of 17° C.

7.78. When table salt is dissolved in water, the degree of dissociation of its molecules is 40 per cent. The osmotic pressure of the solution is equal to 1.21 kgf/cm^2 at a temperature of 27° C. How much of the salt is dissolved in 1 litre of water?

7.79. Table salt in an amount of 2.5 grammes is dissolved in 1 litre of water at a temperature of 18°C. The osmotic pressure of the solution is 1.6×10^5 N/m^2. (1) What is the degree of dissociation of the salt molecules? (2) How many particles of the dissolved substance are there in 1 cm^3 of the solution?

7.80. Forty grammes of sugar ($C_{12}H_{22}O_{11}$) are dissolved in 0.5 litre of water. The temperature of the solution is 50° C. What is the pressure of the saturated water vapours above the solution?

7.81. The pressure of saturated vapours above a solution is 31.5 mm Hg at a temperature of 30° C. Find their pressure at a temperature of 60° C.

7.82. The pressure of saturated vapours above a solution is 1.02 times smaller than that of pure water. How many molecules of water are there per molecule of the dissolved substance?

7.83. One hundred grammes of an unvolatile substance are dissolved in 1 litre of water. The temperature of the solution is 90° C and the pressure of the saturated vapours above the solution 515.9 mm Hg. Determine the mass of one kilomole of the dissolved substance.

7.84. An unvolatile substance with a mass of one kilomole of $\mu = 60$ kg/kmole is dissolved in water. The temperature of the solution is 80° C and the pressure of the saturated vapours above the solution 353 mm Hg. Find the osmotic pressure of the solution.

8. Solids

The change in the melting point dT upon a change in the pressure by dp is described by the Clausius-Clapeyron equation

$$dT = T\frac{V_l - V_s}{q_0}dp$$

where q_0 = molecular heat of fusion
V_l = volume of one kilomole of liquid
V_s = volume of one kilomole of solid
T = melting point.

When the temperatures are not too low, solids obey Dulong and Petit's law, according to which the atomic heat of all chemically simple solids is approximately equal to $3\,R = 25\times10^3$ J/kg-atom·deg $=6$ cal/g-atom·deg.

The amount of heat transferred by conduction during the time Δt can be determined from the formula

$$Q = -\lambda\frac{\Delta T}{\Delta x}\Delta A\Delta t$$

where $\frac{\Delta T}{\Delta x}$ = temperature gradient in a direction perpendicular to the area ΔA
λ = coefficient of thermal conductivity.

When the temperature rises, the length of solids increases to a first approximation linearly with the temperature, i.e.,

$$l_t = l_0(1+\alpha t)$$

where l_t = length of a solid body at the temperature t
l_0 = length of the body at the temperature 0° C
α = coefficient of linear thermal expansion.

For isotropic solids $\alpha = \frac{1}{3}\gamma$, where γ is the coefficient of volume thermal expansion.

In deformation due to longitudinal tension (or unilateral compression) of a rod, the relative change in its length according to Hooke's law is

$$\frac{\Delta l}{l}=\frac{p_l}{E}$$

where p_l = specific load, i.e., $p_l=\frac{F}{A}$ (here F is the tensile or compressive force, and A is the cross-sectional area)
E = modulus of elasticity (Young's modulus).

The relative change in the thickness of a rod in longitudinal tension is

$$\frac{\Delta d}{d}=\beta p_l$$

where β is the coefficient of lateral compression. The quantity

$$\mu=\beta E$$

is known as Poisson's ratio.

To twist a rod (a wire) through a certain angle φ the moment of a couple of forces should be applied

$$M=\frac{\pi G r^4 \varphi}{2l}$$

where l = length of the wire
r = radius of the wire
G = shear modulus of the wire material.

8.1. When 1 kmole of ice is melted, the change in the entropy is 22.2 kJ/deg. Find the change in the melting point of the ice when the external pressure is increased by 1×10^5 N/m^2.

8.2. The melting point of tin is 231.9° C at a pressure of 10^5 N/m^2, and 232.2° C at a pressure of 10^7 N/m^2. The density of liquid tin is 7.0 g/cm^3. Find the increase in entropy when 1 kmole of tin is melted.

8.3. The melting point of iron changes by 0.012° C when the pressure changes by 1 kgf/cm^2. Find the change in the volume of one kilomole of iron.

8.4. By using Dulong and Petit's law, find the specific heat of: (1) copper, (2) iron, (3) aluminium.

8.5. By using Dulong and Petit's law, find the material which a metallic ball 0.025 kgf in weight is made of if 117 J of heat are required to heat it from 10° C to 30° C.

8.6. By using Dulong and Petit's law, find how many times the specific heat of aluminium is greater than that of platinum.

8.7. A lead bullet strikes a wall with a velocity of 400 m/s and penetrates into it. Assuming that 10 per cent of the kinetic energy of the bullet is spent to heat it, find by how many degrees the temperature of the bullet is raised. Determine the specific heat of lead from Dulong and Petit's law.

8.8. A copper plate (with a thickness of $d_1=9$ mm) and an iron plate ($d_2=3$ mm) are put together. The external surface of the copper plate is maintained at a constant temperature of $t_1=50°$ C, and that of the iron plate at $t_2=0°$ C. Find the temperature t_x of the contacting surface. The area of the plates is much greater than their thickness.

8.9. The external surface of a wall has a temperature of $t_1=-20°$ C and the internal one—$t_2=+20°$ C. The wall is 40 cm thick. Determine the thermal conductivity coefficient of the material of the wall if each cubic metre of its surface lets through 110 kcal per hour.

8.10. How much heat is lost by a room with a floor area of 4×5 m^2 and 3 m high every minute through its four brick walls? The temperature in the room $t_1=15°$ C and the outside temperature $t_2=-20°$ C. The thermal conductivity coefficient of the brick is 0.002 cal/deg·cm·s and the walls are 50 cm thick. Disregard the loss of heat through the floor and the ceiling.

8.11. One end of an iron rod is maintained at a temperature of 100° C, while the other rests on ice. The rod is 14 cm long, and its cross-sectional area is 2 cm^2. The rod is heat insulated, and the loss of heat through the walls may therefore be neglected. Find: (1) the velocity with which the heat flows along the rod, (2) the amount of ice melted during 40 minutes.

8.12. What amount of heat passes in one second through a copper rod 10 cm^2 in area and 50 cm long if the temperature difference at the ends of the rod is 15° C? Disregard the heat losses.

8.13. An aluminium pan 15 cm in diameter filled with water is put on a stove. The water boils producing 300 grammes of vapour a minute. Find the temperature of the external surface of the pan bottom if it is 2 mm thick. Disregard the heat losses.

8.14. A metal cylindrical vessel with a radius of 9 cm is filled with ice at a temperature of 0° C. The vessel is insulated thermally with a layer of cork 1 cm thick. In how much time will all the ice in the vessel melt if the ambient temperature of the air is 25° C? Assume that heat exchange occurs only through the sides of the vessel having a mean radius of 9.5 cm.

8.15. What force should be applied to the ends of a steel rod with a cross-sectional area of $A=10$ cm^2 to prevent its expanding when heated from $t_1=0°$ C to $t_2=30°$ C?

8.16. A load is suspended from a steel wire with a radius of 1 mm. The load extends the wire the same amount as heating by 20° C. Find the weight of the load.

8.17. A hot copper wire is stretched at a temperature of 150° C between two strong fixed walls. At what temperature will the wire break when it cools? Assume that Hooke's law is true up to failure of the wire.

8.18. A metal is heated from 0° C to 500° C and its density reduces 1.027 times. Determine the coefficient of linear thermal expansion for this metal, considering it constant within the given temperature range.

8.19. What should the length of a steel and a copper rods be at 0° C for the steel rod to be 5 cm longer than the copper one at any temperature?

8.20. Thirty three kilocalories are spent to heat a copper blank weighing 1 kgf from a temperature of 0° C. How many times will its volume increase? Find the heat capacity of copper from Dulong and Petit's law.

8.21. When a copper wire with a cross section of 1.5 mm^2 was tensioned, a permanent set was observed to begin at a load of 4.5 kgf. What is the elastic limit of the wire material?

8.22. What should the minimum diameter of a steel wire rope be for it to resist a load of 1 tonf?

8.23. Determine the length of a copper wire which when suspended vertically breaks under its own weight.

8.24. Solve the previous problem for a lead wire.

8.25. A load suspended from a steel wire rope is lowered from a ship to measure the depth of the sea. Neglecting the weight of the load as compared with that of the wire rope, find the maximum depth that can be measured by this method. Assume the density of sea water to be 1 g/cm^3.

8.26. A steel wire 40 m long and 2 mm in diameter hangs down from the roof of a house. (1) What maximum load can be suspended from this wire without its breaking? (2) How much will this wire stretch under a weight of 70 kgf? (3) Will there be permanent set when the weight is removed? Assume the elastic limit of steel to be $2.94 \times \times 10^8$ N/m^2.

8.27. A load of 981 N is suspended from a steel wire with a radius of 1 mm. What is the maximum angle through which the wire with the load can be deflected so that it does not break when the load passes through the position of equilibrium?

8.28. A load of 1 kgf is attached to an iron wire 50 cm long and 1 mm in diameter. At what maximum number of revolutions per second can the wire with the load be revolved uniformly in a vertical plane so that the wire does not break?

8.29. A homogeneous copper rod 1 m long is revolved uniformly around a vertical axis passing through one of its ends. What is the velocity of rotation at which the rod will break?

8.30. A homogeneous rod revolves uniformly around a vertical axis passing through its centre. The rod breaks when its end attains a linear velocity of 380 m/s. Find the ultimate strength of the rod material. The density of the rod material is 7,900 kg/m^3.

8.31. A load of 100 kgf is suspended from a steel wire 1 m long and with a radius of 1 mm. What is the work of expansion of the wire.

8.32. A boy's catapult is made of a rubber cord 42 cm long and with a radius of 3 mm. The boy stretches the cord by 20 cm. Find Young's modulus for the rubber if a stone weighing 0.02 kgf when catapulted flies with a velocity of 20 m/s. Disregard the cross section of the cord in stretching.

8.33. A rubber hose 50 cm long and with an internal diameter of 1 cm is stretched until its length becomes greater by 10 cm. Find the internal diameter of the stretched hose if Poisson's ratio for rubber is 0.5.

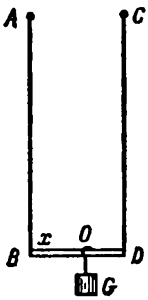

Fig. 15

8.34. In Fig. 15, AB is an iron wire, CD—a copper wire of the same length and the same cross section as AB, and BD—a rod 80 cm long. A load $G=2$ kgf is suspended from the rod. At what distance x should the load be suspended from point B for the rod to remain in a horizontal position?

8.35. Find the moment of a couple of forces required to twist a wire 10 cm long and with a radius of 0.1 mm through an angle of 10′. The shear modulus of the wire material is 5×10^3 kgf/mm^2.

8.36. The mirror of a galvanometer is suspended from a wire $L=10$ cm long and $d=0.01$ mm in diameter. Find the twisting moment which corresponds to a deviation of the reflection of $l=1$ mm on a scale removed from the mirror by $D=1$ m. The shear modulus G of the wire material is 4×10^{11} dyne/cm^2.

8.37. Find the potential energy of a wire 5 cm long and 4×10^{-2} cm in diameter twisted through an angle of 10′. The shear modulus of the wire material is 5.9×10^{11} dyne/cm^2.

8.38. When an electric current flows through the winding of a galvanometer, its frame carrying a mirror is acted upon by a twisting moment equal to 2×10^{-6} dyne·cm. The frame turns through a small angle φ. The work done in twisting is 8.7×10^{-16} J. Over what distance will the reflection from the mirror shift on a scale one metre from the galvanometer?

8.39. Find the value of Poisson's ratio at which the volume of a wire does not change in tension.

8.40. Find the relative change in the density of a cylindrical copper rod when it is compressed by a pressure of $p=1{,}000$ kgf/cm^2. Poisson's ratio is $\mu=0.34$ for copper.

8.41. An iron wire 5 m long is suspended vertically. By how much will the volume of the wire change if a load of 10 kgf is attached to it? Poisson's ratio for iron is 0.3.

Chapter 3

ELECTRICITY AND MAGNETISM

ELECTRICAL AND MAGNETIC UNITS

The International System of Units (SI) incorporates the MKSA system used to measure electrical and magnetic quantities (USSR State Standard GOST 8033-56).

The basic units in this system are the metre (m), kilogramme (kg), second (s) and ampere (A). The derived units of the MKSA system are formed on the basis of the laws which establish the relationship between the relevant physical quantities. For example, the unit of the quantity of electricity—the coulomb (C)—is determined from the equation $Q=It$ as the quantity of electricity which flows through the cross section of a conductor in one second when the current intensity is one ampere, i.e., $1\ \text{C}=1\ \text{A}\times 1\ \text{s}$. The unit of potential difference—volt (V)—can be found from the equation $P=U\times I$, where P is the power of the current. Hence $1\ \text{V}=\frac{1\text{W}}{1\text{A}}$. By proceeding in this way, we can find the units for the other derived quantities in the MKSA system (see Table 11).

The MKSA units are used to rationalize the formulas. Many equations relating to the theory of electric and magnetic phenomena contain the numerical factor 4π (for example, the Gauss theorem, the capacitance of a plane capacitor, the magnetic field intensity inside a solenoid, etc.). The equations are rationalized to dispense with this factor in the formulas most frequently used in electrical and radio engineering. The factor 4π, however, will enter other formulas used less frequently, and in which its presence can be explained by geometrical considerations. The USSR standard specifies the electrical and magnetic units of the SI system for the rationalized form of equations of the electromagnetic field. Accordingly, all the equations in the introductions to the sections of Chapter 3 are given in the rationalized form.

Besides the MKSA system, GOST 8033-56 permits the CGS (Gaussian) system to be used for electrical and magnetic measurements. For this reason the numerical data in the problems will not always be given in the MKSA system. But, bearing in mind the advantages of a unified system, we shall, as before, solve the problems only in the

TABLE 11

Quantity and symbol	Formula	Unit	Symbol of unit	Dimension of quantity
		Basic Units		
Length l	—	metre	m	l
Mass m	—	kilogramme	kg	m
Time t	—	second	s	t
Current intensity, I	—	ampere	A	I
		Derived Units		
Quantity of electricity	$Q=It$	coulomb or ampere-second	C(A·s)	tI
Flux of electric displacement (electric induction flux)	$\psi=N_D=\Sigma Q$	coulomb	C	tI
Linear density of electric charge	$\tau=\frac{Q}{l}$	coulomb per metre	C/m	$l^{-1}tI$
Surface density of electric charge	$\sigma=\frac{Q}{A}$	coulomb per square metre	C/m²	$l^{-2}tI$
Electric displacement (electric induction)	$D=\sigma$	coulomb per square metre	C/m²	$l^{-2}tI$
Volume density of electric charge	$\delta=\frac{Q}{V}$	coulomb per cubic metre	C/m³	$l^{-3}tI$
Potential difference; electromotive force	$\Delta U=\frac{W}{Q}$	volt	V	$l^2mt^{-3}I^{-1}$
Electric field intensity	$E=\frac{U}{l}$	volt per metre	V/m	$lmt^{-3}I^{-1}$
Electric resistance	$R=\frac{U}{I}$	ohm	Ω	$l^2mt^{-3}I^{-2}$
Resistivity	$\rho=\frac{RA}{l}$	ohm-metre	Ω·m	$l^3mt^{-3}I^{-2}$
Electric capacitance	$C=\frac{Q}{U}$	farad	F	$l^{-2}m^{-1}\times t^4I^2$
Current density	$J=\frac{I}{A}$	ampere per square metre	A/m²	$l^{-2}I$
Magnetic flux	$\|d\Phi\|=Edt$	weber	Wb	$l^2mt^{-2}I^{-1}$
Magnetic induction	$B=\frac{\Phi}{A}$	tesla (weber per square metre)	T	$mt^{-2}I^{-1}$
Inductance	$\|L\|=\frac{E}{dI/dt}$	henry	H	$l^2mt^{-2}I^{-2}$
Magnetic field intensity	$H=\frac{I}{2\pi r}$	ampere per metre	A/m	$l^{-1}I$
Magnetic moment of current-carrying circuit (of pointer)	$p=IA$	ampere-square metre	A·m²	l^2I

MKSA system. For this purpose the numerical data in the problems should be converted into MKSA units. Table 12 shows the relationships between certain units of the CGS and MKSA systems according to GOST 8033-56.

Since in the CGS system most of the units have no names, we shall denote the unit of a physical quantity by the symbol of this system with the corresponding subscript. Thus, the unit of current intensity will be designated CGS_I and the unit of capacitance by CGS_C, etc.

The relations in Table 12 are between CGS units for non-rationalized equations and MKSA units for rationalized equations of the electromagnetic field. For the relationship between these equations see the Appendix.

Let us introduce the relative permittivity (dielectric constant) of a medium $\varepsilon_r = \frac{\varepsilon}{\varepsilon_0}$, where ε is the absolute permittivity, whose numerical value depends both on the properties of the medium and the choice of the system of units, and ε_0 is the permittivity of a vacuum. The numerical value of ε_0 depends only on the choice of the system of units. Hence ε in all equations may be replaced by the numerically equal quantity $\varepsilon_0\varepsilon_r$, where ε_0 is the permittivity of a vacuum and ε_r is the relative permittivity (dielectric constant) of the medium with respect to a vacuum, i.e., the usual tabulated value of the permittivity. In the CGS system

$$\varepsilon_0 = 1 \text{ and } \varepsilon = \varepsilon_r$$

and in the MKSA system

$$\varepsilon_0 = \frac{1}{4\pi c^2} \times 10^7 \text{ F/m} = 8.85 \times 10^{-12} \text{ F/m} \quad (c \approx 3 \times 10^8 \text{ m/s})$$

Similarly, let us take instead of the absolute permeability of a medium μ the numerically equal quantity $\mu_0\mu_r$, where μ_0 is the permeability in a vacuum and μ_r is the relative permeability of the medium with respect to a vacuum, i.e., the usual tabulated value of the permeability. In the CGS system

$$\mu_0 = 1 \text{ and } \mu = \mu_r$$

In the MKSA system

$$\mu_0 = 4\pi \times 10^{-7} \text{ H/m} = 12.57 \times 10^{-7} \text{ H/m}$$

EXAMPLES OF SOLUTIONS

Example 1. Find the radius of a ball in air if the surface density of its charge is equal to

$$\sigma = 0.138 \frac{CGS_Q}{cm^2}$$

when the ball is charged to a potential of $U = 4$ CGS_U.

TABLE 12

Quantity	Unit and its conversion factor to SI units
Current intensity	$1\ \mathrm{CGS}_I=\frac{10}{c}\mathrm{A}=\frac{1}{3}\times10^{-9}\mathrm{A}$
Quantity of electricity	$1\ \mathrm{CGS}_Q=\frac{10}{c}\mathrm{C}=\frac{1}{3}\times10^{-9}\mathrm{C}$
Flux of electric displacement (flux of electric induction)	$1\ \mathrm{CGS}_\psi=\frac{10}{4\pi c}\mathrm{C}=\frac{1}{4\pi\times3}\times10^{-9}\mathrm{C}$
Electric displacement (electric induction)	$1\ \mathrm{CGS}_D=\frac{10^5}{4\pi c}\mathrm{C/m^2}=\frac{1}{4\pi\times3}\times10^{-5}\mathrm{C/m^2}$
Surface density of electric charge	$1\ \mathrm{CGS}_\sigma=\frac{10^5}{c}\mathrm{C/m^2}=\frac{1}{3}\times10^{-5}\mathrm{C/m^2}$
Potential difference	$1\ \mathrm{CGS}_U=c\times10^{-8}\mathrm{V}=3\times10^2\mathrm{V}$
Electric field intensity	$1\ \mathrm{CGS}_E=c\times10^{-6}\mathrm{V/m}=3\times10^4\mathrm{V/m}$
Electric resistance	$1\ \mathrm{CGS}_R=c^2\times10^{-9}\,\Omega=9\times10^{11}\,\Omega$
Resistivity	$1\ \mathrm{CGS}_\rho=c^2\times10^{-11}\,\Omega\cdot\mathrm{m}=9\times10^9\,\Omega\cdot\mathrm{m}$
Electric capacitance	$1\ \mathrm{CGS}_C=\frac{1}{c^2}\times10^9\mathrm{F}=\frac{1}{9}\times10^{-11}\mathrm{F}$
Current density	$1\ \mathrm{CGS}_J=\frac{10^5}{c}\mathrm{A/m^2}=\frac{1}{3}\times10^{-5}\mathrm{A/m^2}$
Magnetic flux	$1\ \mathrm{CGS}_\Phi=1$ maxwell (Mx) $=10^{-8}$ Wb
Magnetic induction	$1\ \mathrm{CGS}_B=1$ gauss (Gs) $=10^{-4}$ T
Inductance	$1\ \mathrm{CGS}_L=c^2\times10^{-9}\mathrm{H}=9\times10^{11}\mathrm{H}$ *
Magnetic field intensity	$1\ \mathrm{CGS}_H=1$ oersted (Oe) $=\frac{1}{4\pi}\times10^3\mathrm{A/m}$

Note. In this table the numerical value of the velocity of light in a vacuum is expressed in centimetres per second, i. e., $c=3\times10^{10}$ cm/s.

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* The unit of inductance can be determined either by the equation $\varepsilon=-L\frac{dI}{dt}$ or the equation $\Phi=LI$; in the CGS system $\Phi=\frac{1}{c}LI$. From the first equation $|L|=\frac{\varepsilon dt}{dI}$, and thus

$$1\mathrm{H}=\frac{1\mathrm{V}\times1\mathrm{s}}{1\mathrm{A}}=\frac{\left(\frac{1}{c}\times10^8\mathrm{CGS}_U\right)1\mathrm{s}}{\frac{c}{10}\mathrm{CGS}_I}=\frac{1}{c^2}\times10^9\ \mathrm{CGS}_L$$

whence $1\ \mathrm{CGS}_L=c^2\times10^{-9}$ H This relation is given in Table 12 in conformity with GOST 8033-56. If we take the second equation, $L=\frac{c\Phi}{I}$, and thus

$$1\mathrm{H}=\frac{c\times1\mathrm{Wb}}{1\mathrm{A}}=\frac{c\,(10^8\mathrm{CGS}_\Phi)}{\frac{c}{10}\mathrm{CGS}_I}=10^9\ \mathrm{CGS}_L$$

whence $1\ \mathrm{CGS}_L=10^{-9}$ H. This relation is given in almost all text-books on physics. In this book the inductance will be expressed only in SI units—henries (H).

Solution. The charge of the ball Q, its capacitance C and potential U are related by the equation

$$C = \frac{Q}{U} \tag{1}$$

where

$$Q = \sigma 4\pi r^2 \tag{2}$$

Besides, the capacitance of a ball is

$$C = 4\pi \varepsilon_0 \varepsilon_r r \tag{3}$$

From Eqs. (1), (2) and (3) we have

$$r = \frac{\varepsilon_0 \varepsilon_r U}{\sigma} \tag{4}$$

In our case $\varepsilon_0 = 8.85 \times 10^{-12}$ F/m, $\varepsilon_r = 1$, $U = 4\ \text{CGS}_U = 12 \times 10^2$ V, $\sigma = 0.138 \frac{\text{CGS}_Q}{\text{cm}^2} = \frac{0.138}{3} \times 10^{-5}\ \text{C/m}^2$. Upon inserting these data in Eq. (4), we obtain

$$r = \frac{8.85 \times 10^{-12} \times 12 \times 10^2 \times 3}{0.138 \times 10^{-5}}\ \text{m} = 23 \times 10^{-3}\ \text{m} = 2.3\ \text{cm}$$

Example 2. The electric induction (displacement) in a plane capacitor is 10^{-5} C/m². What is the surface density of the charge on the plates of this capacitor?

Solution. We have $D = \varepsilon_0 \varepsilon_r E$, but $E = \frac{\sigma}{\varepsilon_0 \varepsilon_r}$, hence

$$D = \varepsilon_0 \varepsilon_r \frac{\sigma}{\varepsilon_0 \varepsilon_r} = \sigma \tag{1}$$

i.e., the induction of the electric field is numerically equal to the surface density of the charge on the capacitor plates. In our case $D = 10^{-5}$ C/m². Hence $\sigma = 10^{-5}$ C/m². Let us now convert the quantities D and σ into CGS units. According to Table 12

$$1\ \text{CGS}_D = \frac{10^5}{4\pi c}\ \text{C/m}^2 \text{ or } 1\ \text{C/m}^2 = \frac{4\pi c}{10^5}\ \text{CGS}_D \tag{2}$$

Hence

$$D = 10^{-5}\ \text{C/m}^2 = 10^{-5} \frac{4\pi c}{10^5}\ \text{CGS}_D = 37.7\ \text{CGS}_D \tag{3}$$

Further, $\sigma = 10^{-5}$ C/m². From Table 12, $1\ \text{C} = \frac{c}{10}\ \text{CGS}_Q$. Besides, 1 m = 10^2 cm, and thus

$$1\ \text{C/m}^2 = \frac{c \times \text{CGS}_Q}{10 \times 10^4\ \text{cm}^2} = 3 \times 10^5 \frac{\text{CGS}_Q}{\text{cm}^2} \tag{4}$$

and

$$\sigma = 10^{-5}\,\mathrm{C/m^2} = 10^{-5}\times 3\times 10^{5}\,\frac{\mathrm{CGS}_Q}{\mathrm{cm^2}} = 3\,\frac{\mathrm{CGS}_Q}{\mathrm{cm^2}} \qquad (5)$$

Hence, the quantities D and σ are numerically equal only in the rationalized MKSA system, and do not coincide in the non-rationalized CGS system. Therefore, when the "coulomb per square metre" is converted into the CGS system, pay attention to the quantity it follows, since, according to Eqs. (2) and (4), we have

$$1\,\mathrm{C/m^2} = 1\,\mathrm{MKSA}_D = \frac{4\pi c}{10^5}\,\mathrm{CGS}_D$$

and

$$1\,\mathrm{C/m^2} = 1\,\mathrm{MKSA}_\sigma = 3\times 10^5\,\mathrm{CGS}_Q/\mathrm{cm^2}$$

Example 3. When a current $I = 4$ A is passed through the winding of a long coil without a core, the magnetic flux through this coil is $\Phi = 250$ Mx. The cross-sectional area of the coil $A = 5\ \mathrm{cm^2}$. What is the number of turns in this coil per unit of length?

Solution. The magnetic flux through a solenoid is determined from the formula $\Phi = \mu_0\mu_r InA$, whence

$$n = \frac{\Phi}{\mu_0\mu_r IA} \qquad (1)$$

In our case, $\Phi = 250\ \mathrm{Mx} = 250\times 10^{-8}$ Wb, $\mu_0 = 12.57\times 10^{-7}$ H/m, $\mu_r = 1$, $I = 4$ A, and $A = 5\ \mathrm{cm^2} = 5\times 10^{-4}\ \mathrm{m^2}$. Upon inserting these data into Eq. (1), we obtain

$$n = \frac{250\times 10^{-8}}{12.57\times 10^{-7}\times 4\times 5\times 10^{-4}}\,\frac{1}{\mathrm{m}} = 1{,}000\ \mathrm{m^{-1}}$$

Example 4. A plane capacitor is periodically charged by a storage battery to the potential difference $U = 80$ V and discharged through a solenoid (without a core). The capacitor is switched over 100 times a second. The area of the capacitor plates $A = 100\ \mathrm{cm^2}$, and the distance between the plates $d = 4.7$ mm. The space between the plates is filled with paraffin ($\varepsilon_r = 2.1$). The solenoid $l = 25$ cm long has $N = 250$ turns. Find the mean magnetic induction in the solenoid.

Solution. Upon each discharge of the capacitor, the solenoid will let through electricity amounting to $Q = CU$, where C is the capacitance equal to $\frac{\varepsilon_0\varepsilon_r A}{d}$. The mean intensity of the current passing through the solenoid is $I = Qn$, where n is the number of capacitor discharges a second. The intensity of the magnetic field inside the solenoid $H = I\frac{N}{l}$ and the magnetic induction in the solenoid $B = \mu_0\mu_r H$.

From all these equations we finally obtain

$$B=\frac{\mu_0\mu_r\varepsilon_0\varepsilon_r AUnN}{ld} \tag{1}$$

In our case $\mu_0=12.57\times10^{-7}$ H/m, $\mu_r=1$, $\varepsilon_0=8.85\times10^{-12}$ F/m, $\varepsilon_r=2.1$, $A=100$ cm^2$=100\times10^{-4}$ m^2, $U=80$ V, $n=100\,\frac{1}{\text{s}}$, $N=250$, $l=25$ cm$=0.25$ m and $d=4.7$ mm$=4.7\times10^{-3}$ m. Upon inserting these data into Eq. (1), we get

$$B=\frac{12.57\times10^{-7}\times1\times8.85\times10^{-12}\times2.1\times10^{-2}\times80\times10^{2}\times250}{0.25\times4.7\times10^{-3}}\ \text{T}=$$
$$=3.97\times10^{-10}\ \text{T}$$

By using Table 12, the answer can also be obtained in gausses

$$B=3.97\times10^{-6}\ \text{Gs}$$

9. Electrostatics

According to the Coulomb law, the force acting between two charged bodies whose dimensions are small with respect to the distance between them can be found from the formula

$$F=\frac{Q_1Q_2}{4\pi\varepsilon_0\varepsilon_r r^2}$$

where Q_1 and Q_2= electric charges of the bodies
r = distance between them
ε_r= relative permittivity of the medium
ε_0= permittivity in a vacuum in the MKSA system equal to 8.85×10^{-12} F/m.

The intensity of an electric field is determined from the formula

$$E=\frac{F}{Q}$$

where F is the force acting on the charge Q.

The intensity of the field of a point charge is

$$E=\frac{Q}{4\pi\varepsilon_0\varepsilon_r r^2}$$

The intensity of an electric field induced by several charges (for example, a dipole field) can be found by the geometrical summation of the fields.

From the Gauss theorem, the intensity flux through any closed surface is

$$N_E=\frac{\sum Q}{\varepsilon_0\varepsilon_r}$$

where ΣQ is the algebraic sum of the charges present inside this surface. Accordingly, the electric induction flux through any closed surface is equal to

$$N_D = \sum Q$$

The Gauss theorem can be used to find the intensity of the electric field formed by various charged bodies.

The intensity of a field formed by a charged infinitely long filament is

$$E = \frac{\tau}{2\pi\varepsilon_0\varepsilon_r a}$$

where τ is the linear density of the charge on the filament and a is the distance from the filament. If the filament is finite in length, the intensity of the field at a point on the perpendicular erected from the middle of the filament at a distance of a from it will be

$$E = \frac{\tau \sin\theta}{2\pi\varepsilon_0\varepsilon_r a}$$

where θ is the angle between the normal to the filament and the radius-vector drawn from the point to the end of the filament.

The intensity of the field formed by a charged infinitely long plane is

$$E = \frac{\sigma}{2\varepsilon_0\varepsilon_r}$$

where σ is the surface density of the charge on a plane.

If the surface is made in the form of a disk with a radius of R, the intensity of the field at a point on the perpendicular erected from the centre of the disk at the distance a from it is equal to

$$E = \frac{\sigma}{2\varepsilon_0\varepsilon_r}\left(1 - \frac{a}{\sqrt{R^2 + a^2}}\right)$$

The intensity of the field formed by oppositely charged parallel and infinite planes (the field of a plane capacitor) will be

$$E = \frac{\sigma}{\varepsilon_0\varepsilon_r}$$

The intensity of the field formed by a charged ball is

$$E = \frac{Q}{4\pi\varepsilon_0\varepsilon_r r^2}$$

where Q is the charge of the ball with a radius of R, and r is the distance from the ball centre; $r > R$.

The electric induction of the field D can be found from the formula

$$D = \varepsilon_0\varepsilon_r E = \sigma$$

The difference of potentials between two points of an electric field is determined by the work which must be performed to transfer a unit of positive charge from one point to the other

$$U_1 - U_2 = \frac{W}{Q}$$

The potential of the field of a point charge is

$$U = \frac{Q}{4\pi\varepsilon_0\varepsilon_r r}$$

where r is the distance from the charge.

The intensity of an electric field and the potential are related by the formula

$$E = -\frac{dU}{dr}$$

For a homogeneous field—the field of a plane capacitor

$$E = \frac{U}{d}$$

where U is the difference of potentials between the capacitor plates and d is the distance between them.

The potential of an isolated conductor and its charge are related by the formula

$$Q = CU$$

where C is the capacitance of the conductor.

The capacitance of a plane capacitor is

$$C = \frac{\varepsilon_0\varepsilon_r A}{d}$$

where A is the area of each plate of the capacitor.

The capacitance of a spherical capacitor

$$C = \frac{4\pi\varepsilon_0\varepsilon_r rR}{R - r}$$

where r is the radius of the internal sphere and R is the radius of the external one. In a particular case, when $R = \infty$,

$$C = 4\pi\varepsilon_0\varepsilon_r r$$

is the capacitance of an isolated sphere.

The capacitance of a cylindrical capacitor

$$C = \frac{2\pi\varepsilon_0\varepsilon_r l}{\log_e \frac{R}{r}}$$

where l is the height of the coaxial cylinders, and r and R are the radii of the internal and external cylinders, respectively.

The capacitance of a system of capacitors is

$$C=C_1+C_2+C_3+\ \dots$$

when the capacitors are connected in parallel, and

$$\frac{1}{C}=\frac{1}{C_1}+\frac{1}{C_2}+\frac{1}{C_3}+\dots$$

when the capacitors are connected in series.

The energy of an isolated charged conductor can be found from one of the following three formulas

$$W_e=\frac{1}{2}QU, \qquad W_e=\frac{1}{2}CU^2, \qquad W_e=\frac{Q^2}{2C}$$

In the particular case of a plane capacitor

$$W_e=\frac{\varepsilon_0\varepsilon_r AU^2}{2d}=\frac{\varepsilon_0\varepsilon_r E^2Ad}{2}=\frac{\sigma^2Ad}{2\varepsilon_0\varepsilon_r}$$

where A = area of each plate
σ = surface density of charge on the plates
U = difference of potentials between the plates.

The quantity

$$W_0=\frac{\varepsilon_0\varepsilon_r E^2}{2}=\frac{ED}{2}$$

is known as the volume energy density of an electric field.

The force of attraction of the plates in a plane capacitor is

$$F=\frac{\varepsilon_0\varepsilon_r E^2A}{2}=\frac{\varepsilon_0\varepsilon_r AU^2}{2d^2}=\frac{\sigma^2A}{2\varepsilon_0\varepsilon_r}$$

9.1. Find the force of attraction between the nucleus of a hydrogen atom and an electron. The radius of the hydrogen atom is 0.5×10^{-8} cm and the charge of the nucleus is equal in magnitude and opposite in sign to that of the electron.

9.2. Two point charges in air at a distance of 20 cm from each other interact with a certain force. At what distance from each other should these charges be placed in oil to obtain the same force of interaction?

9.3. Plot a diagram showing how the force of interaction between two point charges depends on the distance between them within the limits of $2\leqslant r\leqslant 10$ cm at intervals of 2 cm. The charges are equal to $2\times\times 10^{-8}$ C and 3×10^{-8} C, respectively.

9.4. How many times is the force of Newtonian attraction between two protons smaller than the force of Coulomb repulsion? The charge of a proton is numerically equal to that of an electron.

9.5. Calculate the force of electrostatic repulsion between the nucleus of a sodium atom and a proton bombarding it, assuming that the proton approaches the sodium atom nucleus by 6×10^{-12} cm. The charge of the sodium nucleus is 11 times greater than that of the proton. Disregard the influence of the electron shell of the sodium atom.

9.6. Two identical charged metal balls weighing 0.2 kgf each are at a certain distance from each other. Find the charge of the balls if their electrostatic energy at this distance is one million times greater than their mutual gravitational energy.

9.7. How many times is the energy of electrostatic interaction of two particles with the charge Q and the mass m greater than their gravitational interaction? Solve the problem for: (1) electrons, and (2) protons.

9.8. Plot a diagram showing how the potential electrostatic energy of two point charges depends on the distance between them within the limits $2\leqslant r\leqslant 10$ cm at intervals of 2 cm. The charges are $Q_1=10^{-9}$ C and $Q_2=3\times10^{-9}$ C; $\varepsilon_r=1$. Plot a diagram for (1) like charges, and (2) unlike charges.

9.9. Find the intensity of an electric field at a point lying at the middle between the point charges $Q_1=8\times10^{-9}$ C and $Q_2=-6\times10^{-9}$ C. The distance between the charges is $r=10$ cm; $\varepsilon_r=1$.

9.10. A negative charge is placed in the centre of a square each vertex of which contains a charge of 7 CGS_Q. Find the magnitude of the negative charge if the resulting force acting on each charge is equal to zero.

9.11. The vertices of a regular hexagon have three positive and three negative charges. Find the intensity of the electric field at the centre of the hexagon when these charges are arranged in different combinations. The magnitude of each charge is $Q=4.5$ CGS_Q. Each side of the hexagon is 3 cm.

9.12. Solve the previous problem if all the six charges at the vertices of the hexagon are positive.

9.13. The distance between two point charges $Q_1=22.5$ CGS_Q and $Q_2=-44.0$ CGS_Q is 5 cm. Find the intensity of the electric field at a point which is at a distance of 3 cm from the positive charge and 4 cm from the negative one.

9.14. Two balls of the same radius and weight are suspended on threads so that their surfaces are in contact. A charge of $Q_0=4\times10^{-7}$ C applied to the balls makes them repel each other to an angle of 60°. Find the weight of the balls if the distance from the point of suspension to the centre of a ball is 20 cm.

9.15. Two balls of the same radius and weight are suspended on two threads so that their surfaces are in contact. What charge should be applied to the balls for the tension of the threads to become equal to

0.098 N? The distance from the point of suspension to the centre of a ball is 10 cm and each ball weighs 5×10^{-3} kgf.

9.16. Find the density of the material of the balls in Problem 9.14 if the angle of divergence of the threads becomes 54° when the balls are immersed in kerosene.

9.17. Two charged balls of the same radius and weight suspended on threads of equal length are immersed into a liquid dielectric having a density of ρ_1 and a dielectric constant (relative permittivity) of ε_r. What should the density ρ of the material of the balls be for the angles of divergence of the threads in the air and in the dielectric to be the same?

9.18. In Fig. 16, line AA is a charged infinite plane with a charge surface density of 4×10^{-9} C/cm², and B is a ball with a like charge having a mass of 1 g and a charge of 3 CGS_Q. What angle will be formed between plane AA and the thread which the ball is suspended from?

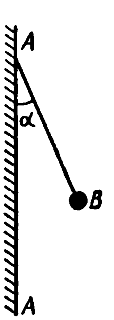


Fig. 16

9.19. In Fig. 16, line AA is a charged infinite plane and B is a ball with a like charge weighing $G=4\times10^{-5}$ kgf and having a charge of $Q=6.67\times10^{-10}$ C. The tension of the thread which the ball is suspended from is $F=4.9\times10^{-4}$ N. Find the charge surface density on plane AA.

9.20. Determine the force acting on a charge of 2 CGS_Q if it is located: (1) at a distance of 2 cm from a charged thread with a charge linear density of 2×10^{-9} C/cm, (2) in the field of a charged plane with a charge surface density of 2×10^{-9} C/cm², (3) at a distance of 2 cm from the surface of a charged ball with a radius of 2 cm and a charge surface density of 2×10^{-9} C/cm². In all three cases the dielectric constant of the medium $\varepsilon_r = 6$.

9.21. Plot on one diagram curves showing how the intensity of an electric field depends on the distance within the limits $1\leqslant r\leqslant5$ cm at intervals of 1 cm if the field is formed by: (1) a point charge of 100 CGS_Q, (2) an infinitely long charged thread with a charge linear density of 1.67×10^{-8} C/cm, (3) an infinite charged plane with a charge surface density of 2.5×10^{-9} C/cm².

9.22. Find the intensity of an electric field at a distance of 2×10^{-8} cm from a monovalent ion. The ion is point-charged.

9.23. What force does the electric field of a charged infinite plane act with on each metre of a charged infinitely long filament placed in this field? The charge linear density on the filament is 3×10^{-8} C/cm and its surface density on the plane is 2×10^{-9} C/cm².

9.24. Determine the force (per unit length) which two like-charged infinitely long filaments with the same charge linear density of 3×10^{-8} C/cm and spaced 2 cm apart will repel each other with. What work (per unit length) must be performed to move these filaments together until they are 1 cm apart?

9.25. Two long like-charged filaments are spaced $a=10$ cm apart. The charge linear density on the filaments is $\tau_1=\tau_2=10^{-7}$ C/cm. Find the magnitude and direction of the intensity of the resulting electric field at a point 10 cm from each filament.

9.26. Determine the force (per unit area) which two like-charged infinitely long planes having the same charge surface density of $3\times\times10^{-8}$ C/cm^2 will repel each other with.

9.27. A copper ball 1 cm in diameter is immersed in oil with a density of $\rho=800$ kg/m^3. What is the charge of the ball if in a homogeneous electric field it will be suspended in the oil? The electric field is directed vertically upward and its intensity $E=36{,}000$ V/cm.

9.28. A charged drop of mercury is in equilibrium in a plane horizontal capacitor and the intensity of the electric field $E=600$ V/cm. The charge of the drop is 2.4×10^{-9} CGS_Q. Find the radius of the drop.

9.29. Prove that the electric field formed by a charged filament of finite length changes, in the limit cases, into the electric field of: (1) an infinitely long filament, and (2) a point charge.

9.30. A charged filament is 25 cm long. At what maximum distance from the filament (for points lying on a perpendicular erected from its middle) can the electric field be regarded as the field of an infinitely charged filament? The error should not exceed 5 per cent.

Note. The error will be $\delta=\frac{E_2-E_1}{E_2}$, where E_2 is the intensity of the electric field of the infinitely long filament and E_1 that of the field of the filament with a finite length.

9.31. The intensity of an electric field is 1,500 V/cm at point A at a distance of 5 cm from an infinitely long charged filament. (1) At what limit length of the filament will the determined intensity be true to an accuracy of 2% if point A is on a perpendicular erected from the middle of the filament? (2) What will the intensity of the electric field be at point A if the filament is 20 cm long? The charge linear densities on the filament and on an infinitely long filament are assumed to be the same. (3) Find the charge linear density on the filament.

9.32. A ring made of wire with a radius of $R=10$ cm is charged negatively and carries a charge of $Q=-5\times10^{-9}$ C. (1) Find the intensity of an electric field on the axis of the ring at points lying at a distance of l equal to 0, 5, 8, 10 and 15 cm from the ring centre. Plot a diagram $E=f(l)$. (2) At what distance l from the ring centre will the intensity of the electric field be maximum?

9.33. The intensity of an electric field on the axis of a charged ring reaches its maximum at a distance of $l=l_{max}$ from the centre of the ring. How many times will the intensity of the electric field at a point located at a distance of $l=0.5\ l_{max}$ from the centre of the ring be smaller than the maximum intensity?

9.34. Prove that the electric field formed by a charged disk transforms, in the limit cases, into the electric field of: (1) an infinitely long plane, and (2) a point charge.

9.35. The diameter of a charged disk is 25 cm. At what maximum distance from the disk along a normal to its centre may an electric field be regarded as the field of an infinitely long plane? The error should not exceed 5%.

Note. The error $\delta=\frac{E_2-E_1}{E_2}$, where E_1 is the intensity of the field induced by the disk, and E_2 that of the field induced by an infinite plane.

9.36. Find the intensity of an electric field at point A, at a distance of $a=5$ cm from a charged disk (on a normal to its centre). (1) What is the maximum radius of the disk at which the field at point A does not differ by more than 2% from that of an infinitely long plane? (2) What is the intensity of the field at point A if the disk radius R is 10 times greater than distance a? (3) How many times is the intensity determined at this point smaller than that of an infinitely long plane?

9.37. Two parallel unlike-charged disks with the same charge surface density are at a distance of $h=1$ cm from each other. (1) What can the limit value of the radii R of the disks be for the field between the centres of the disks to differ from the field of a plane capacitor by not more than 5%? (2) What error is made in assuming the field for these points to be equal to that of a plane capacitor when $\frac{R}{h}=10$?

9.38. A ball with a mass of 40 mg having a positive charge of 10^{-9} C moves at a velocity of 10 cm/s. Up to what distance can the ball approach a positive point charge equal to 4 CGS_Q?

9.39. Up to what distance can two electrons approach each other if they are moving toward each other with a relative velocity of 10^8 cm/s?

9.40. A proton (hydrogen atom nucleus) moves with a velocity of 7.7×10^8 cm/s. Up to what minimum distance can it approach the nucleus of an aluminium atom? The charge of the nuclei of aluminium atoms $Q=Ze_0$, where Z is the ordinal number of the atom in the Mendeleyev (Periodic) Table and e_0 the charge of a proton numerically equal to that of an electron. Consider the mass of the proton to be equal to that of a hydrogen atom. The proton and the nucleus of the aluminium atom are to be regarded as point charges. Disregard the effect of the electronic shell of the aluminium atom.

9.41. In bombarding an immobile sodium nucleus with an α-particle, the repelling force between them is 14 kgf. (1) Up to what minimum distance did the α-particle approach the sodium atom nucleus?

(2) What was the velocity of the α-particle? Disregard the effect of the electronic shell of the sodium atom.

9.42. Two balls with charges of $Q_1=20$ CGS_Q and $Q_2=40$ CGS_Q are at a distance of $r_1=40$ cm from each other. What work must be performed to reduce this distance to $r_2=25$ cm?

9.43. A ball with a radius of 1 cm and a charge of 4×10^{-8} C is immersed in oil. Plot a diagram of the ratio $U=f(x)$ for the points of a field at a distance of x equal to 1, 2, 3, 4 and 5 cm from the ball surface.

9.44. Determine the potential of a field point which is at a distance of 10 cm from the centre of a charged ball with a radius of 1 cm. Solve the problem when: (1) the charge surface density on the ball is 10^{-11} C/cm^2, and (2) the potential of the ball is 300 V.

9.45. What work is performed when a point charge of 2×10^{-8} C is transferred from infinity to a point at a distance of 1 cm from the surface of a ball with a radius of 1 cm and a charge surface density of $\sigma=10^{-9}$ C/cm^2?

9.46. A ball with a mass of 1 gramme and a charge of 10^{-8} C moves from point A whose potential is 600 V to point B whose potential is zero. What was the velocity of the ball at point A if at point B it is 20 cm/s?

9.47. Find the velocity v of an electron which passed through a potential difference of U equal to 1, 5, 10, 100 and 1,000 V.

9.48. During radioactive decay an α-particle flies from the nucleus of a polonium atom at a velocity of 1.6×10^9 cm/s. Find the kinetic energy of the α-particle and the difference of potentials of a field in which such a particle from a state of rest can be accelerated to the same velocity.

9.49. A point charge $Q=2$ CGS_Q is at a distance of $r_1=4$ cm from an infinitely long charged filament. Under the action of a field the charge moves to the distance $r_2=2$ cm, and the work $W=50$ erg is performed. Find the linear density of the filament charge.

9.50. An electric field is formed by a positively charged infinitely long filament. An α-particle moving under the action of this field from a point at a distance of $x_1=1$ cm from the filament to a point $x_2=4$ cm changes its velocity from 2×10^5 to 3×10^6 m/s. Find the linear density of the charge on the thread.

9.51. An electric field is formed by a positively charged infinitely long filament with a charge linear density of 2×10^{-9} C/cm. What velocity will be imparted to an electron by the field if the electron approaches the filament from a distance of 1 cm to a distance of 0.5 cm from the filament?

9.52. A point charge $Q=2$ CGS_Q is near a charged infinitely long plane. The action of the field moves the charge along a force line over a distance of 2 cm and the work $W=50$ erg is performed. Find the charge surface density on the plane.

9.53. The difference of potentials between the plates of a plane (plane-parallel) capacitor is 90 V. The area of each plate is 60 cm^2 and its charge 10^{-9} C. What is the distance between the plates?

9.54. A plane capacitor can be used as a sensitive microbalance. A particle with a charge of $Q=1.44\times10^{-9}$ CGS_Q is inside a horizontal plane capacitor with the plates spaced $d=3.84$ mm apart. For the particle to be in equilibrium between the capacitor plates, a potential difference of $U=40$ V must be applied. Determine the mass of the particle.

9.55. A charged drop with a mass of $m=5\times10^{-11}$ gramme is in a plane horizontal capacitor with the plates spaced $d=1$ cm apart. When an electric field is absent, the air resistance causes the drop to fall with a certain constant velocity. If a potential difference of $U=600$ V is applied to the capacitor plates, the drop will fall with half the velocity. Find the charge of the drop.

9.56. A speck of dust falls between two vertical plates at an equal distance from them. In view of the air resistance, the speck of dust falls at a constant velocity of 2 cm/s. What time is required for the speck of dust to reach one of the plates after a potential difference of $U=3{,}000$ V is applied to the plates? What vertical distance l will be covered by the speck of dust before it reaches the plate? The distance between the plates $d=2$ cm, the mass of the speck of dust $m=2\times10^{-9}$ g and its charge $Q=6.5\times10^{-17}$ C.

9.57. Solve the previous problem, disregarding the force of friction (a vacuum capacitor).

9.58. A charged drop of oil is in a plane horizontal capacitor whose plates are spaced $d=1$ cm apart. When an electric field is absent, the drop falls with a constant velocity of $v_1=0.011$ cm/s. If a difference of potentials of $U=150$ V is applied to the plates, the drop will fall with the velocity $v_2=0.043$ cm/s. Find the radius of the drop and its charge. The dynamic viscosity of the air $\eta=1.82\times10^{-5}$ $N\cdot s/m^2$. The density of the oil is $\Delta\rho=900$ kg/m^3 greater than that of the gas in which the drop falls.

9.59. A charged elder ball with a mass of 0.1 gramme hangs on a thread between two vertical plates spaced 1 cm apart. A potential difference of 1,000 V applied to the plates deflects the thread with the ball through an angle of 10°. Find the charge of the ball.

9.60. A soap bubble with a charge of 2.22×10^{-10} C is in equilibrium in the field of a horizontal plane capacitor. Find the difference of potentials between the capacitor plates if the mass of the bubble is 0.01 gramme and the distance between the plates 5 cm.

9.61. The distance between the plates of a plane capacitor is 4 cm. An electron begins to move from the negative plate at the same moment as a proton begins to move from the positive plate. At what distance from the positive plate will they meet?

9.62. The distance between the plates of a plane capacitor is 1 cm. A proton and an α-particle begin to move simultaneously from one of the plates. What distance will be covered by the α-particle during the time in which the proton travels the entire path from one plate to the other?

9.63. Having covered the path from one plate to the other in a plane capacitor, an electron acquires a velocity of 10^8 cm/s. The distance between the plates is 5.3 mm. Find: (1) the difference of potentials between the plates, (2) the intensity of the electric field inside the capacitor, (3) the charge surface density on the plates.

9.64. An electric field is formed by two parallel plates at a distance of 2 cm from each other. The difference of potentials between them is 120 V. What velocity will be imparted to an electron under the action of the field after it has covered a distance of 3 mm along a force line?

9.65. An electron in a homogeneous electric field receives an acceleration of 10^{14} cm/s^2. Find: (1) the intensity of the electric field, (2) the velocity imparted to the electron during 10^{-6} second of its motion if its initial velocity is zero, (3) the work done by the forces of the electric field during this time, (4) the potential difference passed by the electron.

9.66. An electron flies from one plate of a plane capacitor to the other. The difference of potentials between the plates is 3 kV and the distance between them 5 mm. Find: (1) the force acting on the electron, (2) the electron acceleration, (3) the velocity with which the electron reaches the second plate, (4) the charge surface density on the capacitor plates.

9.67. An electron flies with a certain initial velocity v_0 into a plane capacitor parallel to the plates and at an equal distance from them. A potential difference of $U=300$ V is applied to the capacitor plates. The distance between the plates $d=2$ cm and the capacitor length $l=10$ cm. What is the greatest initial velocity v_0 of the electron at which it will not fly out of the capacitor? Solve the same problem also for an α-particle.

9.68. An electron flies into a plane horizontal capacitor parallel to the plates and at an equal distance from them. The distance between the plates $d=4$ cm and the intensity of the electric field in the capacitor $E=1$ V/cm. (1) In how much time will the electron get onto one of the plates after it has entered the capacitor? (2) At what distance from the beginning of the capacitor will the electron get onto the plate if it has been accelerated by a potential difference of 60 V?

9.69. An electron flies into a plane horizontal capacitor parallel to the plates with a velocity of 9×10^6 m/s. Find the total, normal and tangential accelerations of the electron in 10^{-8} second after it begins to move in the capacitor. The potential difference between them is 1 cm.

9.70. A proton and an α-particle move with the same velocity and fly into a plane capacitor parallel to the plates. How many times will the deflection of the proton by the capacitor field be greater than that of the α-particle?

9.71. A proton and an α-particle accelerated by the same potential difference fly into a plane capacitor parallel to the plates. How many times will the deflection of the proton by the capacitor field be greater than that of the α-particle?

9.72. An electron flies into a plane horizontal capacitor parallel to its plates with a velocity of $v_x=10^7$ m/s. The intensity of the field in the capacitor $E=100$ V/cm and the length of the capacitor $l=5$ cm. Find the magnitude and direction of the velocity of the electron when it flies out of the capacitor.

9.73. A beam of electrons accelerated by a potential difference of $U=300$ V passes through an uncharged horizontal plane capacitor parallel to its plates and produces a luminous spot on a fluorescent screen arranged at a distance of $l_1=12$ cm from the end of the capacitor. When the capacitor is charged, the spot on the screen is displaced by $y=3$ cm. Find the potential difference U_1 applied to the capacitor plates. The length of the capacitor $l=6$ cm and the distance between its plates $d=1.4$ cm.

9.74. An electron moves in a plane horizontal capacitor parallel to its plates with a velocity of 3.6×10^4 km/s. The intensity of the field inside the capacitor is 37 V/cm and its plates are 20 cm long. Over what distance will the electron be displaced in a vertical direction under the action of the electric field during its motion in the capacitor?

9.75. A proton flies into a plane horizontal capacitor parallel to its plates with a velocity of 1.2×10^5 m/s. The intensity of the field inside the capacitor is 30 V/cm and the length of its plates 10 cm. How many times will the velocity of the proton when it flies out of the capacitor be greater than its initial velocity?

9.76. A potential difference of 150 V is applied between the plates of a plane capacitor spaced 5 mm apart. One of the plates is in contact with a plane-parallel porcelain plate 3 mm thick. Find the intensity of the electric field in air and in the porcelain.

9.77. Find the capacitance of the Earth. The radius of the Earth is 6,400 km. By how much will the Earth's potential change if it receives a quantity of electricity equal to 1 C?

9.78. A ball with a radius of 2 cm is charged negatively to a potential of 2,000 V. Find the mass of all the electrons forming the charge imparted to the ball during its charging.

9.79. Eight charged water drops each with a radius of 1 mm and a charge of 10^{-10} C merge into a single drop. Find the potential of the big drop.

9.80. Two balls of the same radius $R=1$ cm and weight $G=4\times\times 10^{-5}$ kgf are so suspended on threads of equal length that their surfaces are in contact. After the balls are charged, the threads move apart over a certain angle and their tension becomes $F=4.9\times 10^{-4}$ N. Find the potential of the charged balls if the distance from the point of suspension to the centre of each ball is $l=10$ cm.

9.81. A ball charged to a potential of 792 V has a charge surface density equal to 3.33×10^{-7} C/m^2. What is the radius of the ball?

9.82. Find: (1) the relationship between the radius R of a ball and the maximum potential U which it can be charged to in air if under normal pressure its discharge in air occurs when the intensity of the electric field $E_0=30$ kV/cm, (2) the maximum potential of a ball with a diameter of 1 m.

9.83. Two balls of the same radius $R=1$ cm and weight $G=0.15$ kgf are charged to the same potential $U=3$ kV and are at a certain distance r_1 from each other. Their mutual gravitational energy is 10^{-11} J. The balls approach each other until the distance between them is r_2. The work necessary to bring the balls closer together is 2×10^{-6} J. Find the electrostatic energy of the balls when the distance between them is r_2.

9.84. The area of each plate in a plane capacitor is 1 m^2 and the distance between the plates is 1.5 mm. Find the capacitance of this capacitor.

9.85. The capacitor of the previous problem is charged to a potential of 300 V. Find the charge surface density on its plates.

9.86. To make a capacitor with a capacitance of 2.5×10^{-4} μF, tinfoil disks are glued onto both sides of paraffined paper 0.05 mm thick. Determine the diameter of these disks.

9.87. The area of the plates in a plane air capacitor is 100 cm^2 and the distance between them is 5 mm. A potential difference of 300 V is applied to the plates. After the capacitor is disconnected from the source of power, the space between the plates is filled with ebonite. (1) What is the potential difference between the plates after they are filled? (2) What is the capacitance of the capacitor before and after it is filled? (3) What is the surface density of the charge on the plates before and after filling?

9.88. Solve the previous problem for the case when the space between the plates is filled with the insulator with the source of power connected.

9.89. A potential difference of $U=300$ V is applied between the plates of a plane capacitor spaced $d=1$ cm apart. A plane-parallel glass plate with a thickness of $d_1=0.5$ cm and a plane-parallel paraffin plate with a thickness of d_2=0.5 cm are placed in the space between the capacitor plates. Find: (1) the intensity of the electric field in each layer, (2) the drop of potential in each layer, (3) the capacitance of the capacitor if the area of the plates is A=100 cm^2, (4) the surface density of the charge on the plates.

9.90. A potential difference of 100 V is applied between the plates of a plane capacitor spaced 1 cm apart. One of the plates is in contact with a plane-parallel plate of crystalline thallium bromide (ε_r=173) 9.5 mm thick. After the capacitor is disconnected from the source of power, the crystalline plate is removed. What will the potential difference between the plates be after this is done?

9.91. A coaxial electric cable consists of a core and a concentric cylindrical sheath with insulation between them. Find the capacitance of a unit length of such a cable (in microfarads per metre) if the radius of the core is 1.3 cm, the radius of the sheath is 3.0 cm, and the dielectric constant of the insulation (ε_r) is 3.2.

9.92. The radius of the core of a coaxial cable is 1.5 cm and the radius of the sheath is 3.5 cm. A potential difference of 2,300 V is applied between the core and the sheath. Calculate the intensity of the electric field at a distance of 2 cm from the cable axis.

9.93. The radius of the internal cylinder of a cylindrical air capacitor is r=1.5 cm and that of the external cylinder R=3.5 cm. A potential difference of U=2,300 V is applied between the cylinders. What velocity will be imparted to an electron moving from a distance of l_1=2.5 cm to a distance of l_2=2 cm from the cylinder axis under the action of the field of this capacitor?

9.94. A cylindrical capacitor consists of an internal cylinder with a radius of r=3 mm, two layers of insulation and an external cylinder with a radius of R=1 cm. The first insulation layer with a thickness of d_1=3 mm adjoins the internal cylinder. Find the ratio between the potential drops in these layers.

9.95. Photoelectric phenomena are studied with the aid of a concentric spherical capacitor consisting of a central cathode—a metal ball with a diameter of 1.5 cm—and an anode—the internal surface of a spherical flask 11 cm in diameter silver plated inside. The air is pumped out of the flask. Find the capacitance of such a capacitor.

9.96. What is the potential of a sphere with a radius of 3 cm if: (1) it receives a charge of 10^{-9} C, (2) it is surrounded by another sphere with a radius of 4 cm concentric with the first one and connected to the earth?

9.97. Find the capacitance of a concentric spherical capacitor consisting of two spheres with radii of R_1=10 cm and R_2=10.5 cm. The space between the spheres is filled with oil. What should the radius of a ball placed in the oil be for it to have the same capacitance?

9.98. The radius of the internal sphere of a concentric spherical air capacitor is $R_1=1$ cm and the radius of the external sphere $R_2=$ $=4$ cm. A potential difference of $U=3{,}000$ V is applied between the spheres. Find the intensity of the electric field at a distance of $x=3$ cm from the centre of the spheres.

9.99. The radius of the internal sphere of a concentric spherical air capacitor is $R_1=1$ cm and the radius of the external sphere $R_2=4$ cm. A potential difference of $U=3{,}000$ V is applied between the spheres. What velocity will be imparted to an electron when it approaches the centre of the spheres from a distance of $r_1=3$ cm to a distance of $r_2=2$ cm?

9.100. Find the capacitance of a system of capacitors (Fig. 17). The capacitance of each capacitor is 0.5 μF.

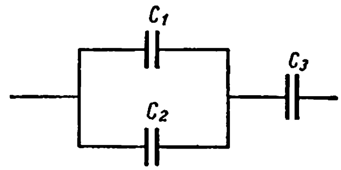


Fig. 17

9.101. The capacitance of two capacitors was compared with the aid of an electrometer. The capacitors were charged to potentials of $U_1=300$ V and $U_2=100$ V, and were connected in parallel. The potential difference between the plates measured by the electrometer was $U=250$ V. Find the capacitance ratio $\frac{C_1}{C_2}$.

9.102. The potential difference between points A and B (Fig. 18) is 0.02 CGS_U. The capacitance of the first capacitor is 2 μF and that of the second 4 μF. Find the charge and the potential difference on the plates of each capacitor.

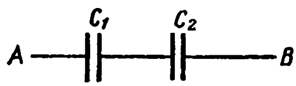


Fig. 18

9.103. Within what limits can the capacitance of a system consisting of two capacitors change if the capacitance of one is constant and equal to 3.33×10^{-9} F, and that of the other can vary from 20 CGS_C to 500 CGS_C?

9.104. Within what limits can the capacitance of a system consisting of two adjustable capacitors change if the capacitance of each can vary from 10 to 450 pF?

9.105. A capacitor with a capacitance of 20 μF is charged to a potential of 100 V. Find the energy of this capacitor.

9.106. A sphere with a radius of 1 m is charged to a potential of 30,000 V. Find the energy of the charged ball.

9.107. A sphere immersed in kerosene has a potential of 4,500 V and a charge surface density of 3.4 CGS_Q/cm^2. Find: (1) the radius, (2) the charge, (3) the capacitance and (4) the energy of the ball.

9.108. After the power source is cut off, sphere A with a radius of 10 cm charged to a potential of 3,000 V is connected by means of a wire (whose capacitance may be neglected) first to a remote uncharged sphere B and then, after it is disconnected from B, to a remote uncharged sphere C. The radii of spheres C and B are 10 cm. Find: (1) the initial energy of sphere A, (2) the energy of spheres A and B after connection, and the work of the discharge during connection (3) the energy of spheres A and C after connection, and the work of the discharge during connection.

9.109. Two metal spheres, one with a charge of 10^{-8} C and a radius of 3 cm, and the other with a radius of 2 cm and a potential of 9,000 V, are connected by means of a wire whose capacitance may be neglected. Find: (1) the potential of the first sphere before discharge, (2) the charge of the second sphere before discharge, (3) the energy of each sphere before discharge, (4) the charge and potential of the first sphere after discharge, (5) the charge and potential of the second sphere after discharge, (6) the energy of the spheres connected by the wire, (7) the work of the discharge.

9.110. A charged sphere A having a radius of 2 cm is brought into contact with an uncharged sphere B having a radius of 3 cm. After the spheres are disconnected, the energy of sphere B is 0.4 J. What charge was on sphere A before the contact?

9.111. The plates of a plane capacitor, each 100 cm^2 in area, are attracted towards each other with a force of 3×10^{-3} kgf. The space between the plates is filled with mica. Find: (1) the charges on the plates, (2) the intensity of the field between the plates, (3) the energy in a unit volume of the field.

9.112. A thin sheet of mica is inserted between the plates of a plane capacitor. What pressure will act on this sheet when the intensity of the electric field is 10 kV/cm?

9.113. An absolute electrometer is made in the form of a plane capacitor whose lower plate is immobile and whose upper one is suspended from a balance beam. When the capacitor is not charged, the distance between the plates $d=1$ cm. What potential difference was applied between the plates if a weight of $G=5.1\times10^{-3}$ kgf had to be placed on the other pan of the balance to preserve the same distance $d=1$ cm? The area of the plates $A=50$ cm^2.

9.114. The difference of potentials between the plates of a plane capacitor each with an area of 100 cm^2 is 280 V. The charge surface density on the plates is 4.95×10^{-11} C/cm^2. Find: (1) the intensity of the field inside the capacitor, (2) the distance between the plates, (3) the velocity imparted to an electron after it has traveled from one plate to the other in the capacitor, (4) the energy of the capacitor, (5) the capacitance of the capacitor, (6) the force of attraction of the capacitor plates.

9.115. The area of the plates in a plane air capacitor is 100 cm^2 and the distance between them 5 mm. Find the difference of potentials applied to the plates if 4.19×10^{-3} J of heat was evolved during the discharge of the capacitor.

9.116. A plane air capacitor with its plates spaced 2 cm apart is charged to a potential of 3,000 V. What will the field intensity of the capacitor be if the plates are moved apart to a distance of 5 cm without disconnecting the power source? Calculate the energy of the capacitor before and after the plates are moved apart. The area of the plates is 100 cm^2.

9.117. Solve the previous problem if the power source is first disconnected and then the plates of the capacitor are moved apart.

9.118. A plane air capacitor in which the plates 100 cm^2 in area are spaced 1 mm apart is charged to 100 V. The plates are then moved apart to a distance of 25 mm. Find the energy of the capacitor before and after the plates are moved apart if the power source before the plates are moved apart: (1) is not disconnected, (2) is disconnected.

9.119. A plane capacitor is filled with a dielectric and a certain potential difference is applied to its plates. The energy of the capacitor is 2×10^{-5} J. After the capacitor is disconnected from the power source, the dielectric is extracted from the capacitor. The work performed against the forces of the electric field in extracting the dielectric is 7×10^{-5} J. Find the dielectric constant (relative permittivity) of the dielectric.

9.120. A plane air capacitor in which the plates are spaced 5 mm apart is charged to a potential of 6 kV. The area of the capacitor plates is 12.5 cm^2. The plates are moved apart to a distance of 1 cm in one of two ways: (1) the capacitor remains connected to the power source, and (2) before the plates are moved apart the capacitor is disconnected from the power source. Find in each of these cases: (a) the change in the capacitance of the capacitor, (b) the change in the intensity flux through the area of the electrodes, and (c) the change in the volume density of the energy of the electric field.

9.121. Find the volume density of the energy of an electric field at a point: (1) at a distance of 2 cm from the surface of a charged sphere with a radius of 1 cm, (2) near an infinitely long charged plane, (3) at a distance of 2 cm from an infinitely long charged filament. The charge surface density on the sphere and the plane is 1.67×10^{-5} C/m^2 and the charge linear density on the filament is 1.67×10^{-7} C/m. For all the three cases the dielectric constant $\varepsilon_r=2$.

9.122. A potential difference of $U=1{,}000$ V is applied to the plates of a plane capacitor. The distance between the plates is $d=3$ cm. The space between the plates is filled with a dielectric ($\varepsilon_r=7$). Find: (a) the surface density of the bound (polarization) charges and (b) the change in the charge surface density on the plates when the capacitor is filled with the dielectric. Solve the problem when: (1) the capacitor is filled with the dielectric while the source of the potential difference is connected, (2) the capacitor is filled with the dielectric after it is disconnected from the power source.

9.123. The space between the plates of a plane capacitor is filled with a dielectric whose dielectric susceptibility is 0.08. A potential difference of 4 kV is applied to the capacitor plates. Find the surface density of the charge on the plates and on the dielectric. The distance between the plates is 5 mm.

9.124. The space between the plates of a plane capacitor is filled with glass. The distance between the plates is 4 mm. A voltage of 1,200 volts is applied to the plates. Find: (1) the field in the glass, (2) the charge surface density on the capacitor plates, (3) the surface density of the bound charge on the glass, and (4) the dielectric susceptibility of the glass.

9.125. The space between the plates of a plane capacitor is filled with oil. The distance between the plates is 1 cm. What potential difference should be applied to the plates of this capacitor for the surface density of the bound charges on the oil to be 6.2×10^{-10} C/cm^2?

9.126. A glass segment is clamped between the plates of a plane capacitor. The area of the plates is 100 cm^2. The plates are attracted towards each other with a force equal to 4.9×10^{-3} N. Find the surface density of the bound charges on the surface of the glass.

9.127. The space between the plates of a plane capacitor is filled with paraffin. When the plates are connected to a source of power, the pressure of the plates on the paraffin became 5 N/m^2. Find: (1) the intensity of the electric field and the electric induction in the paraffin, (2) the surface density of the bound charges on the paraffin, (3) the surface density of the charges on the capacitor plates, (4) the volume density of the electric field energy in the paraffin, and (5) the dielectric susceptibility of the paraffin.

9.128. A dielectric is so placed as to completely fill the space between the plates of a plane capacitor spaced 2 mm apart. A potential difference of 600 V is applied to the plates. If the power source is disconnected and the dielectric is removed from the capacitor, the potential difference on the capacitor plates will rise to 1,800 V. Find: (1) the surface density of the bound charges on the dielectric, (2) the susceptibility of the dielectric.

9.129. The space between the plates of a plane capacitor with a volume of 20 cm^3 is filled with a dielectric ($\varepsilon_r=5$). The capacitor plates are connected to a power source. The surface density of the bound charges on the dielectric is equal to 8.35×10^{-6} C/m^2. What work must be performed against the forces of the electric field to pull the dielectric out of the capacitor? Solve the problem for two cases: (1) the dielectric is removed with the power source connected, and (2) the dielectric is removed after the power source is disconnected.

10. Electric Current

The intensity I of a current is equal numerically to the quantity of electricity flowing through the cross section of a conductor in a unit of time

$$I = \frac{dQ}{dt}$$

If I=const, then

$$I = \frac{Q}{t}$$

The density of an electric current is

$$J = \frac{I}{A}$$

where A is the cross-sectional area of the conductor.

The intensity of a current flowing through a homogeneous conductor conforms with Ohm's law

$$I = \frac{U}{R}$$

where U=difference of potentials at the ends of the conductor
R=resistance of the conductor.

The resistance of a conductor is

$$R = \rho \frac{l}{A} = \frac{l}{\sigma A}$$

where ρ=resistivity of the conductor
σ=its conductivity
l=its length
A=its cross-sectional area.

The resistivity of metals depends on the temperature as follows

$$\rho_t = \rho_0 (1 + \alpha t)$$

where ρ_0=resistivity at 0° C
α=temperature coefficient of resistance.

The work of an electric current on a section of a circuit can be determined from the formula

$$W = IUt = I^2Rt = \frac{U^2}{R} t$$

For a closed circuit Ohm's law has the form

$$I = \frac{\mathcal{E}}{R + r}$$

where $\mathscr{E}$ = e.m.f. of the generator
R = external resistance
r = internal resistance (resistance of the generator).

The total power evolved in a circuit is

$$P=\mathscr{E}I$$

Circuits composed of a network of conductors obey one of Kirchhoff's two laws.

Kirchhoff's first law states that: "The algebraic sum of the currents at any junction of conductors (branch point) must be zero", i.e.,

$$\Sigma I=0$$

Kirchhoff's second law states that: "In any closed circuit the algebraic sum of the potential drops in various sections of the circuit is equal to the algebraic sum of the electromotive forces in this circuit", i.e.,

$$\Sigma IR=\Sigma\mathscr{E}$$

The following rules should be observed in using Kirchhoff's laws. The directions of the currents are indicated arbitrarily by arrows on the diagram near the respective resistors. Moving along the circuit in any direction, let us assume that the currents whose direction coincides with the direction of this movement are positive, and that the oppositely directed currents are negative. The e.m.f.s will be positive if they increase the potential in the direction of the movement, i.e., when passing from the minus to the plus of a generator. After the equations have been solved, the sought quantities may be negative. If currents are being determined, a negative value indicates only that the actual direction of the current in the given section of the circuit is opposite to that sl own by the arrow. If resistances are being determined, a negative value indicates a wrong result (since ohmic resistance is always positive). In this case the direction of the current in the given resistor should be reversed and the problem solved for these conditions.

Faraday established two laws for an electric current. According to the first law, the mass m of a substance liberated during electrolysis is equal to

$$m=KIt=KQ$$

where Q is the quantity of electricity that passed through the electrolyte, and K is the electrochemical equivalent.

According to Faraday's second law, the electrochemical equivalent is proportional to the chemical equivalent, i.e.,

$$K=\frac{1}{F}\frac{m_A}{Z}$$

where m_A = mass of one kg-atom
Z = valency
$\frac{m_A}{Z}$ = mass of a kg-equivalent
F = Faraday's number, equal to 9.65×10^7 C/kg-eq.

The conductivity of an electrolyte can be found from the formula

$$\sigma = \frac{1}{\rho} = \alpha CZF(u_+ + u_-)$$

where α = degree of dissociation
C = concentration, i.e., the number of kg-moles in a unit volume
Z = valency
F = Faraday's number
u_+ and u_- = ion mobilities.

Here the quantity $\alpha = \frac{n_d}{n}$ is the ratio of the number of dissociated molecules in a unit volume n_d to the total number of molecules of the substance dissolved in this volume n. The quantity $\eta = CZ$ is known as the equivalent concentration. Hence $\Lambda = \frac{\sigma}{\eta}$ is the equivalent conductivity.

With small densities J of a current flowing in a gas, Ohm's law is true

$$J = Qn(u_+ + u_-)E = \sigma E$$

where E = intensity of the field
σ = conductivity of the gas
Q = charge of an ion
u_+ and u_- = ion mobilities
n = number of ions of each sign (number of pairs of ions) in a unit volume of the gas.

Here $n = \sqrt{\frac{N}{\gamma}}$, where N is the number of pairs of ions produced by the ionizing agent in a unit volume and in unit time, and γ is the coefficient of molecular formation from the ions.

With a saturation current in a gas, the density of this current can be determined from the formula

$$J_s = NQd$$

where d is the distance between the electrodes.

To be able to fly out from a metal, an electron must have the kinetic energy

$$\frac{mv^2}{2} \geqslant W$$

where W is the work done by the electron to leave the metal.

The density of the saturation current in thermionic emission (the emissivity) is found from the formula

$$J_s = BT^2 e^{-\frac{W}{kT}}$$

where T=absolute temperature of the cathode
e=base of natural logarithms
W=work of emission
k=Boltzmann's constant
B=a certain (emission) constant different for different metals.

10.1. The current intensity I in a conductor changes with time t according to the equation $I=4+2t$, where I is in amperes and t in seconds. (1) What quantity of electricity flows through a cross section of the conductor during the time from $t_1=2$ seconds to $t_2=6$ seconds? (2) At what intensity of a constant current does the same quantity of electricity pass through the cross section of the conductor during the same time?

10.2. A rheostat consists of five electric lamps connected in parallel. Find the resistance of the rheostat: (1) when all the lamps are burning, (2) when (a) one, (b) two, (c) three, and (d) four lamps are turned out. The resistance of each lamp is 350 Ω.

10.3. How many turns of a nichrome wire 1 mm in diameter should be wound around a porcelain cylinder with a radius of 2.5 cm to obtain a furnace with a resistance of 40 Ω?

10.4. A copper wire coil has a resistance of R=10.8 Ω. The wire weighs G=3.41 kgf. How many metres of the wire and of what diameter d are wound on the coil?

10.5. Find the resistance of an iron rod 1 cm in diameter and weighing 1 kgf.

10.6. Two cylindrical conductors, one of copper and the other of aluminium, have the same length and the same resistance. How many times is the copper conductor heavier than the aluminium one?

10.7. The resistance of the tungsten filament of an electric lamp is 35.8 Ω at 20° C. What will the temperature of the filament be if a current of 0.33 A flows through it when the lamp is connected to 120-V mains? The temperature coefficient of resistance of tungsten is 4.6×10^{-3} deg^{-1}.

10.8. An iron-wire rheostat, a milliammeter and a current generator are connected in series. The resistance of the rheostat is 120 Ω at 0° C and that of the milliammeter 20 Ω. The milliammeter shows 22 mA. What will the reading of the milliammeter be if the rheostat is heated to 50° C? The temperature coefficient of resistance of iron is 6×10^{-3} deg^{-1}. Disregard the resistance of the generator.

10.9. A copper wire winding has a resistance of 10 Ω at a temperature of 14° C. After a current flowed through the coil, the resistance of the winding rose to 12.2 Ω. What temperature was the winding heated to? The temperature coefficient of resistance of copper is 4.15×10^{-3} deg^{-1}.

10.10. Find the potential drop along a copper wire 500 m long and 2 mm in diameter if the current in it is equal to 2 A.

10.11. Determine the potential drop in resistors R_1, R_2 and R_3 (Fig. 19) if the ammeter shows 3 A; $R_1=4$ Ω, $R_2=2$ Ω and $R_3=4$ Ω. Find I_2 and I_3—the current intensities in resistors R_2 and R_3.

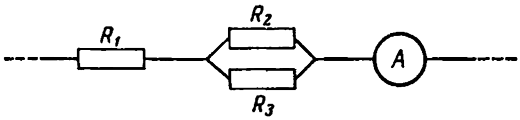


Fig. 19

10.12. An element with an e.m.f. of 1.1 V and an internal resistance of 1 Ω is connected across an external resistance of 9 Ω. Find: (1) the current intensity in the circuit, (2) the potential drop in the external circuit, (3) the potential drop inside the element, (4) the efficiency of the element.

10.13. Plot a diagram showing how the potential drop in the external circuit depends on the external resistance for the circuit of the previous problem. Take the external resistance within $0\leqslant R\leqslant10$ Ω at intervals of 2 ohms.

10.14. An element with an e.m.f. of 2 V has an internal resistance of 0.5 Ω. Determine the potential drop inside the element with a current of 0.25 A in the circuit. Also find the external resistance of the circuit in these conditions.

10.15. The e.m.f. of an element is 1.6 V and its internal resistance 0.5 Ω. What is the efficiency of the element at a current of 2.4 A?

10.16. The e.m.f. of an element is 6 V. When the external resistance is 1.1 Ω, the current in the circuit is equal to 3 A. Find the potential drop inside the element and its resistance.

10.17. What part of the e.m.f. of an element falls to the potential difference across its ends if the resistance of the element is $1/n$ of the external resistance. Solve the problem for: (1) $n=0.1$, (2) $n=1$, (3) $n=10$.

10.18. An element, a rheostat and an ammeter are connected in series. The e.m.f. of the element is 2 V and its internal resistance 0.4 Ω. The ammeter shows a current of 1 A. What is the efficiency of the element?

10.19. Two identical elements have an e.m.f. of 2 V and an internal resistance of 0.3 Ω. How should these elements be connected (in series or in parallel) to obtain a higher current intensity if: (1) the external resistance is 0.2 Ω, (2) the external resistance is 16 Ω? Calculate the current intensity for each of these cases.

10.20. Assuming the resistance of a voltmeter to be infinitely great, the resistance of rheostat R is determined from the readings of the ammeter and voltmeter in the circuit in Fig. 20. Find the relative error of the determined resistance if the resistance of the voltmeter is actually R_V. Solve the problem for $R_V=1{,}000$ Ω and R equal to: (1) 10 Ω, (2) 100 Ω, (3) 1,000 Ω.

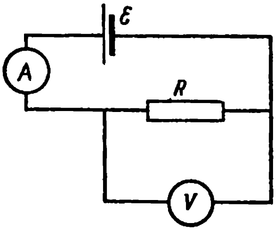


Fig. 20

10.21. Assuming the resistance of an ammeter to be infinitely small, the resistance of the rheostat R is determined from the readings of the ammeter and voltmeter in the circuit in Fig. 21. Find the relative error of the determined resistance if the actual resistance of the ammeter is R_A. Solve the problem for $R_A=0.2$ Ω and R equal to: (1) 1 Ω, (2) 10 Ω, and (3) 100 Ω.

10.22. In the circuit in Fig. 22, the resistance $R=1.4$ Ω, and $\mathcal{E}_1$ and $\mathcal{E}_2$ are two elements whose e.m.f.s are the same and equal to 2 V. The internal resistances of these elements are $r_1=1$ Ω and $r_2=1.5$ Ω, respectively. Find the current intensity in each element and in the entire circuit.

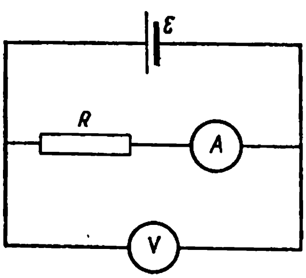


Fig. 21

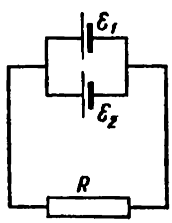


Fig. 22

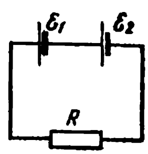


Fig. 23

10.23. In the circuit in Fig. 23, the resistance $R=0.5$ Ω, and $\mathcal{E}_1$ and $\mathcal{E}_2$ are two elements whose e.m.f.s are the same and equal to 2 V. The internal resistances of these elements are $r_1=1$ Ω and $r_2=1.5$ Ω, respectively. Find the potential difference across the terminals of each element.

10.24. In the circuit in Fig. 24, $\mathcal{E}$ is a battery whose e.m.f. is equal to 20 V, and R_1 and R_2 are rheostats. When rheostat R_1 is cut out, the ammeter shows a current of 8 A in the circuit. When the rheostat is cut in, the ammeter shows 5 A. Find the resistance of the rheostats and the potential drop in them when rheostat R_1 is cut in completely. Disregard the resistance of the battery and the ammeter.

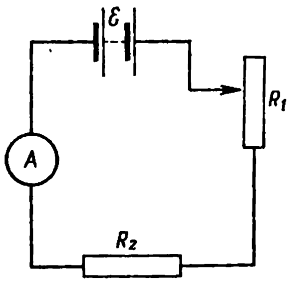


Fig. 24

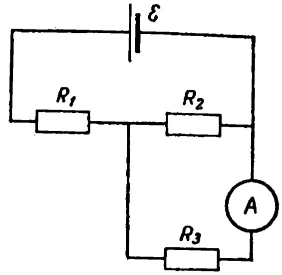


Fig. 25

10.25. An element, an ammeter and a resistor are connected in series. The resistor is a copper wire 100 m long and 2 mm^2 in cross section. The resistance of the ammeter is 0.05 Ω, and it shows 1.43 A.

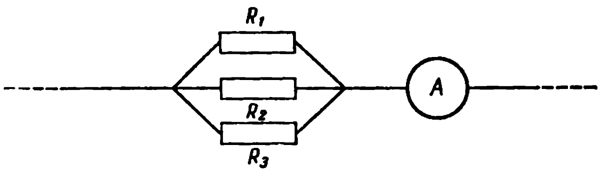


Fig. 26

If we take a resistor made of aluminium wire 57.3 m long and 1 mm^2 in cross section, the ammeter will show 1 A. Find the e.m.f. of the element and its internal resistance.

10.26. Determine the current intensity indicated by the ammeter in the circuit in Fig. 25. The voltage across the terminals of the element in the closed circuit is 2.1 V; $R_1=5$ Ω, $R_2=6$ Ω and $R_3=3$ Ω. Disregard the resistance of the ammeter.

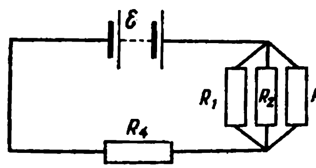


Fig. 27

10.27. In the circuit in Fig. 26, we have $R_2=20$ Ω, $R_3=15$ Ω, and the current flowing through resistor R_2 is 0.3 A. The ammeter shows 0.8 A. Find the resistance R_1.

10.28. In the circuit in Fig. 27, $\mathscr{E}$ is a battery whose e.m.f. is equal to 100 V; the resistances $R_1=R_3=40$ Ω, $R_2=80$ Ω and $R_4=34$ Ω. Find: (1) the intensity of the current flowing through the resistor R_2,

(2) the potential drop across this resistor. Disregard the resistance of the battery.

10.29. In the circuit in Fig. 28, $\mathscr{E}$ is a battery with an e.m.f. equal to 120 V; the resistances $R_3=20\ \Omega$ and $R_4=25\ \Omega$, and the potential drop across resistor R_1 is 40 V. The ammeter shows 2 A. Find the resistance R_2. Disregard the resistance of the battery and the ammeter.

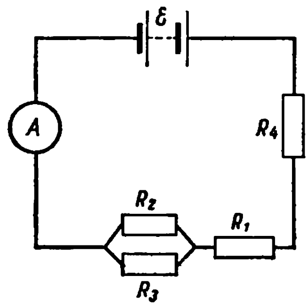


Fig. 28

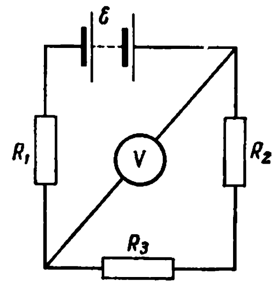


Fig. 29

10.30. (1) What current intensity will be shown by the ammeter in the circuit in Fig. 28 if $\mathscr{E}=10$ V, $r=1\ \Omega$ and the efficiency is 0.8? (2) What is the potential drop across resistor R_2 if that across resistor R_1 is 4 V and across resistor R_4 is 2 V?

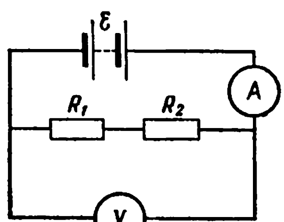


Fig. 30

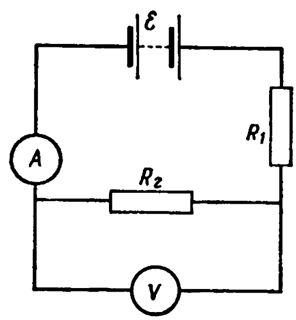


Fig. 31

10.31. In the circuit in Fig. 29, $\mathscr{E}$ is a battery with an e.m.f. equal to 100 V; the resistances $R_1=100\ \Omega$, $R_2=200\ \Omega$, and $R_3=300\ \Omega$. What voltage will the voltmeter show if its resistance is 2,000 Ω? Disregard the resistance of the battery.

10.32. In the circuit in Fig. 29, the resistances $R_1=R_2=R_3=200\ \Omega$. The voltmeter shows 100 V and its resistance $R_V=1{,}000\ \Omega$. Find the e.m.f. of the battery, neglecting its resistance.

10.33. Find the readings of the ammeter and voltmeter in the circuits in Figs. 30-33. The resistance of the voltmeter is 1,000 Ω, the e.m.f of the battery is 110 V, the resistances R_1=400 Ω and R_2= =600 Ω. Disregard the resistance of the battery and the ammeter.

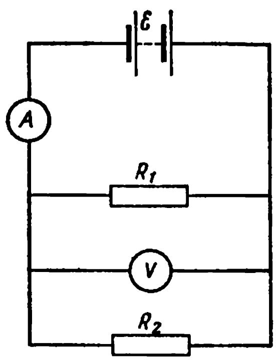


Fig. 32

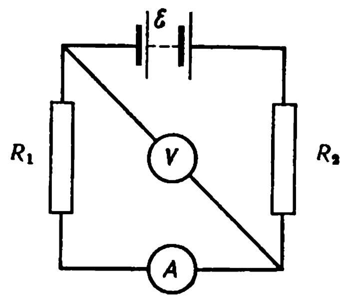


Fig. 33

10.34. An ammeter with a resistance of 0.16 Ω is shunted by a resistance of 0.04 Ω, and shows 8 A. What is the current intensity in the mains?

10.35. The scale of an ammeter with a resistance of 0.18 Ω intended to measure currents up to 10 A has 100 graduations. (1) What resistance should be selected and connected to permit the ammeter to be used for measuring currents up to 100 A? (2) How will the value of an ammeter graduation change in this case?

10.36. The scale of a voltmeter with a resistance of 2,000 Ω intended for measuring a potential difference up to 30 V has 150 graduations. (1) What resistance should be selected and how should it be connected to measure a potential difference up to 75 V by means of this voltmeter? (2) How will the value of a voltmeter graduation change in this case?

10.37. A milliammeter with a scale ranging from 0 to 15 mA has a resistance of 5 Ω. How should the instrument be connected together with a resistor (and with what resistance) to measure: (1) a current from 0 to 0.15 A, (2) a potential difference from 0 to 150 V?

10.38. A 120-volt lamp is rated at 40 W. What additional resistance should be connected in series with the lamp to produce normal glow with a voltage of 220 V in the mains? How many metres of a nichrome wire 0.3 mm in diameter are required to obtain this resistance?

10.39. Three electric lamps designed for a voltage of 110 V each are rated at 40, 40 and 80 W, respectively. How should all the three lamps be connected to produce a normal glow with a voltage of 220 V in the mains? Find the intensity of the current flowing through the lamps with a normal glow. Draw a connection diagram.

10.40. An electric heater consuming 10 A is switched on in a laboratory 100 metres away from a generator. How much will the voltage drop across the terminals of an electric lamp burning in this laboratory? The cross section of the feeding copper wires is 5 mm^2.

10.41. Power has to be transmitted over a distance of 2.5 km from a battery with an e.m.f. of 500 V. The power consumed is 10 kW. Find the minimum losses of power in the mains if the diameter of the feeding copper wires is 1.5 cm.

10.42. Power has to be transmitted over a distance of 2.5 km from a generator with an e.m.f. of 110 V. The power consumed is 1 kW. Find the minimum section of the feeding wires if the losses of power in the mains should not exceed 1%.

10.43. A copper and a steel wires of the same length and diameter are connected to a circuit in series. Find: (1) the ratio of the heats evolved in these wires, (2) the ratio of the voltage drops in these wires.

10.44. Solve the previous problem when the wires are connected in parallel.

10.45. An element with an e.m.f. equal to 6 V produces a maximum current of 3 A. Find the maximum heat which can be evolved in an external resistor in one minute.

10.46. Find: (1) the total power, (2) the net power and (3) the efficiency of a battery with an e.m.f. of 240 V if the external resistance is 23 Ω and the resistance of the battery is 1 Ω.

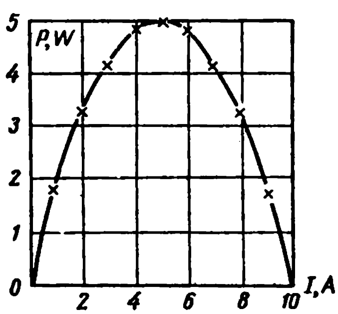


Fig. 34

10.47. Find the internal resistance of a generator if the power produced in the external circuit is the same when the external resistance is $R_1=5\ \Omega$ and $R_2=0.2\ \Omega$. Determine the generator efficiency in each of these cases.

10.48. Figure 34 shows the net power versus the current intensity in a circuit. For the points shown by crosses on the curve find: (1) the internal resistance of the element, (2) the e.m.f. of the element, (3) plot a diagram showing how the efficiency of this element and the potential drop in the external circuit depend on the current intensity in the circuit.

10.49. Use the data given by the curve in Fig. 34 to plot the relation between (1) the efficiency of the given element, (2) the total power P_1, and (3) the net power P_2, and the external resistance of the circuit R. Plot the curves for values of the external resistance R equal to 0, r, $2r$, $3r$, $4r$ and $5r$, where r is the internal resistance of the element.

10.50. An element is first connected across an external resistance $R_1=2\ \Omega$, and then across an external resistance $R_2=0.5\ \Omega$. Find the e.m.f. of the element and its internal resistance if in each of these cases the power evolved in the external circuit is the same and equal to 2.54 W.

10.51. An element with an e.m.f. of 2 V and an internal resistance of 0.5 Ω is connected across an external resistance of R. Plot diagrams showing how (1) the current intensity in the circuit, (2) the potential difference across the ends of the external circuit, (3) the power evolved in the external circuit, and (4) the total power depend on the resistance. The resistance R is to be taken within $0 \leqslant R \leqslant 4\ \Omega$ at intervals of 0.5 Ω.

10.52. An element with an e.m.f. of $\mathscr{E}$ and an internal resistance of r is connected across an external resistance R. The maximum power in the external circuit is 9 W. The current flowing through the circuit in these conditions is 3 A. Find $\mathscr{E}$ and r.

10.53. In the circuit in Fig. 35, $\mathscr{E}$ is a battery whose e.m.f. is 120 V, the resistances $R_3=30$ ˆ and $R_2=60\ \Omega$. The ammeter shows 2 A. Find the power dissipated in the resistor R_1. Disregard the resistance of the battery and the ammeter.

10.54. Determine the reading of the ammeter in the circuit in Fig. 35. The e.m.f. of the battery is 100 V and its internal resistance 2 Ω. The resistances R_1 and R_3 are equal to 25 Ω and 78 Ω, respectively.

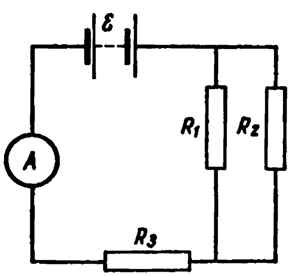


Fig. 35

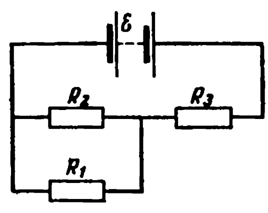


Fig. 36

The power dissipated in the resistor R_1 is equal to 16 W. Disregard the resistance of the ammeter.

10.55. In the circuit in Fig. 36, $\mathscr{E}$ is a battery whose e.m.f. is 120 V, the resistances $R_1=25\ \Omega$ and $R_2=R_3=100\ \Omega$. Find the power dissipated in the resistor R_1. Disregard the resistance of the battery.

10.56. In the circuit in Fig. 36, the resistance $R_1=100\ \Omega$ and the power dissipated in the resistor $P=16$ W. The generator efficiency is 80%. Find the e.m.f. of the generator if the potential drop across the resistor R_3 is 40 V.

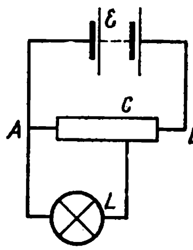


Fig. 37

10.57. In the circuit in Fig. 37, $\mathscr{E}$ is a battery with an e.m.f. of 120 V, AB is a potentiometer with a resistance of 120 Ω and L is an electric lamp. The resistance of the lamp changes upon heating from 30 to 300 Ω. How much will the potential difference across the lamp terminals change if sliding contact C is in the middle of the potentiometer? Also determine the change in the power consumed by the lamp.

10.58. The potential difference between points A and B is 9 V. There are two conductors whose resistances are 5 and 3 Ω, respectively. Find the quantity of heat produced in each conductor in one second if the conductors between A and B are connected: (1) in series, (2) in parallel.

10.59. Two electric lamps are connected to mains in parallel. The resistance of the first lamp is 360 Ω and that of the second 240 Ω. Which of the lamps consumes more power, and how many times?

10.60. The coil of a calorimeter C has a resistance of $R_1=60\ \Omega$. The coil R_1 is connected to the circuit as shown in Fig. 38. How many degrees will 480 grammes of water poured into the calorimeter be heated in 5 minutes during which a current flows through the coil if the ammeter shows 6 A? The resistance $R_2=30\ \Omega$. Disregard the resistances of the generator and the ammeter, and the heat losses.

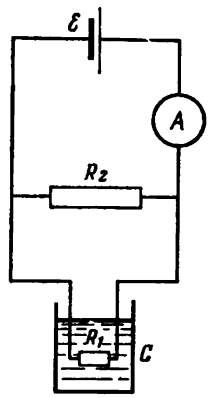


Fig. 38

10.61. How much water can be boiled by 3 hW-h of electric energy? The initial temperature of the water is 10° C. Disregard the heat losses.

10.62. (1) How many watts are consumed by the heater of an electric kettle if one litre of water begins to boil in 5 minutes? (2) What is the resistance of the heater if the voltage in the mains is 120 V? The initial temperature of the water is 13.5° C. Disregard the heat losses.

10.63. A kettle filled with one litre of water having a temperature of 16° C is placed on an electric stove rated at 0.5 kW. The water in the kettle begins to boil in 20 minutes after the stove is switched on.

What amount of heat is lost in this case for heating the kettle, for radiation, etc.?

10.64. The winding in an electric pot consists of two identical sections. The resistance of each section is 20 Ω. In how much time will 2.2 litres of water begin to boil if: (1) one section is switched on, (2) both sections are switched on in series, (3) both sections are switched on in parallel? The initial temperature of the water is 16° C, the voltage of the mains 110 V and the heater efficiency 85%.

10.65. An electric kettle has two windings. When one of them is switched on the water in the kettle begins to boil in 15 minutes, and when the other is switched on—in 30 minutes. In how much time will the water in the kettle begin to boil if the two windings are switched on: (1) in series, (2) in parallel?

10.66. In the circuit in Fig. 39, $\mathscr{E}$ is a battery with an e.m.f. of 120 V, $R_2=10$ Ω and K is an electric kettle. The ammeter shows 2 A. In how much time will 0.5 litre of water in the kettle with an initial temperature of 4° C begin to boil? Disregard the resistance of the battery and the ammeter. The efficiency of the kettle is 76%.

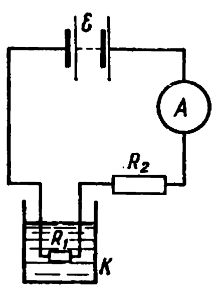


Fig. 39

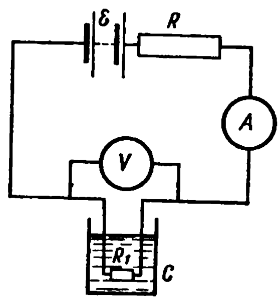


Fig. 40

10.67. In the circuit in Fig. 40, $\mathscr{E}$ is a battery with an e.m.f. of 110 V, and C is a calorimeter containing 500 grammes of kerosene. The ammeter shows 2 A and the voltmeter 10.8 V. (1) What is the resistance of the coil? (2) What is the specific heat of the kerosene, if after passing a current through heater coil R_1 for 5 minutes the kerosene is heated by 5° C? Assume that 80% of the heat evolved in the coil is used to heat the kerosene. (3) What is the resistance of rheostat R? Disregard the resistance of the battery and the ammeter. Consider the resistance of the voltmeter to be infinitely great.

10.68. A heater consumes 0.5 kW-h of electric energy to heat 4.5 litres of water from a temperature of 23° C to the boiling point. What is the efficiency of the heater?

10.69. A room is heated by an electric stove connected to 120-V mains. The room loses 20,800 kcal of heat a day. The temperature in the room must be kept constant. Find: (1) the resistance of the stove, (2) how many metres of nichrome wire 1 mm in diameter must be taken for the winding of the stove, (3) the power of the stove.

10.70. The temperature of a water thermostat with a capacity of one litre is kept constant with the aid of a 26-W heater, 80% of whose power is used to heat the water. How many degrees will the temperature of the water in the thermostat drop during 10 minutes if the heater is switched off?

10.71. How much should be paid for the use of electric energy during 30 days if two electric lamps consuming 0.5 A at 120 V burn six hours a day? Besides, 3 litres of water with an initial temperature of 10° C are boiled every day. The cost of one kilowatt-hour is 4 kopecks. The efficiency of the heater is 80%.

10.72. An electric kettle containing 600 cm^3 of water at 9° C and with a heater coil resistance equal to 16 Ω was left connected to the mains. In how much time will all the water in the kettle boil away? The voltage in the mains is 120 V and the efficiency of the kettle 60%.

10.73. One hundred grammes of mercury are evaporated every minute in a mercury diffusion pump. What should the resistance of the pump heater be if the heater is connected to 127-V mains? The heat of vaporization of mercury is 2.96×10^5 J/kg.

10.74. A lead fuse with a cross section of $A_2=1$ mm^2 is connected to a circuit consisting of a copper wire with a cross section of $A_1=3$ mm^2. What increase in the temperature of the wires upon a short circuit is the fuse designed for. Assume that all the heat evolved is spent to heat the circuit owing to the instantaneous nature of the short circuit. The initial temperature of the fuse is $t_0=17°$ C.

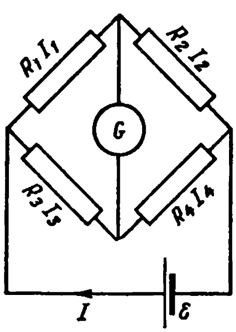


Fig. 41

10.75. Find the quantity of heat evolved every second in a unit volume of a copper conductor with a current density of 30 A/cm^2.

10.76. Find the current intensity in the separate arms of a Wheatstone bridge (Fig. 41) if no current passes through the galvanometer. The e m.f. of the generator is 2 V, $R_1=30$ Ω, $R_2=45$ Ω and $R_3=200$ Ω. Disregard the resistance of the generator.

10.77. In the circuit in Fig. 42, $\mathcal{E}_1$ is an element with an e.m.f. equal to 2.1 V, $\mathcal{E}_2=1.9$ V, $R_1=45$ Ω, $R_2=10$ Ω and $R_3=10$ Ω. Determine the current intensity in all the sections of the circuit. Disregard the internal resistance of the elements.

10.78. What difference of potentials is produced across the terminals of two elements connected in parallel if their e.m.f.s are equal to $\mathscr{E}_1=1.4$ V and $\mathscr{E}_2=1.2$ V, respectively, and the internal resistances are $r_1=0.6\ \Omega$ and $r_2=0.4\ \Omega$?

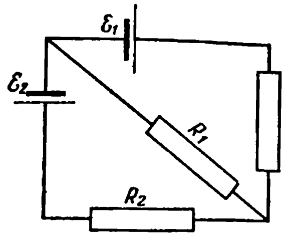


Fig. 42

10.79. In the circuit in Fig. 43, $\mathscr{E}_1$ and $\mathscr{E}_2$ are two elements with equal e.m.f.s of 2 V. The internal resistances of these elements are $r_1=1\ \Omega$ and $r_2=2\ \Omega$, respectively. What is the external resistance R if the current I_1 flowing through $\mathscr{E}_1$ is 1 A? Determine the intensity of the current I_2 flowing through $\mathscr{E}_2$. Find the intensity of the current I_R flowing through resistor R.

10.80. Solve the previous problem if $\mathscr{E}_1=\mathscr{E}_2=4$ V, $r_1=r_2=0.5\ \Omega$ and $I_1=2$ A.

10.81. In the circuit illustrated in Fig. 44, we have $\mathscr{E}_1=110$ V, $\mathscr{E}_2=220$ V, $R_1=R_2=100\ \Omega$, and $R_3=500\ \Omega$. Find the reading of the ammeter. Disregard the internal resistance of the battery and the ammeter.

10.82. In the circuit in Fig. 44, we have $\mathscr{E}_1=2$ V, $\mathscr{E}_2=4$ V, $R_1=0.5\ \Omega$ and the potential drop across the resistor R_2 is 1 V. What is the reading of the ammeter? Disregard the internal resistance of the elements and the ammeter.

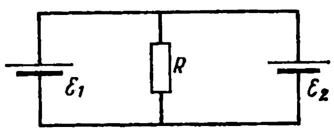


Fig. 43

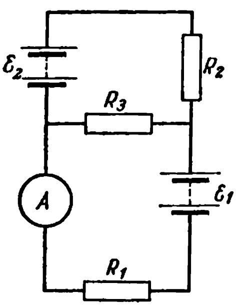


Fig. 44

10.83. In the circuit in Fig. 44, we have $\mathscr{E}_1=30$ V, $\mathscr{E}_2=10$ V, $R_2=20\ \Omega$, and $R_3=10\ \Omega$. A current of 1 A flows through the ammeter. Find the resistance R_1. Disregard the resistance of the battery and the ammeter.

10.84. What current intensity is shown by the milliammeter mA in the circuit in Fig. 45 if $\mathscr{E}_1=2$ V, $\mathscr{E}_2=1$ V, $R_1=10^3\ \Omega$, $R_2=500\ \Omega$, $R_3=200\ \Omega$ and the resistance of the ammeter is $R_A=200\ \Omega$? Disregard the internal resistance of the elements.

10.85. What current intensity is shown by the milliammeter mA in the circuit in Fig. 45 if $\mathscr{E}_1=1$ V, $\mathscr{E}_2=2$ V, $R_3=1{,}500\ \Omega$, $R_A=500\ \Omega$

and the potential drop across resistor R_2 is 1 V? Disregard the resistance of the elements.

10.86. In the circuit in Fig. 46, we have $\mathscr{E}_1=2$ V, $\mathscr{E}_2=4$ V, $\mathscr{E}_3=6$ V, $R_1=4\ \Omega$, $R_2=6\ \Omega$ and $R_3=8\ \Omega$. Find the current intensity in all the sections of the circuit. Disregard the resistance of the elements.

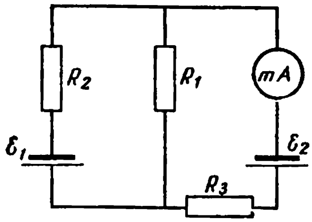


Fig. 45

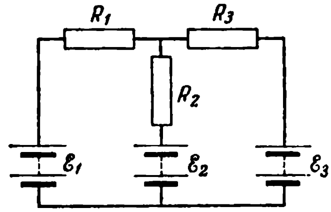


Fig. 46

10.87. In the circuit in Fig. 46, we have $\mathscr{E}_1=\mathscr{E}_2=\mathscr{E}_3$, $R_1=20\ \Omega$, $R_2=12\ \Omega$ and the potential drop across the resistor R_2 is 6 V. Find the current intensity in all the sections of the circuit, and also the resistance R_3. Disregard the internal resistance of the elements.

10.88. In the circuit in Fig. 46, we have $\mathscr{E}_1=25$ V. The potential drop of 10 V across resistor R_1 is equal to the drop across resistor R_3 and double that across resistor R_2. Find $\mathscr{E}_2$ and $\mathscr{E}_3$. Disregard the resistance of the battery.

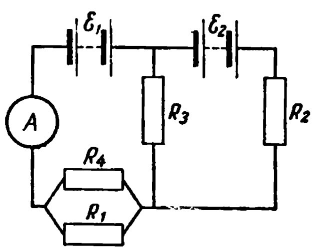


Fig. 47

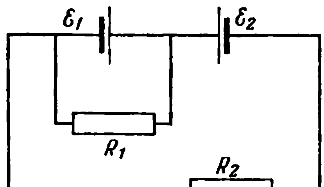


Fig. 48

10.89. In the circuit in Fig. 47, we have $\mathscr{E}_1=\mathscr{E}_2=100$ V, $R_1=20\ \Omega$, $R_2=10\ \Omega$, $R_3=40\ \Omega$ and $R_4=30\ \Omega$. Find the reading of the ammeter. Disregard the resistance of the battery and the ammeter.

10.90. In the circuit in Fig. 47, we have $\mathscr{E}_1=2\mathscr{E}_2$, $R_1=R_3=20\ \Omega$, $R_2=15\ \Omega$ and $R_4=30\ \Omega$. The ammeter shows 1.5 A. Find $\mathscr{E}_1$ and $\mathscr{E}_2$ and the intensities I_2 and I_3 of the current flowing through resistors R_2 and R_3, respectively. Disregard the resistance of the battery and the ammeter.

10.91. In the circuit in Fig. 48, $\mathscr{E}_1$ and $\mathscr{E}_2$ are two elements with the same e.m.f. of 2 V and the same internal resistance equal to 0.5 Ω. Find the intensity of the current flowing: (1) through the resistance $R_1=0.5$ Ω, (2) through the resistance $R_2=1.5$ Ω, (3) through the element $\mathscr{E}_1$.

10.92. In the circuit in Fig. 48, $\mathscr{E}_1$ and $\mathscr{E}_2$ are two elements with the same e.m.f. and the same internal resistance. The resistance $R_2=1$ Ω. The potential drop across the terminals of element $\mathscr{E}_1$, which is equal to 2 V, is double that across the terminals of element $\mathscr{E}_2$. The potential drop across resistor R_2 is equal to the drop across element $\mathscr{E}_2$.

Find the e.m.f. and the internal resistance of the elements.

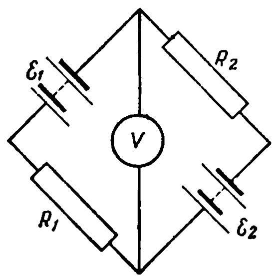


Fig. 49

10.93. In the circuit in Fig. 49, we have $\mathscr{E}_1=\mathscr{E}_2=110$ V, $R_1=R_2=200$ Ω and the resistance of the voltmeter is 1,000 Ω. Find the reading of the voltmeter. Disregard the resistance of the batteries.

10.94. In the circuit in Fig. 49, we have $\mathscr{E}_1=\mathscr{E}_2$, and $R_1=R_2=$ $=100$ Ω. The voltmeter shows 150 V and its resistance is 150 Ω. Find the e.m.f. of the batteries, disregarding their resistance.

10.95. Find the reading of the milliammeter mA in the circuit in Fig. 50 if $\mathscr{E}_1=\mathscr{E}_2=1.5$ V, $r_1=r_2=0.5$ Ω, $R_1=R_2=2$ Ω and $R_3=1$ Ω. The resistance of the milliammeter is 3 Ω.

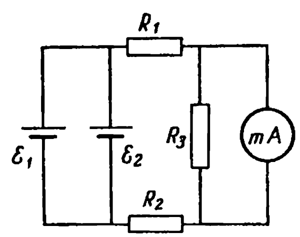


Fig. 50

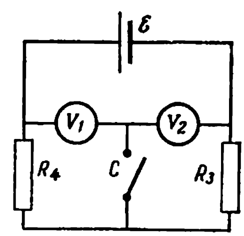


Fig. 51

10.96. In the circuit in Fig. 51, V_1 and V_2 are two voltmeters whose resistances are $R_1=3{,}000$ Ω and $R_2=2{,}000$ Ω, respectively; in addition, $R_3=3{,}000$ Ω, $R_4=2{,}000$ Ω, and $\mathscr{E}=200$ V. Find the readings of the voltmeters V_1 and V_2 when: (1) contact C is open, and (2) contact C is closed. Disregard the resistance of the battery. Solve the problem with the aid of Kirchhoff's laws.

10.97. In how much time will 4.74 g of copper be produced on the cathode during electrolysis of an aqueous solution of cupric chloride ($CuCl_2$)? The current is 2 A.

10.98. A copper plate with a total area of 25 cm^2 serves as a cathode in the electrolysis of blue vitriol. After current with a density of 0.02 A/cm^2 had been passed through the electrolyzer for a certain time, the mass of the plate increased by 99 mg. Find: (1) the time during which the current was passed through the electrolyzer, (2) the thickness of the copper layer formed on the plate.

10.99. Half a gramme of copper is deposited an hour in the electrolysis of blue vitriol. The area of each electrode is 75 cm^2. Find the density of the current.

10.100. Find the electrochemical equivalent of hydrogen.

10.101. An ammeter connected in series with an electrolytic bath containing a solution of $AgNO_3$ shows a current of 0.90 A. Does the ammeter show the correct reading if 316 mg of silver was deposited during 5 minutes with the current flowing through the bath?

10.102. Two electrolytic baths with solutions of $AgNO_3$ and $CuSO_4$ are connected in series. How much copper will be deposited during the time in which 180 mg of silver are produced?

10.103. When aluminium was obtained by the electrolysis of a solution of Al_2O_3, a current of 2×10^4 A with a potential difference of 5 V across the electrodes was passed through molten cryolite. (1) Find the time in which 10^3 kg of aluminium will be deposited. (2) How much electric energy will be used in this case?

10.104. How much electric energy must be spent to deposit 500 mg of silver in the electrolysis of a solution of $AgNO_3$? The potential difference across the electrodes is 4 V.

10.105. The reaction of water formation from hydrogen and oxygen occurs with the liberation of heat

$$2H_2+O_2=2H_2O+5.75\times 10^5 \text{ J}$$

Determine the minimum potential difference at which water will be decomposed by electrolysis.

10.106. Calculate the equivalent conductivity for a very weak solution of nitric acid.

10.107. A current of $I=2$ A is passed through a solution of nitric acid. What quantity of electricity is transferred in one minute by the ions of each sign?

10.108. The equivalent conductivity of a solution of KCl at a certain concentration is equal to 122 $cm^2/\Omega\cdot$g-eq, its specific conductivity at the same concentration is 0.00122 $\Omega^{-1}\cdot cm^{-1}$ and its equivalent conductivity in infinite dissolution is 130 $cm^2/\Omega\cdot$g-eq. Find: (1) the degree of dissociation of the KCl at this concentration, (2) the equivalent concentration of the solution, (3) the sum of the mobilities of the K^+ and Cl^- ions.

10.109. Determine the resistance of a 0.1N solution of $AgNO_3$ filling a tube 84 cm long and 5 mm^2 in cross section if 81% of all the $AgNO_3$ molecules are dissociated into ions.

10.110. Determine the resistance of a 0.05 N solution of KNO_3 filling a tube with a length of $l=2$ cm and a cross-sectional area of $A=7$ cm^2 if the equivalent conductivity of the solution is $1.1\times\times10^{-2}$ $m^2/\Omega\cdot$kg-eq.

10.111. A tube 3 cm long and 10 cm^2 in cross section is filled with a solution containing 0.1 kmole of $CuSO_4$ in 1 m^3 The resistance of the solution is 38 Ω. Find the equivalent conductivity of the solution.

10.112. The specific conductivity of a decinormal solution of hydrochloric acid is 0.035 $\Omega^{-1}\cdot cm^{-1}$. Find the degree of dissociation.

10.113. Find the number of ions of each sign in a unit volume of the solution of the previous problem.

10.114. When a vessel with a gas is subjected to the action of X-rays, 10^{10} molecules are ionized in each millimetre of its volume a second. Recombination produces equilibrium in the vessel and each cubic centimetre contains 10^8 ions of each sign. Find the recombination coefficient.

10.115. A potential difference of 5 V is applied to the electrodes of a discharge tube spaced 10 cm apart. The gas in the tube is singly ionized and the number of ionic pairs in 1 m^3 is 10^8. Also, $u_+=3\times\times10^{-2}$ $m^2/V\cdot s$ and $u_-=3\times10^2$ $m^2/V\cdot s$. Find: (1) the current density in the tube, (2) the part of the total current transferred by the positive ions.

10.116. The area of each electrode in an ionization chamber is 100 cm^2 and the distance between the electrodes is 6.2 cm. Determine the saturation current in this chamber if the ionizer produces 10^9 ions of each sign a second in 1 cm^3. The ions are monovalent.

10.117. Find the maximum possible number of ionic pairs in 1 cm^3 of the chamber of the previous problem if the recombination coefficient is 10^{-6}.

10.118. Find the resistance of a tube 84 cm long and 5 mm^2 in cross section if it is filled with air so ionized that 10^7 pairs of ions are in equilibrium in 1 cm^3. The ions are monovalent and their mobility is $u_+=1.3\times10^{-4}$ $m^2/V\cdot s$ and $u_-=1.8\times10^{-4}$ $m^2/V\cdot s$.

10.119. What current will flow between the electrodes of the ionization chamber of Problem 10.116 if a potential difference of 20 V is applied to them? The mobility of the ions $u_+=u_-=1$ $cm^2/V\cdot s$ and the recombination coefficient $\alpha=10^{-6}$. What part of the saturation current is the determined current?

10.120. What is the minimum velocity of an electron to ionize an atom of hydrogen? The ionization potential of the atom is 13.5 V.

10.121. At what temperature do mercury atoms have a mean kinetic energy of translational motion sufficient for ionization? The ionization potential of a mercury atom is 10.4 V.

10.122. The ionization potential of a helium atom is 24.5 V. Determine the work of ionization.

10.123. What minimum velocity should the free electrons possess in: (1) cesium and (2) platinum to be able to leave the metal?

10.124. How many times will the thermionic emissivity of tungsten at a temperature of 2400° K change if the temperature of the tungsten grows by 100°?

10.125. How many times is the emissivity of a cathode of thoriated tungsten at a working temperature of 1800° K higher than that of a cathode made of pure tungsten at the same temperature? The emission constant B for pure tungsten is assumed equal to 60 A/cm^2·deg^2 and for thoriated tungsten 3 A/cm^2·deg^2.

10.126. At what temperature will thoriated tungsten produce the same specific emission as pure tungsten at $T=2500°$ K? Take the necessary data from the previous problem.

11. Electromagnetism

According to Ampere's law (also called Laplace's law or the Biot-Savart relation), a current I flowing through an element of a circuit dl induces at a certain point A in space a magnetic field with an intensity dH equal to

$$dH = \frac{I \sin\alpha \, dl}{4\pi r^2}$$

where r=distance from the current element dl to point A

α=angle between the radius-vector r and the current element dl.

By applying Ampere's law to circuits of various shapes, we can find:

The intensity of a magnetic field in the centre of a circular current

$$H = \frac{I}{2R}$$

where R is the radius of the circular circuit through which the current is flowing.

The intensity of a magnetic field induced by an infinitely long rectilinear conductor

$$H = \frac{I}{2\pi a}$$

where a is the distance from the point at which the intensity is being determined to the current-carrying conductor.

The intensity of a magnetic field on the axis of the circular current

$$H = \frac{R^2 I}{2(R^2 + a^2)^{3/2}}$$

where R=radius of the circular circuit through which the current is flowing

a=distance from the point at which the intensity is being determined to the plane of the circuit.

The intensity of a magnetic field inside a toroid and an infinitely long solenoid

$$H = In$$

where n is the number of turns per unit length of the solenoid (toroid).

The intensity of a magnetic field on the axis of a solenoid with a finite length

$$H = \frac{In}{2}(\cos\beta_1 - \cos\beta_2)$$

where β_1 and β_2 are the angles between the axis of the solenoid and a radius-vector drawn from the point under consideration to the ends of the solenoid. The magnetic induction B is related to the magnetic field intensity H by the formula

$$B = \mu_0 \mu_r H$$

where μ_r is the relative magnetic permeability of the medium and μ_0 is the permeability in a vacuum, equal in the MKSA system to

$$4\pi \times 10^{-7}\ \text{H/m} = 12.57 \times 10^{-7}\ \text{H/m}$$

For ferromagnetic bodies $\mu_r = \varphi(H)$ and, therefore, $B = f(H)$.

Problems in which the relation $B = f(H)$ must be known should be solved with the aid of the diagram given in the Appendix.

The volume density of the energy of a magnetic field

$$W_0 = \frac{HB}{2}$$

The magnetic induction flux through a circuit

$$\Phi = BA\cos\varphi$$

where A=cross-sectional area of the circuit

φ=angle between a normal to the plane of the circuit and the direction of the magnetic field.

The magnetic induction flux through a toroid is

$$\Phi = \frac{INA\mu_0\mu_r}{l}$$

where N = total number of turns of the toroid
l = length of the toroid
A = cross-sectional area of the toroid
μ_r = relative magnetic permeability of the core material
μ_0 = permeability in a vacuum.

If a toroid has an air-gap, then

$$\Phi = \frac{IN}{\frac{l_1}{A\mu_0\mu_{r1}} + \frac{l_2}{A\mu_0\mu_{r2}}}$$

where l_1 = length of the air-gap
l_2 = length of the iron core
μ_{r2} = magnetic permeability of the iron core
μ_{r1} = magnetic permeability of air.

The element dl of a current-carrying conductor placed in a magnetic field is acted upon by the Ampere force

$$dF = BI \sin\alpha \, dl$$

where α is the angle between the directions of the current and the magnetic field.

A closed circuit through which a current is flowing and a magnetic needle in a magnetic field are acted upon by a couple of forces with the rotational moment

$$M = pB \sin\alpha$$

where p is the magnetic moment of the circuit (or of the magnetic needle) and α is the angle between the direction of the magnetic field and a normal to the plane of the circuit (or the axis of the needle).

The magnetic moment of a circuit through which a current is flowing is

$$p = IA$$

where A is the area of the circuit, and thus

$$M = BIA \sin\alpha$$

Two parallel rectilinear conductors with currents I_1 and I_2 mutually interact with the force

$$F = \frac{\mu_0\mu_r I_1 I_2 l}{2\pi d}$$

where l is the length of the conductors and d the distance between them.

The work performed in moving a conductor with current in a magnetic field is

$$dW = I d\Phi$$

where $d\Phi$ is the magnetic induction flux crossed by the conductor during its motion.

The force acting on a charged particle moving with the velocity v through a magnetic field can be determined from Lorentz's formula

$$F = QBv \sin\alpha$$

where Q is the charge of the particle and α is the angle between the directions of the particle velocity and the magnetic field.

A current I flowing along a conducting plate perpendicular to the magnetic field produces a transverse potential difference of

$$U = R_h \frac{IB}{a} = \frac{IB}{Nqa}$$

where a = thickness of the plate
B = magnetic field induction
$R_h = \frac{1}{Nq}$ = the Hall constant, inversely proportional to the density N of the current carriers and their charge q.

The mobility u of the current carriers can be determined if we know R_h and the conductivity of the material $\sigma = \frac{1}{\rho} = Nqu$.

Electromagnetic induction induces an e.m.f. in a circuit each time the magnetic flux Φ changes through the surface enveloped by the circuit. The e.m.f. $\mathscr{E}$ of induction is determined from the equation

$$\mathscr{E} = -\frac{d\Phi}{dt}$$

The magnetic flux can be changed by varying the current intensity in the circuit (self-induction). Here the self-induction e.m.f. can be found from the formula

$$\mathscr{E} = -L\frac{dI}{dt}$$

where L is the inductance (self-inductance) of the circuit.

The inductance of a solenoid is

$$L = \mu_0 \mu_r n^2 l A$$

where l = length of the solenoid
A = cross-sectional area of the solenoid
n = number of turns per unit of the solenoid length.

When the e.m.f. is switched off, the phenomenon of self-inductance will cause the intensity of the current in the circuit to diminish according to the law

$$I = I_0 e^{-\frac{R}{L}t}$$

and when the e.m.f. is switched on again, the current intensity increases according to the law

$$I = I_0\left(1 - e^{-\frac{R}{L}t}\right)$$

where R is the resistance of the circuit.

The magnetic energy of a circuit through which a current is flowing is

$$W_m = \frac{1}{2}LI^2$$

The magnetic flux can also be changed by varying the current intensity in the adjacent circuit (mutual induction). Here the induced e.m.f. is

$$\mathscr{E} = -L_{12}\frac{dI}{dt}$$

where L_{12} is the mutual inductance of the circuits.

The mutual inductance of two solenoids through which a common magnetic flux passes is

$$L_{12} = \mu_0\mu_r n_1 n_2 Al$$

where n_1 and n_2 are the numbers of turns per unit length of the solenoids.

The quantity of electricity passing through the cross section of a conductor when an induced current appears in it is

$$dQ = -\frac{1}{R}d\Phi$$

11.1. Find the intensity of a magnetic field at a point 2 cm away from an infinitely long conductor which a current of 5 A flows through.

11.2. Find the intensity of a magnetic field at the centre of a circular wire coil turn with a radius of 1 cm which a current of 1 A flows through.

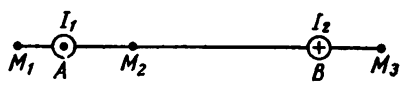


Fig. 52

11.3. Figure 52 shows a cross section of two infinitely long rectilinear conductors with current. The distance AB between the conductors is 10 cm, I_1=20 A, I_2=30 A. Find the intensity of the magnetic field induced by the currents I_1 and I_2 at points M_1, M_2 and M_3. The distance M_1A = 2 cm, AM_2=4 cm and BM_3=3 cm.

11.4. Solve the previous problem if the currents flow in the same direction.

11.5. Figure 53 shows a cross section of three infinitely long rectilinear conductors with current. The distance $AB=BC=5$ cm, $I_1=I_2=$ $=I$ and $I_3=2I$. Find the point on straight line AC at which the intensity of the magnetic field induced by the currents I_1, I_2 and I_3 is zero.

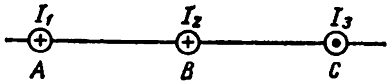


Fig. 53

11.6. Solve tne previous problem if all the three currents flow in the same direction.

11.7. Two infinitely long rectilinear conductors are arranged perpendicular to each other in one plane (Fig. 54). Find the intensity of the magnetic field at points M_1 and M_2 if $I_1=2$ A and $I_2=3$ A. The distances $AM_1=AM_2=1$ cm, and $BM_1=CM_2=2$ cm.

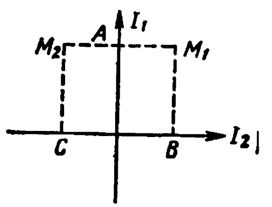


Fig. 54

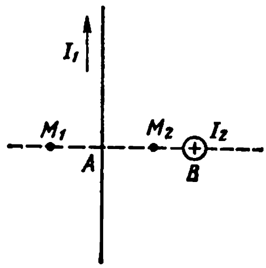


Fig. 55

11.8. Two infinitely long rectilinear conductors are arranged perpendicular to each other and are in mutually perpendicular planes (Fig. 55). Find the intensity of the magnetic field at points M_1 and M_2, if $I_1=2$ A and $I_2=3$ A. The distances $AM_1=AM_2=1$ cm and $AB=2$ cm.

11.9. Two long rectilinear conductors are arranged parallel to each other at a distance of 10 cm. Currents of $I_1=I_2=5$ A flow through the conductors in opposite directions. Find the magnitude and direction of the magnetic field intensity at a point 10 cm from each conductor.

11.10. A current of $I=8$ A flows down a long vertical conductor. At what distance r from the conductor is the intensity of the field produced by summation of the terrestrial magnetic field and the field of the current directed vertically up? The horizontal component of the terrestrial field is $H_h=0.2$ Oe.

11.11. Calculate the intensity of the magnetic field induced by section AB of a rectilinear conductor with current at point C arranged on a perpendicular erected from the middle of this section 5 cm from it. The current in the conductor is 20 A. Section AB of the conductor can be seen from point C at an angle of 60°.

11.12. Solve the previous problem when the current in the conductor is 30 A and the section of the conductor can be seen from point C at an angle of 90°. Point C is at a distance of 6 cm from the conductor.

11.13. A section of a rectilinear conductor with current is 30 cm long. At what maximum distance from it can a magnetic field for points on a perpendicular erected from the middle of the conductor be considered as a field with an infinitely long rectilinear current? The error in this assumption should not exceed 5%.

Note. The permissible error $\delta=\frac{H_2-H_1}{H_2}$, where H_1 is the field induced by the section of the current-carrying conductor, and H_2 is that induced by an infinitely long rectilinear current.

11.14. The intensity of a magnetic field is 400 A/m at point C at a distance of 5 cm from an infinitely long rectilinear conductor with current. (1) At what limit length of the conductor will this magnitude of the intensity be true to an accuracy of 2%? (2) What will the intensity of the magnetic field be at point C if the conductor with current is 20 cm long? Point C lies on a perpendicular erected from the middle of this conductor.

11.15. A current of 20 A flows in a long conductor bent to form a right angle. Find the intensity of the magnetic field at a point on the bisectrix of this angle 10 cm from its vertex.

11.16. A current of $I=20$ A flows in a copper wire ring with a cross section of $A=1.0$ mm^2 and induces a magnetic field intensity of $H=2.24$ Oe at the centre of the ring. What potential difference is applied across the ends of the wire forming the ring?

11.17. Find the intensity of a magnetic field on the axis of a circular contour at a distance of 3 cm from its plane. The contour radius is 4 cm and the current in the contour is 2 A.

11.18. The intensity of a magnetic field at the centre of a circular coil turn with a radius of 11 cm is 0.8 Oe. Find the intensity of the magnetic field on the axis of the turn at a distance of 10 cm from its plane.

11.19. Two circular coil turns each with a radius of 4 cm are arranged in parallel planes 0.1 m apart. Currents of $I_1=I_2=2$ A flow through them. Find the intensity of the magnetic field on the axis of the turns at a point equidistant from them. Solve the problem when: (1) the currents in the turns flow in the same direction, (2) the currents flow in opposite directions.

11.20. Two circular coil turns each with a radius of 4 cm are arranged in parallel planes 5 cm apart. Currents of $I_1=I_2=4$ A flow through the turns. Find the intensity of the magnetic field at the centre of one of the turns. Solve the problem when: (1) the currents in the turns flow in the same direction, (2) the currents flow in opposite directions.

11.21. Find the distribution of the intensity of a magnetic field along the axis of a circular turn 10 cm in diameter which a current of 10 A flows through. Compile a table showing the values of H for those of x within $0 \leqslant x \leqslant 10$ cm at intervals of 2 cm and plot a diagram showing the scale used.

11.22. Two circular coil turns are so arranged in two mutually perpendicular planes that the centres of the turns coincide. The radius of each coil is 2 cm and the currents flowing through the turns $I_1=I_2=5$ A. Find the intensity of the magnetic field at the centre of these turns.

11.23. A square frame is made of wire 1 m long. A current of 10 A flows through the frame. Find the intensity of the magnetic field at the centre of the frame.

11.24. A magnetic field H is produced at the centre of a circular wire coil turn with the potential difference U across its ends. How should the applied potential difference be changed to obtain the same intensity of the magnetic field at the centre of a turn with a radius double that of the first turn and made of the same wire?

11.25. A current of $I=2$ A flows through a wire frame having the form of a regular polygon. A magnetic field with an intensity of $H=33$ A/m is formed at the centre of the frame. Find the length L of the wire which the frame is made of.

11.26. An infinitely long conductor has a circular loop tangent to it. A current of 5 A flows through the conductor. Find the radius of the loop if the intensity of the magnetic field at its centre is 41 A/m.

11.27. A coil 30 cm long consists of 1,000 turns. Find the intensity of the magnetic field inside the coil if the current flowing through it is 2 A. Consider the diameter of the coil to be small as compared with its length.

11.28. The winding of a coil is made of wire 0.8 mm in diameter. The turns contact each other closely. Assuming the coil to be sufficiently long, find the intensity of the magnetic field inside it at a current of 1 A.

11.29. A solenoid with a magnetic field intensity of 300 Oe has to be wound from wire 1 mm in diameter. The maximum current which can flow through the wire is 6 A. How many winding layers are required on the solenoid if the turns are wound close to each other? Consider the diameter of the coil to be small as compared with its length.

11.30. A magnetic field intensity of 12.6 Oe has to be obtained in a solenoid 20 cm long and 5 cm in diameter. Find: (1) the number of ampere-turns required for this solenoid, (2) the potential difference which must be applied across the ends of the winding made of copper wire 0.5 mm in diameter. Regard the solenoid field as a homogeneous one.

11.31. What should the ratio between the length of a coil and its diameter be to allow the intensity of the magnetic field at the coil centre to be found from the formula for the intensity of the field of an infinitely long solenoid? The error in this assumption should not exceed 5%.

Note. The permissible error $\delta = \frac{H_2 - H_1}{H_2}$, where H_2 is the intensity of the magnetic field inside an infinitely long coil, and H_1 is the field intensity inside a coil of finite length.

11.32. What error is made in determining the intensity of a magnetic field at the centre of a solenoid, assuming the solenoid in Problem 11.30 to be infinitely long.

11.33. Find the distribution of the intensity of a magnetic field along the axis of a solenoid 3 cm long and 2 cm in diameter. The current flowing in the solenoid is 2 A. The coil has 100 turns. Compile a table showing the values of H for those of x within $0 \leqslant x \leqslant 3$ cm at intervals of 0.5 cm and plot a diagram with indication of the scale.

11.34. A capacitor with a capacitance of 10^{-6} F is periodically charged from a battery whose e.m.f. is 100 V and discharged through a coil made in the form of a ring 20 cm in diameter and with 32 turns. The plane of the ring coincides with that of the magnetic meridian. A horizontal magnetic needle placed in the centre of the coil deflects through an angle of 45°. The capacitor is switched over 100 times a second. Find the horizontal component of the intensity of the terrestrial magnetic field from the data of these experiments.

11.35. A capacitor with a capacitance of 10 μF is periodically charged from a battery which produces a potential difference of 120 V and is discharged through a solenoid 10 cm long and with 200 turns. The mean intensity of the magnetic field inside the solenoid is 3.02 Oe. How many times is the capacitor switched over a second? Consider the diameter of the solenoid to be small as compared with its length.

11.36. A square frame is placed in a homogeneous magnetic field with an intensity of 1,000 Oe. The plane of the frame forms an angle of 45° with the direction of the magnetic field. The side of the frame is 4 cm. Determine the magnetic flux passing through the frame.

11.37. A rod 1 m long revolves in a magnetic field whose induction is 0.05 T. The axis of rotation passing through one of the rod ends is parallel to the force lines of the magnetic field. Find the magnetic flux intersected by the rod during each revolution.

11.38. A frame whose area is 16 cm^2 revolves in a homogeneous magnetic field making 2 revolutions a second. The axis of rotation is in the plane of the frame and perpendicular to the force lines of the magnetic field. The intensity of the magnetic field is 7.96×10^4 A/m. Find: (1) the relationship between the magnetic flux passing through the frame and the time, (2) the highest magnitude of the magnetic flux.

11.39. An iron specimen is placed in a magnetic field with an intensity of 10 Oe. Determine the permeability of the iron in these conditions.

11.40. How many ampere-turns are required for the volume density of the energy of a magnetic field to be 1.75 J/m^3 inside a small-diameter solenoid 30 cm long?

11.41. How many ampere-turns are required to generate a magnetic flux of 42,000 Mx in a solenoid having an iron core 120 cm long with a cross-sectional area of 3 cm^2?

11.42. The length of the iron core of a toroid is 2.5 m and that of the air-gap is 1 cm. The number of turns of the toroid winding is 1,000. At a current of 20 A, the induction of the magnetic field in the air-gap is 1.6 T. Find the permeability of the iron core in these conditions. The relation $B=f(H)$ is unknown for the given grade of iron.

11.43. The length of the iron core of a toroid is 1 m and that of the air-gap 1 cm. The cross-sectional area of the core is 25 cm^2. Find the number of ampere-turns required to induce a magnetic flux of 1.4×10^6 Mx if in these conditions the permeability of the core material is equal to 800. The relation $B=f(H)$ is unknown for the given grade of iron.

11.44. Determine the magnetic induction in the closed iron core of a toroid 20.9 cm long if the number of ampere-turns of the toroid winding is 1,500. Find the permeability of the core material in these conditions.

11.45. The length of the iron core of a toroid $l_2=1$ m and that of the air-gap $l_1=3$ mm. The number of turns of the toroid winding $N=2{,}000$. Find the intensity of the magnetic field H_1 in the air-gap with a current $I=1$ A in the toroid winding.

11.46. The length of the iron core of a toroid is 50 cm and that of the air-gap 2 mm. The number of ampere-turns of the toroid winding is 2,000. How many times will the intensity of the magnetic field diminish in the air-gap if the length of the latter is doubled with the same number of ampere-turns?

11.47. A solenoid 25.1 cm long and 2 cm in diameter contains an iron core. The solenoid has 200 turns. Plot a diagram for the solenoid and its core showing the magnetic flux Φ versus the current intensity I within $0\leqslant I\leqslant5$ A at intervals of 1 A. Lay off $\Phi\times10^4$ Wb along the axis of ordinates.

11.48. The magnetic flux through a coreless solenoid is 5×10^{-6} Wb. Determine the magnetic moment of this solenoid. The solenoid is 25 cm long.

11.49. A long rectilinear conductor with a current of 25 A passes through the centre of an iron ring perpendicular to its plane. The ring has a tetragonal cross section (Fig. 56) with the dimensions $l_1=18$ mm, $l_2=22$ mm and $h=5$ mm. By assuming approximately that the induction is the same at any point of the ring section and is equal to that on the centre line of the ring, find the magnetic flux Φ passing through the cross-sectional area of the ring.

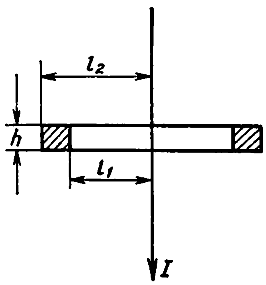


Fig. 56

11.50. Find the magnetic flux Φ passing through the cross-sectional area of the ring of the previous problem, taking into account that the magnetic field is different at different points of the ring cross section. Consider the value of μ_r to be constant and find it from the curve $B=f(H)$ for the value of H on the centre line of the ring.

11.51. A closed iron core 50 cm long has a winding with 1,000 turns which a current of 1 A flows through. What current must be passed through the winding for the induction to remain the same when the core is removed?

11.52. An iron core 50.2 cm long and with an air-gap 0.1 cm long has a winding with 20 turns. What current must be passed through this winding to obtain an induction of 1.2 Wb/m^2 in the gap?

11.53. An iron ring with an average diameter of 11.4 cm has a winding with 200 turns which a current of 5 A flows through. (1) What current must pass through the winding for the induction in the core to remain as before if a slit 1 mm wide is made in the ring? (2) Find the permeability of the core material in these conditions.

11.54. Construct an electromagnet producing a magnetic field induction of 1,400 Gs in the space between the poles. The iron core is 40 cm long, the space between the poles is 1 cm long and the core is 5 cm in diameter. Find: (1) the e.m.f. to be applied across the winding of the electromagnet to produce the necessary magnetic field if copper wire 1 mm^2 in cross section is available, (2) the minimum thickness of the winding if the maximum permissible density of the current is 3 A/mm^2.

11.55. A homogeneous magnetic field with an induction of 1,000 Gs is created between the poles of an electromagnet. A conductor 70 cm long and perpendicular to the force lines carries a current of 70 A. Find the force acting on the conductor.

11.56. Two long parallel rectilinear conductors are 10 cm apart. Currents of $I_1=20$ A and $I_2=30$ A flow through the conductors in one direction. What work is required (per unit length of the conductors) to move the conductors apart to a distance of 20 cm?

11.57. Two long parallel rectilinear conductors are at a certain distance from each other. The conductors carry currents identical in magnitude and direction. Find the intensity of the current flowing in each conductor if work equal to 5.5 erg/cm has to be done (per unit length of the conductors) to move them apart to double this distance.

11.58. A (1) square and a (2) circular circuits are made of wire 20 cm long. Find the rotational moment of the forces acting on each circuit placed in a homogeneous magnetic field with an induction of 1,000 Gs. A current of 2 A flows through the circuits. The plane of each circuit forms an angle of 45° with the direction of the magnetic field.

11.59. An aluminium wire with a cross-sectional area of 1 mm^2 is suspended in a horizontal plane perpendicular to a magnetic meridian, and a current of 1.6 A flows through it (from west to east). (1) What part of the weight of the wire is due to the force of the terrestrial magnetic field acting on it? (2) By how much will the weight of 1 m of the wire be reduced owing to this force? The horizontal component of the terrestrial magnetic field is 0.2 Oe.

11.60. A galvanometer coil consisting of 400 turns of thin wire wound around a rectangular frame 3 cm long and 2 cm wide is suspended from a thread in a magnetic field with an induction of 1,000 Gs. A current of 10^{-7} A flows in the coil. Find the rotational moment acting on the coil if: (1) the plane of the coil is parallel to the direction of the magnetic field, (2) the plane of the coil forms an angle of 60° with the direction of the magnetic field.

11.61. A short magnetic needle with a magnetic moment of 10^{-2} A·m^2 is suspended on a thin thread 100 cm long and 0.1 mm in diameter at a distance of 20 cm from a long vertical rectilinear conductor. The needle is in the plane passing through the conductor and the thread. Through what angle will the needle turn if a current of 30 A is passed through the conductor? The shear modulus of the thread material is 600 kgf/mm^2. The system is screened from the terrestrial magnetic field.

11.62. A galvanometer coil consisting of 600 wire turns is so suspended on a thread 10 cm long and 0.1 mm in diameter in a magnetic field with an intensity of 16×10^4 A/m that the plane of the coil is parallel to the direction of the magnetic field The length of the coil frame $a=2.2$ cm and its width $b=1.9$ cm. What current flows in the winding of the coil if it turns through an angle of 0.5°? The shear modulus of the thread material is 600 kgf/mm^2.

11.63. A square frame is suspended on a wire so that the force lines of a magnetic field form an angle of 90° with a perpendicular to the plane of the frame. One side of the frame is 1 cm. The magnetic induction of the field is 1.37×10^{-2} T. If a current of $I=1$ A is passed through the frame, it will turn through 1° Find the shear modulus of the wire material. The length of the wire is 10 cm and the radius of the thread 0.1 mm.

11.64. A circular contour is so placed in a homogeneous magnetic field that the plane of the contour is perpendicular to the force lines of the field. The intensity of the magnetic field is 2,000 Oe. A current of 2 A flows through the contour. The radius of the contour is 2 cm. What work must be done to turn the contour through 90° around an axis coinciding with its diameter?

11.65. A conductor 10 cm long uniformly moves in a homogeneous magnetic field with an induction of 0.5 Wb/m². A current of 2 A flows in the conductor. The conductor has a velocity of 20 cm/s perpendicular to the direction of the magnetic field. Find: (1) the work done to move the conductor during 10 seconds of its motion, (2) the power spent for this motion.

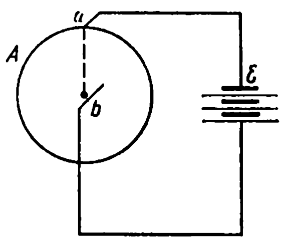


Fig. 57

11.66. Copper disk *A* in Fig. 57 has a radius of $r=5$ cm, and its plane is perpendicular to the direction of a magnetic field with an induction of $B=0.2$ T. A current of $I=5$ A passes along radius *ab* of the disk (*a* and *b* are sliding contacts). The disk rotates with a frequency of $\nu=3$ rev/s. Find: (1) the power of such a motor, (2) the direction of rotation of the disk if the magnetic field is directed towards us from the drawing, (3) the torque (rotational moment) acting on the disk.

11.67. A homogeneous copper disk *A* (see Fig. 57) with a mass of 0.35 kg is so placed in a magnetic field whose induction is 2.4×10^{-2} T that the plane of the disk is perpendicular to the force lines of the field. When circuit *aba* is closed, the disk begins to rotate, and in 30 seconds acquires a velocity of 5 rev/s. Find the current intensity in the circuit.

11.68. Find the magnetic flux intersected by radius *ab* of disk *A* (see Fig. 57) during one minute of rotation. The disk radius $r=10$ cm. The magnetic field induction $B=0.1$ T. The disk revolves at 5.3 rev/s.

11.69. An electron accelerated by a potential difference of 1,000 V flies into a homogeneous magnetic field perpendicular to the direction of its motion. The induction of the field is 1.19×10^{-3} T. Find: (1) the radius of curvature of the electron trajectory, (2) the period of its rotation along the circle, (3) the moment of the momentum of the electron.

11.70. An electron accelerated by a potential difference of 300 V moves parallel to a long rectilinear conductor at a distance of 4 mm from it. What force will act on the electron if a current of 5 A flows through the conductor?

11.71. A stream of α-particles (helium atom nuclei) accelerated by a potential difference of 1 MV flies into a homogeneous magnetic field with an intensity of 15,000 Oe. The velocity of each particle is directed at right angles to the direction of the magnetic field. Find the force acting on each particle.

11.72. An electron flies into a homogeneous magnetic field perpendicular to the force lines. The velocity of the electron $v=4\times 10^7$ m/s. The induction of the magnetic field is 10^{-3} T. What are the tangential and normal accelerations of the electron in the magnetic field?

11.73. Find the kinetic energy of a proton moving along the arc of a circle with a radius of 60 cm in a magnetic field having an induction of 10^4 Gs.

11.74. A proton and an electron move with the same velocity and penetrate into a homogeneous magnetic field. How many times is the radius of curvature of the path of the proton R_1 greater than that of the electron R_2?

11.75. A proton and an electron accelerated by the same potential difference fly into a homogeneous magnetic field. How many times is the radius of curvature of the path of the proton R_1 greater than that of the electron R_2?

11.76. The path of an electron photographed in a Wilson chamber which is placed in a magnetic field takes the form of an arc of a circle with a radius of 10 cm. The induction of the magnetic field is 10^{-2} T. Find the energy of the electron in electron-volts.

11.77. A charged particle moves in a magnetic field over a circle with a velocity of 10^6 m/s. The induction of the magnetic field is 0.3 T. The radius of the circle is 4 cm. Find the charge of the particle if its energy is 12 keV.

11.78. A proton and an α-particle fly into a homogeneous magnetic field. The velocity of the particles is perpendicular to the force lines of the field. How many times is the period of revolution of the proton in the magnetic field greater than that of the α-particle?

11.79. An α-particle with a kinetic energy of 500 eV flies into a homogeneous magnetic field perpendicular to its velocity. The induction of the magnetic field is 1,000 Gs. Find: (1) the force acting on the particle, (2) the radius of the circle along which the particle moves, (3) the period of revolution of the particle.

11.80. An α-particle whose moment of momentum is 1.33×10^{-22} kg·m²/s flies into a homogeneous magnetic field perpendicular to its velocity. The induction of the magnetic field is 2.5×10^{-2} T. Find the kinetic energy of the α-particle.

11.81. Singly charged ions of potassium isotopes with atomic weights of 39 and 41 are accelerated by a potential difference of 300 V. Then they get into a homogeneous magnetic field perpendicular to the direction of their motion. The induction of the magnetic field is 800 Gs. Find the radii of curvature of the trajectories of these ions.

11.82. Find the ratio Q/m for a charged particle if it moves along the arc of a circle with a radius of 8.3 cm upon flying with a velocity of 10^8 cm/s into a homogeneous magnetic field with an intensity of 2,500 Oe. The direction of the velocity of the particle is perpendicular to the direction of the magnetic field. Compare the determined Q/m ratio with that of an electron, a proton and an α-particle.

11.83. A beam of electrons accelerated by a potential difference of $U=300$ V flies into a homogeneous magnetic field (Fig. 58) directed from the drawing towards the observer. The width of the field $l=2.5$ cm. In the absence of a magnetic field, the electron beam produces a spot at point F on a fluorescent screen AA at a distance of $l_1=5$ cm from the edge of the poles of the magnet. When the magnetic field is switched on the spot moves to point B. Find the displacement $x=FB$ of the electron beam if the induction of the magnetic field is 1.46×10^{-5} Wb/m^2.

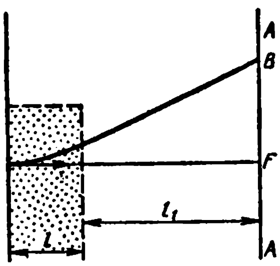


Fig. 58

11.84. A magnetic field with an intensity of $H=8\times10^3$ A/m and an electric field with an intensity of $E=10$ W/cm are directed similarly. An electron flies into such an electromagnetic field with a velocity of $v=10^5$ m/s. Find the normal a_n, tangential a_t and total a accelerations of the electron. Solve the problem when: (1) the velocity of the electron is directed parallel to the force lines, and (2) the velocity of the electron is directed perpendicular to the force lines of the fields.

11.85. A magnetic field with an induction of $B=5$ Gs is perpendicular to an electric field with an intensity of $E=10$ V/cm. A beam of electrons flies with a certain velocity v into the space where these fields are present, the velocity of the electrons being perpendicular to the plane in which the vectors E and B lie. Find: (1) the velocity v of the electrons if the electron beam is not deflected when both fields act simultaneously, (2) the radius of curvature of the trajectory of the electrons when only the magnetic field is switched on.

11.86. An electron accelerated by a potential difference of $U=6$ kV flies into a homogeneous magnetic field at an angle of $\alpha=30°$ to the direction of the field and begins to move helically. The induction of the magnetic field $B=1.3\times10^{-2}$ Wb/m^2. Find: (1) the radius of a helix turn, and (2) the helix pitch.

11.87. A proton flies into a homogeneous magnetic field at an angle of $\alpha=30°$ to the direction of the field and moves along a helix with a radius of 1.5 cm. The induction of the magnetic field is 10^3 Gs. Find the kinetic energy of the proton.

11.88. An electron flies into a plane horizontal capacitor parallel to its plates with a velocity of $v_0=10^7$ m/s. The length of the capacitor $l=5$ cm and the intensity of its electric field $E=100$ V/cm. When the electron leaves the capacitor, it gets into a magnetic field whose force lines are perpendicular to those of the electric field. The induction of the magnetic field $B=10^{-2}$ T. Find: (1) the radius of the helical trajectory of the electron in the magnetic field, and (2) the pitch of the helical trajectory of the electron.

11.89. An electron accelerated by a potential difference of $U=$ $=3{,}000$ V flies into the magnetic field of a solenoid at an angle of $\alpha=30°$ to its axis. The number of solenoid ampere-turns is 5,000 and its length is 25 cm. Find the pitch of the helical trajectory of the electron in the magnetic field of the solenoid.

11.90. A current of $I=20$ A flows through the section $A=ab$ of a copper plate $a=0.5$ mm thick and $b=10$ mm high. When the plate is placed in a magnetic field perpendicular to rib b and to the direction of the current, a transverse potential difference of $U=3.1\times10^{-6}$ V appears. The induction of the magnetic field $B=1$ T. Find: (1) the concentration of the conduction electrons in the copper, and (2) the average velocity of the electrons in these conditions.

11.91. A current of $I=5$ A flows through the section $A=ab$ of an aluminium plate (a is the thickness and b the height). The plate is placed in a magnetic field perpendicular to rib b and to the direction of the current. Determine the resulting transverse potential difference if the induction of the magnetic field $B=0.5$ T and the plate is $a=0.1$ mm thick. The concentration of the conduction electrons is the same as that of the atoms.

11.92. A semiconductor plate $a=0.2$ mm thick is placed in a magnetic field directed along a. The resistivity of the semiconductor $\rho=10^{-5}$ $\Omega\cdot$m and the induction of the magnetic field $B=1$ T. A current of $I=0.1$ A is made to flow along the plate perpendicular to the field, generating a transverse potential difference of $U=3.25\times10^{-3}$ V. Determine the mobility of the current carriers in the semiconductor.

11.93. A conductor 10 cm long moves in a homogeneous magnetic field whose induction is 0.1 T. The velocity of the conductor is 15 m/s and it is directed perpendicular to the magnetic field. What is the e.m.f. induced in the conductor?

11.94. A coil 10 cm in diameter with 500 turns is placed in a magnetic field. What is the mean e.m.f. of induction in this coil if the induction of the magnetic field increases from 0 to 2 Wb/m^2 during 0.1 second?

11.95. The velocity of a jet airplane is 950 km/h. Find the induction e.m.f. induced on the ends of its wings if the vertical component of the intensity of the terrestrial magnetic field is 0.5 Oe and the wing span is 12.5 m.

11.96. A rod 1 m long revolves at a constant angular velocity of 20 rad/s in a magnetic field with an induction of 500 Gs. The axis of rotation passes through the end of the rod and is parallel to the force lines of the magnetic field. Find the induction e.m.f. appearing on the rod ends.

11.97. The principle of operation of an electromagnetic liquid flow meter is explained in Fig. 59. A pipe with a current-conducting liquid is put in a magnetic field. An induction e.m.f. appears on the electrodes *A* and *B*. Find the rate of flow of the liquid in the pipe if the magnetic field induction is 100 Gs, the distance between the electrodes (internal diameter of the pipe) is 50 mm and the resulting e.m.f. is 0.25 mV.

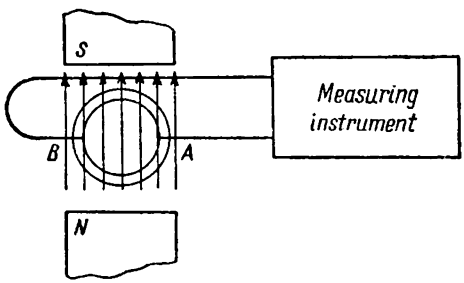


Fig. 59

11.98. A circular wire turn with an area of 100 cm^2 is placed in a homogeneous magnetic field whose induction is 1 Wb/m^2. The plane of the turn is perpendicular to the direction of the magnetic field. What is the mean induction e.m.f. appearing in the turn when the field is switched off during 0.01 second?

11.99. A coil consisting of 100 turns of wire rotates uniformly in a homogeneous magnetic field having an induction of 1,000 Gs. The coil makes 5 rev/s. The cross-sectional area of the coil is 100 cm^2. The axis of rotation is perpendicular to the coil axis and the direction of the magnetic field. Find the maximum induction e.m.f. in the revolving coil.

11.100. A frame rotates uniformly with an angular velocity of 15 rad/s in a homogeneous magnetic field having an induction of 0.8 T. The area of the frame is 150 cm^2. The axis of rotation is in the plane of the frame and forms an angle of 30° with the direction of the force lines of the magnetic field. Find the maximum induction e.m.f. in the rotating frame.

11.101. In Fig. 60, *D* is a copper disk with a radius of 5 cm whose plane is perpendicular to the direction of the magnetic field, and *a* and *b* are sliding contacts allowing a current to flow through the circuit $abAa$. The induction of the magnetic field is 2,000 Gs and the disk rotates at 3 rev/s. Find the e.m.f. of such a generator. Indicate the direction of the electric current if the magnetic field is directed from the observer towards the drawing and the disk rotates counterclockwise.

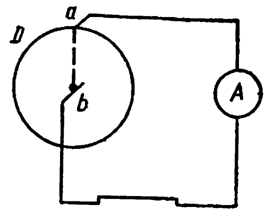


Fig. 60

11.102. A horizontal rod 1 m long rotates about a vertical axis passing through one of its ends. The axis of rotation is parallel to the force lines of a magnetic field with an induction of 5×10^{-5} T. At what number of revolutions per second will the potential difference across the ends of this rod be equal to 1 mV?

11.103. A wire turn is placed on a solenoid 20 cm long and with a cross-sectional area of 30 cm². The solenoid has 320 turns and a current of 3 A flows through it. What mean e.m.f. is induced in the turn placed on the solenoid when the current in the solenoid is switched off during 0.001 second?

11.104. What is the mean e.m.f. induced in the turn if the solenoid in the previous problem has an iron core?

11.105. A wire turn is placed on a solenoid 144 cm long and 5 cm in diameter. The solenoid winding has 2,000 turns and a current of 2 A flows through it. The solenoid has an iron core. What mean e.m.f. is induced in the turn placed on the solenoid if the current is switched off in the latter during 0.002 second?

11.106. A coil consisting of 200 turns rotates in a homogeneous magnetic field whose induction is 0.1 T. The axis of rotation is perpendicular to the axis of the coil and to the direction of the magnetic field. The period of rotation of the coil is 0.2 s and its cross-sectional area is 4 cm². Find the maximum induction e.m.f. in the rotating coil.

11.107. (1) Find the inductance of a coil having 400 turns over a length of 20 cm. The cross-sectional area of the coil is 9 cm². (2) Find the inductance of this coil if an iron core is introduced into it. In conditions of operation, the permeability of the core material is 400.

11.108. A solenoid winding consists of N turns of copper wire with a cross section of $A=1$ mm². The length of the solenoid $l=25$ cm and its resistance $R=0.2\ \Omega$. Find the inductance of the solenoid.

11.109. A coil 20 cm long and 3 cm in diameter has 400 turns. A current of 2 A flows in the coil. Find: (1) the inductance of the coil, (2) the magnetic flux piercing its cross-sectional area.

11.110. How many turns of wire does a single-layer coil winding consist of if the inductance of the coil is 0.001 H. The diameter of the coil is 4 cm and that of the wire 0.6 mm. The turns fit tightly against one another.

11.111. A coil with an iron core has a cross-sectional area of 20 cm^2 and 500 turns. The inductance of the coil with the core is 0.28 H and the current flowing through the winding is 5 A. Find the permeability of the iron core in these conditions.

11.112. A solenoid 50 cm long and with a cross-sectional area of 2 cm^2 has an inductance of 2×10^{-7} H. At what current intensity will the volume density of the energy of the magnetic field inside the solenoid be equal to 10^{-3} J/m^3?

11.113. How many turns does a coil have if its inductance $L=0.001$ H and the magnetic flux through the coil $\Phi=200$ Mx at a current of $I=1$ A.

11.114. The cross-sectional area of a solenoid with an iron core is 10 cm^2. (1) Find the magnetic permeability of the core material in conditions when the magnetic flux piercing the cross section of the solenoid is 1.4×10^{-3} Wb. (2) Determine the intensity of the current flowing through the solenoid which this magnetic flux corresponds to if the inductance of the solenoid in these conditions is 0.44 H. The solenoid is 1 m long.

11.115. A core is inserted into a solenoid 50 cm long. The core is made of iron for which the relation $B=f(H)$ is unknown. The number of turns per unit length of the solenoid is 400 and its cross-sectional area is 10 cm^2. (1) Find the permeability of the core when a current of 5 A flows through the solenoid winding. In these conditions, the magnetic flux piercing the cross-sectional area of the solenoid with the core is equal to 1.6×10^{-3} Wb (2) Find the inductance of the solenoid in these conditions.

11.116. A solenoid with an iron core is 50 cm long, has a cross-sectional area of 10 cm^2 and 1,000 turns. Determine the inductance of this solenoid if the current flowing through the winding of the solenoid is: (1) $I_1=0.1$ A, (2) $I_2=0.2$ A, and (3) $I_3=2$ A.

11.117. Two coils are wound on one common core. The inductance of the first coil is 0.2 H and that of the second 0.8 H. The resistance of the second coil is 600 Ω. What current will flow through the second coil if the current of 0.3 A in the first coil is switched off during 0.001 second?

11.118. A square frame made of copper wire is placed in a magnetic field with an induction of 0.1 T. The cross-sectional area of the wire is 1 mm^2, the area of the frame is 25 cm^2 and a perpendicular to the plane of the frame is directed along the force lines of the field. What quantity of electricity will pass through the frame when the magnetic field disappears?

11.119. A coil consisting of 200 turns of wire is placed in a magnetic field with an induction of 500 Gs. The resistance of the coil is 40 Ω and its cross-sectional area is 12 cm^2. The coil is so placed that its axis forms an angle of 60° with the direction of the magnetic field. What quantity of electricity will pass through the coil when the magnetic field disappears?

11.120. A circular contour with a radius of 2 cm is placed in a homogeneous magnetic field with an induction of 0.2 Wb/m^2. The plane of the contour is perpendicular to the direction of the magnetic field, and the resistance of the contour is 1 Ω. What quantity of electricity will pass through the coil when it turns through 90°?

11.121. A coil consisting of 50 turns is placed on a solenoid 21 cm long with a cross-sectional area of 10 cm^2. The coil is connected with a ballistic galvanometer having a resistance of 10^3 Ω. A current of 5 A flows in the winding of the solenoid, consisting of 200 turns. Find the ballistic constant of the galvanometer if its pointer deflects by 30 scale divisions when the current in the solenoid is switched off. Disregard the resistance of the coil, which is small when compared with that of the galvanometer. The ballistic constant of a galvanometer is a quantity numerically equal to the quantity of electricity which makes the pointer deflect by one scale division.

11.122. A coil consisting of 50 turns of wire and connected to a ballistic galvanometer is placed between the poles of an electromagnet to measure the induction of a magnetic field. The axis of the coil is parallel to the direction of the magnetic field The cross-sectional area of the coil is 2 cm^2 and its resistance may be neglected as compared with that of the galvanometer. The resistance of the galvanometer is 2×10^3 Ω and its ballistic constant is 2×10^{-8} C/div. When the coil is rapidly pulled out of the magnetic field, the pointer on the galvanometer deflects by 50 scale divisions. What is the induction of the magnetic field?

11.123. The relation between the permeability μ_r and the intensity of a magnetic field H was investigated by A. G. Stoletov in his work on investigation of the functions of magnetizing soft iron (published in 1872). An iron specimen was given the form of a toroid. The iron was magnetized by passing a current I through a coil wound around the toroid. A change in the direction of the current in this primary coil caused a deflection of α in a ballistic galvanometer. The galvanometer was connected to the circuit of a secondary coil wound around the same toroid.

The toroid used by A. G. Stoletov had the following parameters: cross-sectional area $A=1.45$ cm^2, length $l=60$ cm, number of turns of the primary coil $N_1=800$, number of turns of the secondary coil $N_2=100$. The ballistic constant of the galvanometer was $C_b=1.2\times 10^{-5}$ C/div and the resistance of the secondary circuit was 12 Ω.

The results of one of Stoletov's experiments are given in the following table.

I, A	0.1	0.2	0.3	0.4	0.5
α (in scale divisions)	48.7	148	208	241	256

Use these data to compile a table and draw a corresponding diagram showing how the permeability μ_r depends on the intensity of the magnetic field H for the iron employed by A. G. Stoletov in his experiment.

11.124. An iron toroid with a length of $l=50$ cm and a cross-sectional area of $A=4$ cm^2 was made to measure the permeability of iron. One of the toroid windings had $N_1=500$ turns and was connected to a source of power, and the other had $N_2=1{,}000$ turns and was connected to a galvanometer. By reversing the direction of the current in the primary winding, we can create an induction current in the secondary winding. Find the permeability of the iron when a current of 1 A is reversed in the primary winding, if a quantity of electricity equal to 0.06 C passes through the galvanometer. The resistance of the secondary winding is 20 Ω.

11.125. An electric lamp whose resistance in a hot state is 10 Ω is connected via a choke to a 12-V storage battery. The inductance of the choke is 2 H and its resistance 1 Ω. In how much time after being switched on will the lamp light if it begins to glow noticeably at a voltage of 6 V?

11.126. A coil 20 cm long and 2 cm in diameter has a winding comprising 200 turns of copper wire with a cross-sectional area of 1 mm^2. The coil is connected to a circuit with a certain e.m.f., which is then switched off and the coil is short circuited. In how much time after the e.m.f. is switched off will the current intensity in the circuit drop to half of its original value.

11.127. The inductance of a coil is 0.2 H and its resistance is 1.64 Ω. How many times will the current intensity in the coil drop in 0.05 second after the e.m.f. is switched off and the coil is short circuited.

11.128. A coil with a resistance of $R=10$ Ω has an inductance of $L=0.144$ H. How much time is required for the current to become equal to half of the steady current in the coil after it is switched on?

11.129. A circuit has a resistance of 2 Ω and an inductance of 0.2 H. Plot a diagram showing the increase of the current intensity in the circuit versus the time elapsed from the moment the e.m.f. is switched on. Plot the ratio between the intensity of the increasing current I and that of the final current I_0 along the axis of ordinates for $0 \leqslant t \leqslant 0.5$ second at intervals of 0.1 second.

11.130. A square frame made of copper wire with a cross section of 1 mm^2 is placed in a magnetic field whose induction changes according to the law $B=B_0 \sin\omega t$, where $B_0=0.01$ T, $\omega=\frac{2\pi}{T}$ and $T=0.02$ s. The area of the frame is 25 cm². The plane of the frame is perpendicular to the direction of the magnetic field. Find the dependence on time and the maximum value of: (1) the magnetic flux piercing the frame, (2) the induction e.m.f. appearing in the frame, (3) the intensity of the current flowing in the frame.

11.131. Through a coil with an inductance of 0.021 H there flows a current that changes with time according to the law $I=I_0 \sin \omega t$, where $I_0=5$ A, $\omega=\frac{2\pi}{T}$ and $T=0.02$ s. Determine the dependence on time of: (1) the e.m.f. of self-induction appearing in the coil, (2) the energy of the magnetic field in the coil.

11.132. Two coils have a mutual inductance of 0.005 H. The intensity of the current in the first coil changes according to the law $I=I_0 \sin\omega t$, where $I_0=10$ A, $\omega=\frac{2\pi}{T}$ and $T=0.02$ s. Find: (1) the dependence on time of the e.m.f. induced in the second coil, (2) the maximum value of this e.m.f.

Chapter 4

OSCILLATIONS AND WAVES

ACOUSTIC UNITS

Acoustic quantities are measured in the MKS system, which is a part of the SI system.

The basic and some derived acoustic units of the MKS system as specified by GOST 8849-58 are given in Table 13.

TABLE 13

Quantity and symbol	Formula	Unit	Symbol of unit	Dimension of quantity
		Basic Units		
Length l	—	metre	m	l
Mass m	—	kilogramme	kg	m
Time t	—	second	s	t
		Derived Units		
Sound pressure p	$p=\frac{F}{S}$	newton per square metre	N/m^2	$l^{-1}mt^{-2}$
Volume velocity	$v_{vol}=vS$	cubic metre per second	m^3/s	l^3t^{-1}
Sound intensity	$I=\frac{E}{St}$	watt per square metre	W/m^2	mt^{-3}
Density of sound energy	$w=\frac{E}{V}$	joule per cubic metre	J/m^3	$l^{-1}mt^{2}$

TABLE 14

Quantity	Unit and its conversion factor to SI units
Sound pressure	$1\ dyn/cm^2=0.1\ N/m^2$
Volume velocity	$1\ cm^3/s=10^{-6}\ m^3/s$
Sound intensity	$1\ erg/cm^2s=10^{-3}\ W/m^2$
Density of sound energy	$1\ erg/cm^3=0.1\ J/m^3$

Table 14 shows some acoustic units of the CGS system and their relation to the SI units.

Some arbitrary acoustic units permitted by GOST 8849-58 are given in Table 15.

TABLE 15

Quantity	Unit	Symbol of unit	Definition of unit
Level of sound pressure	decibel	db	A level of sound pressure, twenty common logarithms of whose ratio to an arbitrary pressure threshold equal to 2×10^{-5} N/m^2 taken as the zero level are equal to unity
Loudness level	phon	phon	A level of sound loudness for which the level of sound pressure of an equally loud sound with a frequency of 10^3 Hz is equal to 1 db

EXAMPLES OF SOLUTIONS

Example 1. The amplitude of harmonic oscillations of a material point is 5 cm, its mass is 10 grammes and the total energy of the oscillations is 3.1×10^{-5} J. Write the equation of the harmonic oscillations of this point (with numerical coefficients) if the initial oscillation phase is 60 degrees.

Solution. The general equation of harmonic oscillations is

$$x = A\sin\left(\frac{2\pi t}{T}+\varphi\right) \qquad (1)$$

In our case $A=5$ cm, $\varphi=60°=\frac{\pi}{3}$. The oscillation period T is unknown, but it can be found from the condition $E=\frac{2\pi^2A^2m}{T^2}=3.1\times10^{-5}$ J. Hence

$$T=\sqrt{\frac{2\pi^2A^2m}{E}} \qquad (2)$$

In our case $A=5\times10^{-2}$ m, the mass $m=10^{-2}$ kg and $E=3.1\times10^{-5}$ J. Upon inserting these data in Eq. (2), we get $T=4$ s. Hence $\frac{2\pi t}{T}=\frac{2\pi t}{4}=\frac{\pi}{2}t$, and equation (1) becomes $x=5\sin\left(\frac{\pi}{2}t+\frac{\pi}{3}\right)$ cm. Since $\sin\left(\frac{\pi}{2}t+\frac{\pi}{3}\right)$ is a dimensionless magnitude, it is not necessary to insert A in metres; x will correspond to the dimension of the amplitude A.

Example 2. The sound pressure level L_1 is 40 db. Find the amplitude of the sound pressure and the sound intensity. Assume the audibility threshold to be $I_0=10^{-12}$ W/m^2.

Solution. The sound pressure level L_1 in decibels is related to the amplitude of the sound pressure Δp by the equation

$$L_1 = 20 \log_{10} \frac{\Delta p}{\Delta p_0} \tag{1}$$

where Δp_0 is the amplitude of the sound pressure at the zero loudness level. In the MKS system we have $\Delta p_0=2\times10^{-5}$ N/m^2. According to the initial condition, $L_1=40$ db. Hence, from Eq. (1) we have $\log_{10}\frac{\Delta p}{\Delta p_0}=2$, whence $\frac{\Delta p}{\Delta p_0}=10^2$; and the sought amplitude of the sound pressure will be $\Delta p=\Delta p_0\times10^2=2\times10^{-5}\times10^2$ N/m^2$=2\times10^{-3}$ N/m^2.

The loudness level L_2 in phons is related to the sound intensity as follows

$$L_2 = 10 \log_{10} \frac{I}{I_0} \tag{2}$$

From the definition of the phon, we have $L_2=40$ phon when $L_1=40$ db. Then from Eq. (2) $\log_{10}\frac{I}{I_0}=4$ or $\frac{I}{I_0}=10^4$, and the sought sound intensity $I=I_0\times10^4=10^{-12}\times10^4$ W/m^2$=10^{-8}$ W/m^2.

12. Harmonic Oscillatory Motion and Waves

The equation of harmonic oscillatory motion can be written as

$$x = A \sin\left(\frac{2\pi t}{T}+\varphi\right) = A\sin(2\pi\nu t+\varphi) = A\sin(\omega t+\varphi)$$

where x=displacement of a point from the position of equilibrium, different for different moments of time

A=amplitude

T=period

φ=initial phase

$\nu=\frac{1}{T}$=oscillation frequency

$\omega=\frac{2\pi}{T}$=angular frequency.

The velocity of an oscillating point is

$$v=\frac{dx}{dt}=\frac{2\pi A}{T}\cos\left(2\pi\frac{t}{T}+\varphi\right)$$

and the acceleration

$$a=\frac{dv}{dt}=\frac{d^2x}{dt^2}=-\frac{4\pi^2A}{T^2}\sin\left(2\pi\frac{t}{T}+\varphi\right)$$

The force which causes a point of mass m to oscillate harmonically is

$$F=ma=-\frac{4\pi^2 A}{T^2}m\sin\left(2\pi\frac{t}{T}+\varphi\right)=-\frac{4\pi^2 m}{T^2}x=-kx$$

where $k=\frac{4\pi^2 m}{T^2}$, whence $T=2\pi\sqrt{\frac{m}{k}}$.

Here T is the period of oscillations of a point oscillating under the action of the force $F=-kx$, where k is the deformation coefficient, numerically equal to the force producing a displacement equal to unity.

The kinetic energy of an oscillating point is

$$E_k=\frac{mv^2}{2}=\frac{2\pi^2 A^2 m}{T^2}\cos^2\left(\frac{2\pi t}{T}+\varphi\right)$$

and its potential energy

$$E_p=\frac{kx^2}{2}=\frac{2\pi^2 A^2 m}{T^2}\sin^2\left(\frac{2\pi t}{T}+\varphi\right)$$

The total energy will be

$$E=\frac{2\pi^2 A^2 m}{T^2}$$

Small oscillations of a pendulum serve as an example of harmonic oscillatory motion. The period of oscillations of a mathematical pendulum is

$$T=2\pi\sqrt{\frac{l}{g}}$$

where l=length of pendulum
g=gravity acceleration.

Upon summation of two identically directed harmonic oscillations having the same period, we obtain a harmonic oscillation of this period with the amplitude

$$A=\sqrt{A_1^2+A_2^2+2A_1A_2\cos(\varphi_2-\varphi_1)}$$

and with the initial phase determined from the equation

$$\tan\varphi=\frac{A_1\sin\varphi_1+A_2\sin\varphi_2}{A_1\cos\varphi_1+A_2\cos\varphi_2}$$

where A_1 and A_2 are the amplitudes of the oscillations being added and φ_1 and φ_2 their initial phases.

Upon summation of two mutually perpendicular oscillations having the same period, the equation of the path of the resulting motion takes the form

$$\frac{x^2}{A_1^2}+\frac{y^2}{A_2^2}-\frac{2xy}{A_1A_2}\cos(\varphi_2-\varphi_1)=\sin^2(\varphi_2-\varphi_1)$$

If, besides the elastic force $F=-kx$, a material point with the mass m is acted upon by the friction force $F_{fr}=-rv$, where r is the coefficient of friction and v is the velocity of the oscillating point, then the oscillations of the point will be damped.

The equation of damped oscillatory motion has the form

$$x=Ae^{-\delta t}\sin(\omega t+\varphi)$$

where δ is the damping coefficient. Here $\delta=\frac{r}{2m}$ and $\omega=\sqrt{\omega_0^2-\delta^2}$, where ω_0 is the angular frequency of natural oscillations.

The quantity δT is known as the logarithmic damping decrement.

If a material point with the mass m and whose oscillation is given in the form

$$x_1=Ae^{-\delta t}\sin\omega_0 t$$

is acted upon by an external periodic force $F=F_0\sin\omega t$, the oscillations of the point will be forced, and the equation of its motion will take the form

$$x_2=A\sin(\omega t+\varphi)$$

where

$$A=\frac{F_0}{m\sqrt{(\omega_0^2-\omega^2)^2+4\delta^2\omega^2}}$$

and

$$\tan\varphi=\frac{2\delta\omega}{\omega_0^2-\omega^2}$$

Resonance sets in when the frequency of the forced oscillations ω is related to the frequency of natural oscillations ω_0 and the damping coefficient δ as follows:

$$\omega=\sqrt{\omega_0^2-2\delta^2}$$

When undamped oscillations propagate with the velocity c along a certain direction called a ray, the displacement of any point lying on the ray and removed from the source of oscillations by a distance l is described by the equation

$$x=A\sin\left(\frac{2\pi t}{T}-\frac{2\pi l}{\lambda}\right)$$

where A=amplitude of oscillating points
λ=wavelength.

Here $\lambda=cT$. Two points lying on the ray at distances l_1 and l_2 from the source of oscillations have the phase difference

$$\varphi_2-\varphi_1=2\pi\frac{l_2-l_1}{\lambda}$$

Upon interference of the waves, the maximum amplitude is obtained when

$$l_2-l_1=2n\frac{\lambda}{2} \quad (n=0,\ 1,\ 2,\ \ldots)$$

where l_2-l_1 is the difference of the ray path.

The amplitude minimum is obtained when

$$l_2-l_1=(2n+1)\frac{\lambda}{2} \quad (n=0,\ 1,\ 2,\ \ldots)$$

12.1. Write the equation of harmonic oscillatory motion with an amplitude of 5 cm if 150 oscillations are performed during one minute and the initial oscillation phase is 45°. Draw a diagram of this motion.

12.2. Write the equation of harmonic oscillatory motion with an amplitude of 0.1 m, a period of 4 s and an initial phase equal to zero.

12.3. The amplitude of harmonic oscillations is 50 mm, the period 4 s and the initial phase $\frac{\pi}{4}$. (1) Write the equation of this oscillation. (2) Find the displacement of an oscillating point from the equilibrium position at $t=0$ and $t=1.5$ s. (3) Draw a diagram of this motion.

12.4. Write the equation of harmonic oscillatory motion if the initial phase is: (1) 0, (2) $\frac{\pi}{2}$, (3) π, (4) $\frac{3}{2}\pi$, (5) 2π. The amplitude of the oscillations is 5 cm and the period 8 s. Draw a diagram of the oscillations for all these cases.

12.5. Draw on one diagram two harmonic oscillations with the same amplitudes ($A_1=A_2=2$ cm) and the same periods ($T_1=T_2=8$ s), but with a phase difference of (1) $\frac{\pi}{4}$, (2) $\frac{\pi}{2}$, (3) π and (4) 2π.

12.6. In what time after motion begins will a harmonically oscillating point be brought out of the equilibrium position by half the amplitude? The oscillation period is 24 s and the initial phase is zero.

12.7. The initial phase of harmonic oscillation is zero. After the elapse of what fraction of the period will the velocity of the point be equal to half its maximum velocity?

12.8. In what time after its motion begins will a point oscillating according to the equation $x=7\sin 0.5\pi t$ move from the position of equilibrium to the maximum displacement?

12.9. The amplitude of harmonic oscillation is 5 cm and the period 4 s. Find the maximum velocity of an oscillating point and its maximum acceleration.

12.10. The equation of motion of a point is given as $x=2\sin\left(\frac{\pi}{2}t+\frac{\pi}{4}\right)$ cm. Find: (1) the period of oscillations, (2) the maximum velocity of the point, (3) its maximum acceleration.

12.11. The equation of motion of a point is given as $x=\sin \frac{\pi}{6} t$. Find the moments of time at which the maximum velocity and acceleration are attained.

12.12. A point performs harmonic oscillation. The period of oscillations is 2 s, the amplitude 50 mm and the initial phase is zero. Find the velocity of the point at the moment when it is displaced from equilibrium by 25 mm.

12.13. Write the equation of harmonic oscillatory motion if the maximum acceleration of a point is 49.3 cm/s^2, the period of oscillations 2 s and the displacement of the point from equilibrium at the initial moment of time 25 mm.

12.14. The initial phase of harmonic oscillation is zero. When the point deviates by 2.4 cm from the position of equilibrium, its velocity is 3 cm/s, and by 2.8 cm—2 cm/s. Find the amplitude and period of this oscillation.

12.15. The equation of oscillation of a material point with a mass of $m=1.6\times 10^{-2}$ kg has the form $x=0.1\sin\left(\frac{\pi}{8}t+\frac{\pi}{4}\right)$ m. Plot a diagram showing how the force F acting on the point depends on the time t (within one period). Find the maximum force.

12.16. A material point with a mass of 10 g oscillates according to the equation $x=5\sin\left(\frac{\pi t}{5}+\frac{\pi}{4}\right)$ cm. Find the maximum force acting on the point and the total energy of the oscillating point.

12.17. The equation of oscillation of a material point with a mass of 16 g has the form $x=2\sin\left(\frac{\pi t}{4}+\frac{\pi}{4}\right)$ cm. Plot a diagram showing how the kinetic, potential and total energies of the point depend on the time (within one period).

12.18. What is the ratio between the kinetic energy of a harmonically oscillating point and its potential energy for the moments of time: (1) $t=\frac{T}{12}$ s, (2) $t=\frac{T}{8}$ s, (3) $t=\frac{T}{6}$ s? The initial phase of oscillations is zero.

12.19. What is the relationship between the kinetic energy of a harmonically oscillating point and its potential energy for the moments when the displacement of the point from the position of equilibrium is: (1) $x=\frac{A}{4}$, (2) $x=\frac{A}{2}$, (3) $x=A$, where A is the amplitude of oscillations.

12.20. The total energy of a harmonically oscillating body is 3×10^{-5} J and the maximum force acting on the body is 1.5×10^{-3} N. Write the equation of motion of this body if the period of oscillations is 2 s and the initial phase is 60°.

12.21. The amplitude of harmonic oscillations of a material point $A=2$ cm and the total energy of the oscillations $E=3\times 10^{-7}$ J. At what displacement from the position of equilibrium will the oscillating point be acted upon by a force of $F=2.25\times 10^{-5}$ N?

12.22. A ball suspended from a thread 2 m long is deflected through an angle of 4 deg and its oscillations are observed. Assuming the oscillations to be undamped and harmonic, find the velocity of the ball when it passes through the position of equilibrium. Check the solution by finding this velocity from the equations of mechanics.

12.23. A load of 10 kgf is suspended on a spring. Determine the period of vertical oscillations of the load if the spring stretches 1.5 cm under a force of 1 kgf.

12.24. A load is suspended on a spring. Find the deformation coefficient of the spring if the maximum kinetic energy of the oscillations of the load is 1 J. The amplitude of oscillations is 5 cm.

12.25. How will the period of vertical oscillations of a load hanging on two identical springs change if instead of tandem connection the springs are connected in parallel?

12.26. A copper ball suspended on a spring performs vertical oscillations. How will the period of oscillations change if an aluminium ball of the same radius is attached to the spring instead of the copper one?

12.27. A pan with a set of weights is attached to a spring. The period of vertical oscillations is 0.5 s. After additional weights are placed on the pan, the period of vertical oscillations becomes 0.6 s. By how much does the spring stretch owing to the additional weight?

12.28. A weight of 0.5 kgf is suspended from a rubber cord 40 cm long with a radius of 1 mm. Find the period of vertical oscillations of the weight if Young's modulus for the rubber is 0.3 kgf/mm^2.

Note. Remember that the deformation coefficient k of rubber is related to Young's modulus E by the equation $k=\frac{SE}{l}$, where S is the cross-sectional area of the rubber and l its length.

12.29. An aerometer weighing $G=0.2$ kgf floats in a liquid. If the aerometer is slightly submerged and then released it begins to oscillate with a period of $T=3.4$ s. Assuming the oscillations to be undamped, use the data of this experiment to find the density of the liquid ρ in which the aerometer is floating. The diameter of the vertical cylindrical tube of the aerometer $d=1$ cm (see also Chapter 1, Sections 2 and 3).

12.30. Write the equation of motion obtained by the summation of two identically directed harmonic oscillatory motions with the same period of eight seconds and the same amplitude of 0.02 m. The difference of phases between the oscillations is $\frac{\pi}{4}$. The initial phase of one of the oscillations is zero.

12.31. Find the amplitude and the initial phase of the harmonic oscillation obtained by the summation of identically directed oscillations conforming to the equations $x_1=0.02\sin\left(5\pi t+\frac{\pi}{2}\right)$ m and $x_2=0.03\sin\left(5\pi t+\frac{\pi}{4}\right)$ m.

12.32. The summation of two identically directed harmonic oscillations with the same amplitudes and periods produces a resulting oscillation with the same period and amplitude. Find the difference of phases of the initial oscillations.

12.33. (1) Find the amplitude and initial phases of the harmonic oscillation obtained by summation of the identically directed oscillations whose equations are $x_1=4\sin\pi t$ cm and $x_2=3\sin\left(\pi t+\frac{\pi}{2}\right)$ cm. (2) Write the equation of the resulting oscillation. (3) Draw a vector diagram showing the summation of the amplitudes.

12.34. Fig. 61 shows a spectrum of a complex oscillation. (1) Write the equations of the constituent oscillations using the data on the drawing. (2) Draw a diagram of these oscillations assuming that the difference of phases between these oscillations is zero at the moment $t=0$. (3) Plot a diagram of the resulting complex oscillation.

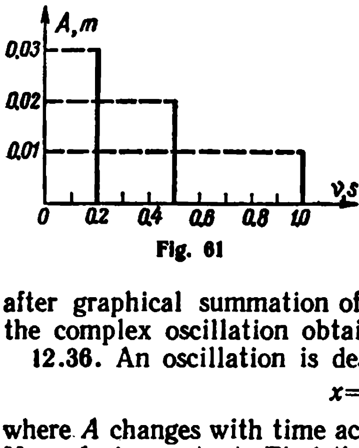


Fig. 61

12.35. There are two harmonic oscillations $x_1=3\sin 4\pi t$ cm and $x_2=6\sin 10\pi t$ cm. Draw a diagram of these oscillations, as well as a diagram of the resulting oscillation after graphical summation of the initial ones. Draw a spectrum of the complex oscillation obtained.

12.36. An oscillation is described by the equation

$$x=A\sin 2\pi\nu_1 t \qquad (1)$$

where A changes with time according to the law $A=A_0(1+\cos 2\pi\nu_2 t)$. Here A_0 is constant. Find the harmonic oscillations forming oscillation (1). Draw a diagram of the constituent and resulting oscillations when $A_0=4$ cm, $\nu_1=2\text{ s}^{-1}$, $\nu_2=1\text{ s}^{-1}$. Plot a spectrum of the complex oscillation.

12.37. Write the equation of the resulting oscillation obtained by summation of two mutually perpendicular oscillations with the same frequency $\nu_1=\nu_2=5$ Hz and the same initial phase $\varphi_1=\varphi_2=60°$. Their amplitudes are $A_1=0.10$ m and $A_2=0.05$ m.

12.38. A point takes part in two oscillations having the same period and the same initial phase. The amplitudes of the oscillations $A_1=3$ cm and $A_2=4$ cm. Find the amplitude of the resulting oscillation if:

(1) the oscillations are in the same direction, (2) the oscillations are mutually perpendicular.

12.39. A point simultaneously participates in two mutually perpendicular oscillations: $x=2\sin\omega t$ m and $y=2\cos\omega t$ m. Find the trajectory of motion of the point.

12.40. A point simultaneously participates in two mutually perpendicular oscillations $x=\cos\pi t$ and $y=\cos\frac{\pi t}{2}$. Find the trajectory of the resulting motion of the point.

12.41. A point simultaneously participates in two mutually perpendicular oscillations $x=\sin\pi t$ and $y=2\sin\left(\pi t+\frac{\pi}{2}\right)$. Find the trajectory of motion of the point, and draw it, showing the scale used.

12.42. A point simultaneously participates in two mutually perpendicular oscillations $x=\sin\pi t$ and $y=4\sin(\pi t+\pi)$. Find the trajectory of motion of the point and draw it showing the scale used.

12.43. The period of damped oscillations is 4 s, the logarithmic damping decrement δT is 1.6 and the initial phase is zero. The displacement of the point is 4.5 cm at $t=\frac{T}{4}$. (1) Write the equation of motion of this oscillation. (2) Plot a diagram of this oscillatory motion within the limits of two periods.

12.44. Plot a diagram of a damped oscillation whose equation is given in the form $x=e^{-0.1t}\sin\frac{\pi}{4}t$ m.

12.45. The equation of damped oscillations is given in the form $x=5e^{-0.25t}\sin\frac{\pi}{2}t$ m. Find the velocity of an oscillating point at the moments of time: 0, T, $2T$, $3T$ and $4T$.

12.46. The logarithmic damping decrement of a mathematical pendulum is $\delta T=0.2$. How will the amplitude of oscillations decrease during one full oscillation of the pendulum?

12.47. What is the logarithmic damping decrement of a mathematical pendulum if the amplitude of the oscillations halved in one minute? The pendulum is 1 m long.

12.48. A mathematical pendulum 24.7 cm long performs damped oscillations. In what time will the energy of the pendulum oscillations decrease 9.4 times? Solve the problem for logarithmic damping decrements of: (1) $\delta T=0.01$, and (2) $\delta T=1$.

12.49. A mathematical pendulum performs damped oscillations with a logarithmic damping decrement equal to 0.2. How many times will the full acceleration of the pendulum decrease in its extreme position during one oscillation?

12.50. The amplitude of damped oscillations of a mathematical pendulum halves in one minute. How will it decrease in 3 minutes?

12.51. A mathematical pendulum 0.5 m long brought out of equilibrium deflects by 5 cm during the first oscillation and by 4 cm during the second one (in the same direction). Find the time of relaxation, i.e., the time during which the amplitude of the oscillations decreases e times, where e is the base of natural logarithms.

12.52. A weight is suspended from a vertically hanging spring, and the spring extends 9.8 cm. The weight is pulled down and released to make it oscillate. What should the damping coefficient δ be for: (1) the oscillations to cease in 10 seconds (assume conditionally that the oscillations cease as soon as their amplitude drops to 1 per cent of the initial magnitude), (2) the weight to return to equilibrium aperiodically, (3) the logarithmic damping decrement to be equal to 6?

12.53. A body with a mass of $m=10$ g performs damped oscillations with a maximum amplitude of 7 cm, an initial phase equal to zero and a damping coefficient of 1.6 s^{-1}. The body is acted upon by an external periodic force which produces forced oscillations whose equation is $x=5\sin(10\pi t-0.75\pi)$ cm. Find: (1) the equation (with numerical coefficients) of the natural oscillations, (2) the equation (with numerical coefficients) of the external periodic force.

12.54. A weight of 0.2 kgf is suspended on a vertical spring and performs damped oscillations with a damping coefficient of 0.75 s^{-1}. The deformation coefficient of the spring is 0.5 kgf/cm. Draw a diagram showing how the amplitude A of the forced oscillations of the weight depends on the frequency ω of an external periodic force if the maximum value of this force is 0.98 N. To plot the diagram, find the values of A for the following frequencies: $\omega=0$, $\omega=0.5\omega_0$, $\omega=0.75\omega_0$, $\omega=\omega_0$, $\omega=1.5\omega_0$ and $\omega=2\omega_0$, where ω_0 is the frequency of the natural oscillations of the suspended weight.

12.55. A tractor leaves tracks on an unpaved road in the form of a number of depressions 30 cm apart. A pram with two identical leaf springs, each of which deflects 2 cm under a load of 1 kgf, is pushed across the tracks. What is the speed of the pram if the resulting jolts cause it to swing heavily in resonance? The pram weighs 10 kgf.

12.56. Find the wavelength of an oscillation with a period of 10^{-14} s. The oscillations propagate with a velocity of 3×10^8 m/s.

12.57. Sound oscillations with a frequency of $\nu=500$ Hz and an amplitude of $A=0.25$ mm propagate in air. The wavelength $\lambda=70$ cm. Find: (1) the velocity of propagation, (2) the maximum velocity of the particles of air.

12.58. An equation of undamped oscillations is $x=10\sin 0.5\pi t$ cm. (1) Find the equation of the wave if the oscillations propagate with a velocity of 300 m/s. (2) Write and plot graphically the equation of oscillation for a point at a distance of 600 m from the source of oscillations. (3) Write and plot graphically the equation of oscillation for the points of the wave at a moment of $t=4$ s after the oscillations begin.

12.59. An equation of undamped oscillations is $x=4 \sin 600\pi t$ cm. Find the displacement from the position of equilibrium of a point 75 cm away from the source of oscillations in 0.01 second after they begin. The oscillations propagate with a velocity of 300 m/s.

12.60. An equation of undamped oscillations is $x=\sin 2.5\pi t$ cm. Find the displacement from the position of equilibrium, the velocity and the acceleration of a point 20 m away from the source of oscillations for a moment of $t=1$ s after the oscillations begin. The oscillations propagate with a velocity of 100 m/s.

12.61. What is the difference of phases between the oscillations of two points at a distance of 10 and 16 m respectively from the source of oscillations? The period of oscillations is 0.04 s and the velocity of their propagation 300 m/s.

12.62. Find the phase difference of the oscillations of two points 2 m apart lying on a ray if the wavelength is 1 m.

12.63. Find the displacement from the position of equilibrium of a point removed from the source of oscillations by $l=\frac{\lambda}{12}$ for the moment $t=\frac{T}{6}$. The amplitude of the oscillations is $A=0.05$ m.

12.64. The displacement from the position of equilibrium of a point 4 cm from a source of oscillations is half the amplitude at the moment $t=\frac{T}{6}$. Find the length of the running wave.

12.65. Find the position of the nodes and antinodes and draw a diagram of a standing wave for two cases: (1) reflection is from a less dense medium, (2) reflection is from a denser medium. The length of the running wave is 12 cm.

12.66. Determine the wavelength of oscillations if the distance between the first and fourth antinodes of a standing wave is 15 cm.

13. Acoustics

The velocity of propagation of acoustic oscillations in a certain medium can be found from the formula

$$c=\sqrt{\frac{E}{\rho}}$$

where E = Young's modulus of the medium

ρ = density of the medium.

In gases, the velocity of propagation is

$$c=\sqrt{\frac{C_p RT}{C_V \mu}}$$

where μ=mass of one kilomole of gas
T=absolute temperature of the gas
R=gas constant
C_p=heat capacity of the gas at constant pressure
C_V=ditto at constant volume.

The sound pressure level L_1 in decibels is related to the sound pressure amplitude Δp by the ratio

$$L_1 = 20 \log_{10} \frac{\Delta p}{\Delta p_0}$$

where Δp_0 is the sound pressure amplitude at a loudness level equal to zero. The loudness level L_2 in phons is related to the sound intensity by the ratio

$$L_2 = 10 \log_{10} \frac{I}{I_0}$$

where I_0 is the zero loudness level. It is assumed that

$$I_0 = 10^{-12}\ \text{W/m}^2 \text{ and } \Delta p_0 = 2\times 10^{-5}\ \text{N/m}^2$$

According to Doppler's principle, the frequency of sound perceived by an observer is determined from the formula

$$\nu' = \frac{c+v}{c-u}\,\nu$$

where ν=frequency of the sound emitted by its source
u=velocity of the sound source
v=speed of the observer
c=velocity of propagation of sound.

The velocity $v>0$ if the observer moves towards the source of sound, and $u>0$ if the source moves towards the observer.

The frequency of the fundamental tone of a string can be determined from the formula

$$\nu = \frac{1}{2l}\sqrt{\frac{F}{\rho S}}$$

where l=length of string
F=its tension
S=its cross-sectional area
ρ=density of the medium material.

13.1. Find the wavelength of the fundamental of the musical tone A (frequency 435 Hz). The velocity of sound is 340 m/s.

13.2. A man's ear can perceive sound with a frequency of 20 to 20,000 Hz. Between what wavelengths does the range of audibility of sound oscillations lie? The velocity of sound in air is 340 m/s.

13.3. Find the velocity of sound propagation in steel.

13.4. Find the velocity of sound propagation in copper.

13.5. Sound propagates in kerosene with a velocity of 1,330 m/s. Find the coefficient of compression of kerosene.

13.6. What is the depth of a sea measured by means of an echo sounder if the time between the moment the sound is produced and received is 2.5 s? The coefficient of compression of water is 4.6×10^{-10} m^2/N and the density of sea water is 1,030 kg/m^3.

13.7. Find the velocity with which sound propagates in air at temperatures of: (1) —20° C, (2) 0° C, (3) +20° C.

13.8. How many times is the velocity of sound propagation in air in summer (temperature +27° C) higher than in winter (temperature —33° C)?

13.9. If the mean quadratic velocity of the molecules of a biatomic gas is 461 m/s in the conditions of an experiment, find the velocity of sound propagation in these conditions.

13.10. Find the velocity of sound propagation in a biatomic gas if the density of this gas is 1.29×10^{-3} g/cm^3 at a pressure of 760 mm Hg.

13.11. The mean kinetic energy of translational motion of the molecules of one kilomole of nitrogen is 3.4×10^3 kJ. Find the velocity of sound propagation in nitrogen in these conditions.

13.12. The temperature of the upper layer of the atmosphere cannot be measured with a thermometer, since it will not get into thermal equilibrium with the environment owing to the low density of the gas. For this purpose use is made of a rocket with grenades which explode at a certain altitude. Find the temperature at an altitude of 20 km from the Earth's surface if the sound produced by an explosion at an altitude of 21 km is detected 6.75 s after that produced by an explosion at an altitude of 19 km.

13.13. Find the refraction of a sound wave on the boundary between air and glass. Young's modulus for glass is 6.9×10^{10} N/m^2, the density of glass is 2.6 g/cm^3, and the air temperature is 20° C.

13.14. Find the limit angle of complete internal reflection of sound waves on the boundary between air and glass. Take the necessary data from the previous problem.

13.15. Two sounds differ in loudness level by 1 phon. Find the ratio of the intensities of these sounds.

13.16. Two sounds differ in sound pressure level by 1 db. Find the ratio of their sound pressure amplitudes.

13.17. A noise in the street with a loudness of 70 phons can be heard in a room as a noise of 40 phons. Find the ratio between the sound intensities in the street and in the room.

13.18. The intensity of a sound increases 1,000 times. (1) By how many decibels does the sound pressure level increase? (2) How many times does the sound pressure amplitude increase?

13.19. The intensity of a sound is 10^{-8} W/m^2. Find: (1) the loudness level, (2) the amplitude of the sound pressure.

13.20. By how many phons will the loudness level increase if the sound intensity grew (1) 3,000 times, and (2) 30,000 times?

13.21. Find the groove pitch on a phonograph record for the musical tone A (435 Hz): (1) at the beginning of recording at a distance of 12 cm from the centre, (2) at the end of recording at a distance of 4 cm from the centre. The record rotates at a speed of 78 rev/min.

13.22. Find the groove pitch on a phonograph record for: (1) $\nu = 100$ Hz and (2) $\nu = 2{,}000$ Hz. The mean distance from the record centre is 10 cm. The record rotates at a speed of 78 rev/min.

13.23. Six antinodes are observed in the air column when a standing wave forms in a Kundt tube. What is the length of the air column if a steel bar is secured: (1) at the middle, (2) at the end? The bar is 1 m long. The velocity of sound in steel is 5,250 m/s and in air 343 m/s.

13.24. What is the length of a glass bar in a Kundt tube if five antinodes are observed in the air column when the bar is secured at the middle? The latter is 0.25 m long. Young's modulus for glass is 6.9×10^{10} N/m^2 and the density of glass is 2.5 g/cm^3. The velocity of sound in air is 340 m/s.

13.25. For what maximum frequencies can Kundt's method be used to determine the velocity of sound if we assume that the minimum detectable distance between antinodes is $l \cong 4$ mm? The velocitv of sound in air is 340 m/s.

13.26. Two trains are traveling towards each other at speeds of 72 km/h and 54 km/h, respectively. The first train whistles emitting a sound with a frequency of 600 Hz. Find the frequency of the sound oscillations which can be heard by a passenger in the second train: (1) before the trains meet, (2) after the trains meet. The velocity of sound is 340 m/s.

13.27. When a whistling train travels past a person standing still, the height of the tone of the whistle sharply changes. What percentage of the actual frequency does the change in tone form if the train moves at 60 km/h.

13.28. A man on the seashore hears the hooting of a ship. When neither is moving, the sound has a frequency of 420 Hz. When the ship moves towards the man, the frequency of the sound he hears is 430 Hz. When the ship moves away from the man, the frequency is 415 Hz. Find the speed of the ship in the first and second cases if the velocity of sound during the experiment is 338 m/s.

13.29. A bullet flies with a velocity of 200 m/s. How many times will the height of the tone of its whistling change for a man standing still past whom the bullet flies? The velocity of sound is 333 m/s.

13.30. Two trains move towards each other with the same speed. What should their speed be if the height of the tone of the whistle of one of them heard on the other changes 9/8 times? The velocity of sound is 335 m/s.

13.31. A bat flies perpendicular to a wall with a speed of $v=6.0$ m/s emitting an ultrasound with a frequency of $\nu=4.5\times10^4$ Hz. What sound of two frequencies can be heard by the bat? The velocity of sound is 340 m/s.

13.32. How long should a steel string with a radius of 0.05 cm be for it to produce a tone with a frequency of 320 Hz when it is stretched by a force of 100 kgf?

13.33. What force should tension a steel string 20 cm long and 0.2 mm in diameter for it to produce the musical note A (frequency 435 Hz)?

13.34. The ultimate strength of steel being known, find the maximum frequency to which a string 1 m long can be tuned.

13.35. A string tensioned with a force of 15 kgf produces eight beats per second in comparison with a tuning fork. When the string is tensioned with a force of 16 kgf, it becomes tuned in unison with the fork. Find the number of oscillations of the tuning fork.

13.36. The tuning fork of the previous problem gives in comparison with another fork 10 beats in five seconds. Find the oscillation frequency of the second fork.

13.37. Find the frequency of the fundamental tone of a string tensioned by a force of $F=6\times10^3$ N. The length of the string $l=0.8$ m and its weight $G=0.03$ kgf.

13.38. Find the frequency of the fundamental tone of: (1) an open tube, (2) a closed tube.

13.39. A closed tube produces the fundamental musical tone C which corresponds to a frequency of 130.5 Hz. The tube is then opened. What fundamental tone will it emit now? What is the length of the tube? Assume the velocity of sound in air to be 340 m/s.

14. Electromagnetic Oscillations and Waves

The period T of electromagnetic oscillations in a circuit consisting of a capacitance C, inductance L and resistance R is determined from the formula

$$T=\frac{2\pi}{\sqrt{\frac{1}{LC}-\left(\frac{R}{2L}\right)^2}}$$

If the resistance of the circuit is so small that

$$\left(\frac{R}{2L}\right)^2\ll\frac{1}{LC}$$

then the period of oscillations will be

$$T=2\pi\sqrt{LC}$$

If the resistance R of the circuit is not zero, the oscillations will be damped, and the difference of potentials across the capacitor plates changes with time according to the law

$$U = U_0 e^{-\delta t} \cos \omega t$$

if the time is counted from the moment corresponding to the maximum potential difference across the capacitor plates. Here $\delta = \frac{R}{2L}$ is the damping coefficient. The quantity δT is called the logarithmic damping decrement.

If $\delta = 0$, the oscillations will be undamped and we have

$$U = U_0 \cos \omega t$$

If the time is counted from the moment when the difference of potentials across the capacitor plates is zero, the following equation will be true

$$U = U_0 \sin \omega t$$

Ohm's law for an alternating current is written as

$$I_{ef} = \frac{U_{ef}}{Z}$$

where I_{ef} and U_{ef} are effective values of the current intensity and voltage, which are related with their amplitude values I_0 and U_0 by the equations

$$I_{ef} = \frac{I_0}{\sqrt{2}} \text{ and } U_{ef} = \frac{U_0}{\sqrt{2}}$$

and Z is the impedance of the circuit. If a circuit includes a resistance R, capacitance C and inductance L connected in series, then

$$Z = \sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}$$

Here the phase shift between the voltage and the current intensity is determined by the formula

$$\tan \varphi = \frac{\omega L - \frac{1}{\omega C}}{R}$$

Formulas for the impedance of a circuit Z and the phase shift φ for various methods of connecting R, C and L are given in the table on p. 334.

In an alternating current circuit, a coil with a resistance R and inductance L corresponds to R and L connected in series. A leaky capacitor with the capacitance C and resistance R corresponds to R and C connected in parallel.

The power of an alternating current is

$$P = I_{ef} U_{ef} \cos \varphi$$

14.1. An oscillatory circuit consists of a capacitor with a capacitance of 800 CGS_C and a coil with an inductance of 2×10^{-3} H. What wavelength is the circuit tuned to? Disregard the resistance of the circuit.

14.2. What wave range can an oscillatory circuit be tuned to if its inductance is 2×10^{-3} H and its capacitance can vary from 62 to 480 CGS_C? The resistance of the circuit is negligibly small.

14.3. What inductance should be connected to an oscillatory circuit to obtain a sound frequency of 1,000 Hz with a capacitance of 2 μF? Disregard the resistance of the circuit.

14.4. A coil with an inductance of $L=3\times 10^{-5}$ H is connected to a plane-parallel capacitor with a plate area of $S=100$ cm^2 and the plates spaced at $d=0.1$ mm. What is the relative permittivity of the medium filling the space between the plates if the circuit resonates to a wavelength of 750 m?

14.5. An oscillatory circuit consists of a capacitor with a capacitance of 0.025 μF and a coil with an inductance of 1.015 H. Neglect the resistance of the circuit. The capacitor is charged with a quantity of electricity equal to 2.5×10^{-6} C. (1) Write for this circuit an equation (with numerical coefficients) showing how the difference of potentials across the capacitor plates and the current intensity in the circuit change with time. (2) Find the difference of potentials across the capacitor plates and the current intensity in the circuit at the moments of time $\frac{T}{8}$, $\frac{T}{4}$ and $\frac{T}{2}$ s. (3) Plot diagrams of these relationships within the limits of one period.

14.6. For the oscillatory circuit of the previous problem: (1) write an equation (with numerical coefficients) showing the change with time of the energy of its electric field, the energy of its magnetic field and the total energy, (2) find the energy of the electric field, the energy of the magnetic field and the total energy at the moments of time $\frac{T}{8}$, $\frac{T}{4}$ and $\frac{T}{2}$ s, (3) plot diagrams of these relations within the limits of one period.

14.7. The equation of the change with time of the difference of potentials across the capacitor plates in an oscillatory circuit is given as $U=50\cos 10^4\pi t$ V. The capacitance of the capacitor is 10^{-7} F. Find: (1) the period of oscillations, (2) the inductance of the circuit, (3) the law showing how the current intensity in the circuit changes with time, (4) the wavelength which corresponds to this circuit.

14.8. The equation of the change with time of the current intensity in a circuit is given as $I=-0.02\times\sin 400\pi t$ A. The inductance of the circuit is 1 H. Find: (1) the period of oscillations, (2) the capacitance of the circuit, (3) the maximum difference of potentials across the capacitor plates, (4) the maximum energy of the magnetic field, (5) the maximum energy of the electric field.

14.9. What is the relationship between the energy of the magnetic field of an oscillatory circuit and the energy of its electric field for the moment of time $\frac{T}{8}$ s?

14.10. An oscillatory circuit consists of a capacitor with a capacitance of 7 μF and a coil with an inductance of 0.23 H and a resistance of 40 Ω. The capacitor is charged with a quantity of electricity equal to 5.6×10^{-4} C. (1) Find the period of oscillations of the circuit. (2) Find the logarithmic damping decrements of the oscillations. (3) Write an equation showing how the difference of potentials across the capacitor plates changes with time. (4) Find the difference of potentials at the moments $\frac{T}{2}$, T, $\frac{3}{2}T$ and $2T$ s. (5) Plot a diagram $U=f(t)$ within the limits of two periods.

14.11. An oscillatory circuit consists of a capacitor with a capacitance of 0.2 μF and a coil with inductance of 5.07×10^{-3} H. (1) At what logarithmic damping decrement will the difference of potentials across the capacitor plates be reduced to one-third in 10^{-3} s? (2) What is the resistance of the circuit in this case?

14.12. An oscillatory circuit consists of an inductance of 10^{-2} H, a capacitance of 0.405 μF and a resistance of 2 Ω. How many times will the difference of potentials across the capacitor plates diminish during one period?

14.13. An oscillatory circuit consists of a capacitor with a capacitance of $C=2.22\times10^{-9}$ F and a coil wound of copper wire with a diameter of $d=0.5$ mm. The length of the coil $l=20$ cm. Find the logarithmic damping decrement of the oscillations.

14.14. An oscillatory circuit has a capacitance of 1.1×10^{-9} F and an inductance of 5×10^{-3} H. The logarithmic damping decrement is 0.005. In what time will 99 per cent of the circuit energy be lost owing to damping?

14.15. An oscillatory circuit consists of a capacitor and a long coil wound of copper wire with a cross-sectional area $S=0.1$ mm^2. The length of the coil $l=40$ cm. What is the capacitance C of the capacitor if the error we admit in calculating the period of oscillations of the circuit by means of the approximate formula $T=2\pi\sqrt{LC}$ is $\varepsilon=1$ per cent?

Note. The error $\varepsilon=\frac{T_2-T_1}{T_2}$, where T_1 is the period of oscillations found approximately and T_2 is that determined by means of the accurate formula.

14.16. A coil with a length of $l=50$ cm and a cross-sectional area of $S=10$ cm^2 is connected to an alternating current circuit with a frequency of $\nu=50$ Hz. The number of turns on the coil is $N=3{,}000$. Find the resistance of the coil if the phase shift between the voltage and the current is 60 degrees.

14.17. The winding of a coil consists of 500 turns of copper wire with a cross-sectional area of 1 mm^2. The coil is 50 cm long and 5 cm in diameter. At what frequency of alternating current will the impedance of the coil be twice its resistance?

14.18. Two capacitors with capacitances of $C_1=0.2$ μF and $C_2=0.1$ μF are connected in series to an alternating current circuit with a voltage of 220 V and a frequency of 50 Hz. Find: (1) the intensity of the current in the circuit, (2) the potential drop across the first and second capacitors.

14.19. A coil 25 cm long with a radius of 2 cm has a winding of 1,000 turns of copper wire with a cross-sectional area of 1 mm^2. The coil is connected to an alternating current circuit with a frequency of 50 Hz. What part of the impedance of the coil will be formed by (1) the resistance, and (2) the inductive reactance?

14.20. A capacitor with a capacitance of 20 μF and a rheostat with a resistance of 150 Ω are connected in series to an alternating current circuit with a frequency of 50 Hz. What part of the voltage applied to this circuit is formed by the voltage drop: (1) across the capacitor, and (2) across the rheostat?

14.21. A capacitor and an electric lamp are connected in series to an alternating current circuit at 400 V and 50 Hz. What capacitance should the capacitor have for a current of 0.5 A to flow through the lamp and the potential drop across the lamp to be 110 V?

14.22. A coil with a resistance of 10 Ω and an inductance of L is connected to an alternating current circuit at 127 V and 50 Hz. Find the inductance of the coil if it consumes a power of 400 W and the phase shift between the voltage and the current is 60 degrees.

14.23. Compile a table of formulas for the impedance of a circuit Z and the phase shift $\tan\varphi$ between the voltage and current with various methods of connecting the resistance R, the capacitance C and the inductance L. Consider cases when: (1) R and C are connected in series, (2) R and C are connected in parallel, (3) R and L are connected in series, (4) R and L are connected in parallel and (5) R, L and C are connected in series.

14.24. A capacitor with a capacitance of 1 μF and a rheostat with a resistance of 3,000 Ω are connected to an alternating current circuit with a frequency of 50 Hz. The inductance of the rheostat is negligibly small. Find the impedance of the circuit if the capacitor and the rheostat are connected: (1) in series, and (2) in parallel.

14.25. A capacitance of 35.4 μF, a resistance of 100 Ω and an inductance of 0.7 H are connected in series to an alternating current circuit at 220 V and 50 Hz. Find the intensity of the current in the circuit and the voltage drop across the capacitance, resistance and inductance.

14.26. An inductance of $L=2.26\times10^{-2}$ H and a resistance R are connected in parallel to an alternating current circuit with a frequency of $\nu=50$ Hz. Find R if the phase shift between the voltage and the current is 60 degrees.

14.27. A resistance R and an inductance L are connected in parallel to an alternating current circuit at 127 V and 50 Hz. Find the resistance R and the inductance L if the power consumed in this circuit is 404 W and the phase shift between the voltage and the current is 60 degrees.

14.28. A capacitance C, a resistance R and an inductance L are connected in series to an alternating current circuit at 220 V. Find the voltage drop U_R across the resistance if the voltage drop across the capacitor $U_C=2U_R$ and that across the inductance $U_L=3U_R$.

Chapter 5

OPTICS

LIGHT UNITS

The basic and some derived units intended for the measurement of light in SI units as specified by GOST 7932-56 are given in Table 16.

TABLE 16

Quantity and symbol	Formula	Unit	Symbol of unit	Dimension of quantity
		Basic Units		
Length l	—	metre	m	l
Time t	—	second	s	t
Luminous intensity I	—	candela	cd	I
		Derived Units		
Luminous flux	$d\Phi = I d\omega$	lumen	lm	I
Quantity of light	$dQ = \Phi dt$	lumen-second	lm·s	tI
Luminous emittance	$M = \frac{d\Phi}{dS}$	lumen per square metre	lm/m²	$l^{-2}I$
Luminance	$L = \frac{dI}{\cos\theta \cdot dS}$	nit (candela per square metre)	nt	$l^{-2}I$
Illumination	$E = \frac{d\Phi}{dS}$	lux	lx	$l^{-2}I$
Quantity of illumination	$dH_e = Edt$	lux-second	lx·s	$l^{-2}tI$

The unit of luminous flux in this system is the lumen (lm) which is the flux emitted by a point source of light of one candela inside a solid angle of one steradian. Thus, 1 lm = 1 cd × 1 sr.

Illumination is measured in luxes. One lux is the illumination of an area of one square metre by a uniformly distributed luminous flux of one lumen. Thus, 1 lx = 1 lm/m².

The luminous emittance of a light source is measured in lumens per square metre; 1 lm/m^2 is the luminous emittance which corresponds to a luminous flux of 1 lm emitted by an area of 1 m^2.

The unit of luminance is the nit (nt), which is the luminance of a uniformly luminescent flat surface producing in a direction normal to it a luminous intensity of 1 cd from an area of one square metre. Thus, 1 nt=1 cd/m^2.

EXAMPLES OF SOLUTIONS

Example 1. The filament of an electric lamp with a luminous intensity of 1,000 cd is enclosed in a spherical frosted bulb 20 cm in diameter. Find: (1) the luminous flux radiated by this light source, (2) the luminous emittance and the luminance of this light source, (3) the illumination, luminous emittance and luminance of a screen receiving 10 per cent of the luminous flux radiated by this light source. The reflection coefficient of the surface of the screen $\rho=0.8$. The area of the screen is 0.25 m^2. Assume that the screen surface diffuses light according to the Lambert law.

Solution. (1) The luminous flux Φ radiated in all directions by the light source is related to the luminous intensity I of this source by the formula

$$\Phi=4\pi I$$

In our case, $I=10^3$ cd, and therefore $\Phi=1.26\times10^4$ lm.

(2) The luminous emittance of the light source is

$$M=\frac{\Phi}{S}=\frac{4\pi I}{4\pi r^2}=\frac{I}{r^2}$$

where r is the radius of the spherical bulb. Upon inserting the numerical data, we find that

$$M=\frac{1,000}{(0.1)^2}=10^5\ lm/m^2$$

The luminance of the light source is

$$L=\frac{I}{\Delta S'}$$

where $\Delta S'$ is the visible area of the luminescent surface. In our case $\Delta S'=\pi r^2$, where r is the bulb radius, and hence

$$L=\frac{I}{\pi r^2}=\frac{1,000}{\pi(0.1)^2}=3.18\times10^4\ nt$$

(3) According to the initial condition, the screen receives a luminous flux of $\Phi_1=0.1\Phi=1.26\times10^3$ lm.

The illumination of the screen will thus be

$$E=\frac{\Phi_1}{S_1}=\frac{1.26\times10^3}{0.25}\ \text{lm/m}^2\cong5\times10^3\ \text{lx}$$

The luminous emittance of the screen is

$$M=\rho\frac{\Phi_1}{S_1}=\rho E=0.8\times5\times10^3\ \text{lm/m}^2=4\times10^3\ \text{lm/m}^2$$

and the luminance of the screen is

$$L=\frac{M}{\pi}=1.3\times10^3\ \text{nt}$$

Example 2. A black body is maintained at a constant temperature of 1000° K. The surface of the body is 250 cm². Find the radiated power of the body.

Solution. According to the Stefan-Boltzmann law, the energy emitted by a unit of surface of a black body per second is

$$M_e=\sigma T^4$$

and all the radiated energy is

$$Q_e=S\tau M_e=S\tau\sigma T^4$$

where S=surface of the black body
τ=duration of radiation
σ=Stefan-Boltzmann constant
T=temperature of body in degrees Kelvin.

The radiation power is

$$P=\frac{Q_e}{\tau}=S\sigma T^4$$

In our case, $S=250\ \text{cm}^2=2.5\times10^{-2}\ \text{m}^2$, $\sigma=5.67\times10^{-8}\ \text{W/m}^2\cdot\text{deg}^4$ and $T=1000°$ K. Upon inserting these data, we obtain $P=1.42\times10^3\ \text{W}=1.42\ \text{kW}$.

15. Geometrical Optics and Photometry

The optical power D for a spherical mirror is determined from the formula

$$\frac{1}{a_1}+\frac{1}{a_2}=\frac{2}{R}=\frac{1}{F}=D$$

where a_1 and a_2=distances of the object and its image from the mirror
R=radius of curvature of the mirror
F=focal length of the mirror.

Distances measured from the mirror along the ray are positive, and in the reverse direction are negative. If F is in metres, D can be expressed in diopters.

When a ray passes from one medium into another, the law of refraction is applied

$$\frac{\sin i}{\sin r} = n = \frac{v_1}{v_2}$$

where i=angle of incidence
r=angle of refraction
n=index of refraction of the second medium with respect to the first one
v_1 and v_2=velocities of light propagation in the first and second media.

For a thin lens placed in a homogeneous medium, the optical power D can be found from the formula

$$-\frac{1}{a_1} + \frac{1}{a_2} = (n-1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right) = \frac{1}{F} = D$$

where a_1 and a_2=distances from the object and its image to the lens
n=relative index of refraction of the lens material
R_1 and R_2=radii of curvature of the lens.

The rule of signs for lenses is the same as for mirrors. The optical power of two thin lenses placed together is

$$D = D_1 + D_2$$

where D_1 and D_2 are the optical powers of the two lenses.

The lateral magnification in mirrors and lenses is determined from the formula

$$k = \frac{y'}{y} = \frac{a_2}{a_1}$$

where y and y' are the heights of the object and the image, respectively.

The magnification of a magnifying glass is

$$k = \frac{d_v}{F}$$

where d_v=best viewing distance
F=principal focal length of the magnifying glass.

The magnification of a microscope is

$$k = d_v d D_1 D_2$$

where d_v=best viewing distance
d=distance between the focuses of the objective and the eyepiece
D_1 and D_2=optical powers of the objective and the eyepiece.

The magnification of a telescope

$$k = \frac{F_1}{F_2}$$

where F_1 and F_2 are the focal lengths of the objective and the eyepiece, respectively.

The luminous flux Φ is determined by the quantity of light carried by the light waves through a given area in a unit time

$$\Phi=\frac{dQ_e}{dt}$$

The luminous intensity I is numerically equal to the luminous flux per unit of a solid angle

$$I=\frac{d\Phi}{d\omega}$$

The illumination E is characterized by the luminous flux per unit area

$$E=\frac{d\Phi}{dS}$$

A point source with a light intensity of I creates the illumination

$$E=\frac{I\cos i}{r^2}$$

on an area at a distance r from it, where i is the angle of incidence of the light rays.

The luminous emittance M is numerically equal to the luminous flux emitted by a unit area of a luminescent body

$$M=\frac{d\Phi}{dS}$$

If the luminous emittance of a body is caused by its illumination, then $M=\rho E$, where ρ is the diffusion (reflection) coefficient.

The luminance L of a luminescent surface is a quantity numerically equal to the ratio between the luminous intensity from an element of a radiating surface and the area of the projection of this element onto a plane perpendicular to the direction of observation (i.e., to the visible surface of the element):

$$L=\frac{dI}{dS\cos\theta}$$

where θ is the angle between the perpendicular to the element of the surface and the direction of observation.

If a body radiates according to the Lambert law, i.e., the luminance does not depend on the direction, then the luminous emittance M and the luminance L are related by the expression

$$M=\pi L$$

15.1. A horizontal ray of light falls onto a vertical mirror. The mirror is turned through an angle α about its vertical axis. Through what angle will the reflected ray turn?

15.2. The radius of curvature of a concave spherical mirror is 20 cm. An object 1 cm high is placed at a distance of 30 cm from the mirror. Find the position and height of the image. Make a drawing.

15.3. At what distance will the image of an object be obtained in a convex spherical mirror with a radius of curvature of 40 cm if the object is placed at a distance of 30 cm from the mirror? How large will the image be if the object is 2 cm in height? Check the calculations by making a drawing on millimetre graph paper.

15.4. A convex spherical mirror has a radius of curvature of 60 cm. An object 2 cm high is placed at a distance of 10 cm from the mirror. Find the position and height of the image. Make a drawing.

15.5. A real image of half size is to be obtained in a concave spherical mirror with a radius of curvature of 40 cm. Where should the object be placed and where will the image be obtained?

15.6. The image of an object in a concave spherical mirror is twice the size of the object. The distance between the object and the image is 15 cm. Find: (1) the focal length, and (2) the optical power of the mirror.

15.7. A burning candle is placed in front of a concave spherical mirror on its principal optical axis at a distance of $\frac{4}{3}F$ from the apex of the mirror. The candle is arranged at right angles to the axis. The image of the candle in the concave mirror impinges upon a convex mirror with a focal length of $F_1=2F$. The distance between the mirrors is $3F$, and their axes coincide. The image of the candle in the first mirror plays the part of a virtual object with respect to the second mirror and gives a real image arranged between the two mirrors. Plot this image and calculate the total linear magnification of the system.

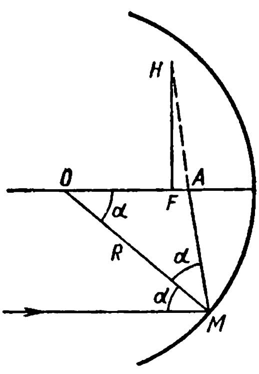


Fig. 62

15.8. Find the position and size of the Sun's image obtained in a spherical reflector with a radius of curvature of 16 m.

15.9. If a broad beam of light (the width of the beam is shown by the angle α in Fig. 62) impinges on a spherical mirror, the ray traveling parallel to the optical axis and falling on the mirror edge will cross the optical axis not at the focus, but at a certain distance AF from it after it has been reflected from the mirror. The distance AF is known as the longitudinal spherical aberration and the distance FH as the transverse spherical aberration. Deduce a formula which relates the magnitudes of these aberrations to α and to the radius of the spherical mirror.

15.10. A concave spherical mirror with an aperture diameter of 40 cm has a radius of curvature of 60 cm. Find the longitudinal and transverse spherical aberrations of the edge rays parallel to the principal axis.

15.11. A concave spherical mirror has a focal length of 20 cm. At what maximum distance h from the optical axis should an object be placed so that the longitudinal spherical aberration does not exceed 2% of the focal length?

15.12. A ray of light falls at an angle of 30° onto a plane-parallel glass plate and leaves it parallel to the initial ray. The refractive index of the glass is 1.5. What is the thickness d of the plate if the distance between the rays is 1.94 cm?

15.13. A ray of light falls onto a plane-parallel glass plate 1 cm thick at an angle of 60°. The refractive index of the glass is 1.73. Some of the light is reflected and the rest, being refracted, passes into the glass, is reflected from the bottom of the plate, refracted a second time and emerges back into the air parallel to the first reflected ray. Determine the distance l between the rays.

15.14. A ray of light falls at an angle i onto a body with a refractive index of n. What should the relationship between i and n be for the reflected ray to be perpendicular to the refracted one?

15.15. The refractive index of glass is 1.52. Find the limit angles of total internal reflection for the surfaces of separation: (1) glass-air, (2) water-air, (3) glass-water.

15.16. In what direction does a person who dived into water see the setting Sun?

15.17. A ray of light emerges from turpentine into air. The limit angle of total internal reflection for this ray is 42°23′. What is the propagation velocity of light in turpentine?

15.18. A glass plate is placed on a glass filled with water. At what angle should a ray of light fall onto the plate for total internal reflection to occur from the surface of separation between the water and the glass? The refractive index of the glass is 1.5.

15.19. A point source of light is placed on the bottom of a vessel filled with water to a height of 10 cm. A circular opaque plate so floats on the surface of the water that its centre is above the source of light. What should the minimum radius of this plate be to prevent all the rays from emerging through the water surface?

15.20. When white light impinges onto a glass plate at an angle of 45°, the following angles of refraction are obtained for rays of different wavelengths:

λ, Å	7,590	6,870	5,890	4,860	3,970
r	24°2′	23°57′	23°47′	23°27′	22°57′

Plot a diagram showing how the refractive index of the plate material depends on the wavelength.

15.21. The refractive indices of a certain grade of glass for red and violet rays are equal to 1.51 and 1.53, respectively. Find the limit angles of total internal reflection when these rays impinge upon the glass-air boundary.

15.22. What will occur when a white ray falls at an angle of 41° upon the glass-air boundary if the glass of the previous problem is taken for the experiment? (Use the results of the solution of the previous problem.)

15.23. A monochromatic ray falls normally onto the side surface of a prism whose refraction angle is 40°. The refractive index of the prism material is 1.5 for this ray. Find the deflection of the ray from the initial position when it leaves the prism.

15.24. A monochromatic ray falls normally onto the side surface of a prism and leaves it deflected by 25°. The refractive index of the prism material is 1.7 for this ray. Find the angle of refraction of the prism.

15.25. The refraction angle of an isosceles prism is 10°. A monochromatic ray falls onto a side face at an angle of 10°. Find the angle of deflection of the ray from the initial direction if the refractive index of the prism material is 1.6.

15.26. The refractive index of the material of a prism is 1.6 for a certain monochromatic ray. What should the maximum angle of incidence of this ray onto the prism be so that no total internal reflection occurs when the ray leaves the prism? The angle of refraction of the prism is 45°.

15.27. A beam of light slides along a side face of an isosceles prism. At what maximum refractive angle of the prism will the refracted rays be subjected to total internal reflection on the second side face? The refractive index of the prism material for these rays is 1.6.

15.28. A monochromatic ray enters through a face of a rectangular isosceles prism and is subjected to total internal reflection from the face corresponding to the hypotenuse and emerges through the face corresponding to the second leg. What should the minimum angle of incidence of the ray onto the prism be for total internal reflection to continue if the refractive index of the prism material for this ray is 1.5?

15.29. A monochromatic ray falls onto the side surface of an isosceles prism, and after refraction travels parallel to its base. When the ray emerges from the prism, it is deflected by an angle δ from its initial direction. Determine the relationship between the refraction angle of the prism γ, the deflection of the ray δ and the refractive index n for this ray.

15.30. A ray of white light falls onto the side surface of an isosceles prism at such an angle that the red ray leaves the prism perpendicular to the second face. Find the deflection of the red and violet rays from the initial direction if the refraction angle of the prism is 45°. The refractive indices of the prism material for red and violet rays are 1.37 and 1.42, respectively.

15.31. Find the principal focal length of a quartz lens for the ultraviolet line of the mercury spectrum ($\lambda=2.59\times10^{-7}$ m) if the principal focal length for the yellow line of sodium ($\lambda=5.89\times10^{-7}$ m) is 16 cm and the refractive indices of quartz for these wavelengths are 1.504 and 1.458, respectively.

15.32. Find the focal length of: (1) a double-convex lens with $R_1=15$ cm and $R_2=-25$ cm, (2) a planoconvex lens with $R_1=15$ cm and $R_2=\infty$, (3) a concavo-convex lens (positive meniscus) with $R_1=15$ cm and $R_2=25$ cm, (4) a double-concave lens with $R_1=-15$ cm and $R_2=25$ cm, (5) a planoconcave lens with $R_1=\infty$, $R_2=-15$ cm, (6) a convexo-concave lens (negative meniscus) with $R_1=25$ cm, $R_2=15$ cm. The refractive index of the lens material $n=1.5$.

15.33. Two glasses with refractive indices of 1.5 and 1.7 are used to make two identical double-convex lenses. (1) Find the ratio between their focal lengths. (2) How will each of these lenses act on a ray parallel to its optical axis if the lenses are submerged into a transparent liquid with a refractive index of 1.6?

15.34. The radii of curvature of the surfaces of a double-convex lens are $R_1=R_2=50$ cm. The refractive index of the lens material $n=1.5$. Find the optical power of the lens.

15.35. An object 2 cm high is placed at right angles to the optical axis 15 cm away from a double-convex lens having an optical power of 10 diopters. Find the position and height of the image. Make a drawing.

15.36. Prove that the principal focuses coincide with the centres of curvature in a double-convex lens with equal radii of curvature and with the refractive index $n=1.5$.

15.37. A lens with a focal length of 16 cm produces a sharp image of an object in two positions which are 60 cm apart. Find the distance from the object to the screen.

15.38. A double-convex lens limited by spherical surfaces having the same radius of curvature of 12 cm is placed at such a distance from the object that the image on a screen is k times greater than the object. Determine the distance from the object to the screen if: (1) $k=1$, (2) $k=20$ and (3) $k=0.2$. The refractive index of the lens material is 1.5.

15.39. The lens of the previous problem is submerged in water. Find its focal length.

15.40. Solve the previous problem if the lens is submerged in carbon bisulphide.

15.41. Find the focal length of a lens submerged in water if its focal length in air is 20 cm. The refractive index of the glass which the lens is made of is 1.6.

15.42. A planoconvex lens with a radius of curvature of 30 cm and a refractive index of 1.5 produces a real image of an object with a magnification equal to 2. Find the distances from the object and the image to the lens. Make a drawing.

15.43. Find the longitudinal chromatic aberration of a double-convex lens made of flint and having the same radii of curvature $|R_1|=|R_2|=8$ cm. The refractive indices of the flint for red ($\lambda_1=7.6\times10^{-5}$ cm) and violet ($\lambda_2=4.3\times10^{-5}$ cm) rays are equal to 1.5 and 1.8, respectively.

15.44. A luminescent point is on the optical axis at a distance of 40 cm in front of the lens of the previous problem. Find the position of the image of this point if it emits monochromatic light with a wavelength of: (1) $\lambda_1=7.6\times10^{-5}$ cm, and (2) $\lambda_2=4.3\times10^{-5}$ cm.

15.45. A flat mirror is arranged in the focal plane of a double-convex lens. An object is placed in front of the lens between the focus and the double focal length. Construct an image of the object.

15.46. Find the magnification produced by a magnifying glass with a focal length of 2 cm: (1) for a normal eye with the best viewing distance of 25 cm, and (2) for a short-sighted eye with the best viewing distance of 15 cm.

15.47. What should the radii of curvature of the surfaces limiting a magnifying glass ($|R_1|=|R_2|$) be for it to give a magnification of $k=10$ for a normal eye? The refractive index of the glass is $n=1.5$.

15.48. A telescope with a focal length of 50 cm is adjusted to infinity. When the eyepiece of the telescope is moved over a certain distance, all objects at a distance of 50 m from the objective become clearly visible. What distance was the eyepiece moved over during adjustment?

15.49. A microscope consists of an objective with a focal length of 2 mm and an eyepiece with a focal length of 40 mm. The distance between the focuses of the objective and the eyepiece is 18 cm. Find the magnification of the microscope.

15.50. A picture with an area of 2×2 m is photographed with a camera from a distance of 4.5 m. The image obtained is 5×5 cm in size. What is the focal length of the camera lens? Assume the distance from the picture to the lens to be great as compared with the focal length.

15.51. A telescope has an objective with a focal length of 150 cm and an eyepiece with a focal length of 10 cm. At what viewing angle can the full Moon be observed through this telescope if it can be seen at an angle of 31′ by a naked eye?

15.52. The image of the Sun is projected onto a screen with the aid of a double-convex lens having a diameter of $D=9$ cm and a focal length of $F=50$ cm. (1) What will the image of the Sun be in size if its angular diameter is 32′? (2) How many times will the illumination produced by the Sun's image be greater than that coming directly from the Sun?

15.53. Light from a 200-cd electric lamp falls at an angle of 45° on a workplace and its illumination is 141 lx. Find: (1) the distance between the workplace and the lamp, (2) the height of the lamp above the workplace.

15.54. A lamp suspended from a ceiling produces a luminous intensity of 60 cd in a horizontal direction. What luminous flux falls on a picture 0.5 m^2 in area hanging vertically on a wall 2 m away from the lamp if a large mirror is on the opposite wall at a distance of 2 m from the lamp?

15.55. A large drawing is first photographed completely and then in parts of full size. How much must the exposure be increased when photographing the separate parts?

15.56. On March 21, the day of vernal equinox, on Severnaya Zemlya the Sun is at an angle of 10° to the horizon at noon. How many times will the illumination of an area placed vertically be greater than that of a horizontal one?

15.57. At noon during the vernal and autumnal equinox the Sun on the equator is at the zenith. How many times is the illumination of the Earth on the equator greater than at a latitude of 60°?

15.58. A lamp hangs in the centre of a square room 25 m^2 in area. Considering the lamp to be a point source of light, find the height from the floor which the lamp should be at to obtain the maximum illumination in the corners of the room.

15.59. A 100-cd lamp hangs above the centre of a round table 2 m in diameter. Considering the lamp to be a point source of light, calculate the change in the illumination of the edge of the table when the lamp is gradually raised from $h=0.5$ to $h=0.9$ m for 10-cm intervals. Plot a diagram $E=f(h)$.

15.60. A desk lamp with one bulb is in the centre of a round table 1.2 m in diameter. The distance from the bulb to the surface of the table is 40 cm. A chandelier with four of the same bulbs as in the desk lamp hangs above the centre of the table at 2 m from its surface. When will the edge of the table be illuminated better (and how many times) — when only the desk lamp or only the chandelier is burning?

15.61. An object being photographed is illuminated by an electric lamp placed 2 m from it. How many times should the exposure be increased if the lamp is moved to 3 m from the object?

15.62. Find the illumination on the Earth's surface due to normally incident sun rays. The luminance of the Sun is 1.2×10^9 nt.

15.63. The filament of a 100-cd electric lamp is enclosed in a frosted spherical bulb with a diameter of (1) 5 cm, and (2) 10 cm. Find the luminous emittance and the luminance of the lamp in both cases. Disregard the loss of light in the shell of the bulb.

15.64. A lamp in which the luminous body is an incandescent ball 3 mm in diameter produces a luminous intensity of 85 cd. Find the luminance of this lamp if its spherical bulb is made of (1) transparent glass, (2) frosted glass. The diameter of the bulb is 6 cm.

15.65. What illumination does the lamp of the previous problem produce at a distance of 5 m with normal light incidence?

15.66. A luminous flux of 120 lm impinges normally onto the surface of a sheet of white paper 20×30 cm in size. Find the illumination, luminous emittance and luminance of the sheet of paper if its diffusion coefficient $\rho = 0.75$.

15.67. What should the illumination of the sheet of paper of the previous problem be for its luminance to be 10^4 nt?

15.68. A sheet of paper 10×30 cm in size is illuminated by a 100-cd lamp and receives 0.5 per cent of all the light emitted by the lamp. Find the illumination of this paper.

15.69. A 100-cd electric lamp sends in all directions 122 J of light energy a minute. Find (1) the mechanical equivalent of light, (2) the luminous efficiency if the lamp consumes 100 W.

16. Wave Optics

According to Doppler's principle, the frequency ν' of light as perceived by a recording device is related to the frequency ν emitted by the light source by the expression

$$\nu' = \nu \sqrt{\frac{1 - \frac{v}{c}}{1 + \frac{v}{c}}}$$

where v = velocity of the recording device relative to the source
c = velocity of light propagation.

When the light source is moving away, v is positive. If $v \ll c$, the previous formula may be written approximately as

$$\nu' \cong \nu \frac{1}{1 + \frac{v}{c}}$$

The distance between the interference bands on a screen parallel to two coherent sources of light is equal to

$$\Delta y = \frac{L}{d} \lambda$$

where λ = wavelength of light
L = distance from the screen to the light sources, which are at a distance d from each other; here $L \gg d$.

The result of light interference in plane-parallel plates (in transmitted light) can be determined from the formulas:

amplification of light

$$2hn \cos r = 2k \frac{\lambda}{2} \quad (k = 0, 1, 2, \ldots)$$

attenuation of light

$$2hn \cos r = (2k+1) \frac{\lambda}{2} \quad (k = 0, 1, 2, \ldots)$$

where h = thickness of the plate
n = index of refraction
r = angle of refraction
λ = wavelength of light.

In reflected light, the conditions of amplification and attenuation are reverse to those in transmitted light.

The radii of Newton's bright rings (in transmitted light) are determined from the formula

$$r_{br} = \sqrt{kR\lambda} \quad (k = 1, 2, \ldots)$$

and those of the dark rings from the formula

$$r_d = \sqrt{(2k-1) R \frac{\lambda}{2}} \quad (k = 1, 2, \ldots)$$

where R is the radius of curvature of the lens.

In reflected light, the arrangement of the bright and dark rings is reverse to that in transmitted light.

The position of the illumination minima with slit diffraction and a normally incident beam of parallel rays can be found from the condition

$$a \sin \varphi = \pm k\lambda \quad (k = 1, 2, 3, \ldots)$$

where a = width of slit
φ = diffraction angle
λ = wavelength of incident light.

The maxima of light are observed in a diffraction grating in the directions forming an angle φ with the normal to the grating, which satisfies the following equation (if light is normally incident on the grating):

$$d \sin \varphi = \pm k\lambda \quad (k = 0, 1, 2, \ldots)$$

where d = grating constant
φ = diffraction angle
λ = wavelength
k = order of spectrum.

The grating constant or period $d=\frac{1}{N_0}$, where N_0 is the number of slits in the grating per unit of length.

The resolving power of a diffraction grating is determined from the formula

$$\frac{\lambda}{\Delta\lambda}=kN$$

where N = total number of slits in the grating
k = order of spectrum
λ and $\lambda+\Delta\lambda$ = wavelengths of two close spectral lines that are still resolved by the grating.

The angular dispersion of a diffraction grating is

$$D=\frac{d\varphi}{d\lambda}$$

The linear dispersion of a diffraction grating is a quantity numerically equal to

$$D_1=FD$$

where F is the focal length of the lens which projects the spectrum onto the screen.

When natural light is reflected from a dielectric mirror, the Fresnel formulas can be applied, namely,

$$I_\perp=0.5I_0\left[\frac{\sin(i-r)}{\sin(i+r)}\right]^2$$

and

$$I_\parallel=0.5I_0\left[\frac{\tan(i-r)}{\tan(i+r)}\right]^2$$

where $I_\perp$ = intensity of the light oscillations in a reflected ray occurring in a direction perpendicular to the plane of light incidence
$I_\parallel$ = ditto in a direction parallel to the plane of light incidence
I_0 = intensity of incident natural light
i = angle of incidence
r = angle of refraction.

If $i+r=90°$, then $I_\parallel=0$. In this case the angle of incidence i and the refractive index n of a dielectric mirror are related by the equation $\tan i=n$ (Brewster's law).

The intensity of light transmitted through a polarizer and an analyzer is equal to (Malus' law)

$$I = I_0 \cos^2\varphi$$

where φ = angle between the basic planes of the polarizer and the analyzer

I_0 = intensity of the light transmitted through the polarizer.

16.1. It was found in photographing the Sun's spectrum that the yellow spectral line (λ=5,890 Å) in the spectra obtained from the left and right edges of the Sun is displaced by 0.08Å. Find the linear velocity of rotation of the solar disk.

16.2. What difference of potentials was applied between the electrodes of a helium discharge tube if during observation along a beam of α-particles the maximum Doppler displacement $\Delta\lambda$ of the helium line (λ=4,922Å) was 8Å?

16.3. It has been found in photographing the spectrum of the star ε Andromedae that the titanium line ($\lambda = 4.954\times10^{-5}$ cm) is displaced towards the violet end by 1.7Å. How is the star moving with respect to the Earth?

16.4. How many times will the distance between adjacent interference bands increase on the screen in Young's experiment if a red light filter ($\lambda = 6.5\times10^{-5}$ cm) is used instead of a green one ($\lambda = 5\times$ $\times10^{-5}$ cm)?

16.5. In Young's experiment the holes were illuminated with monochromatic light having a wavelength of $\lambda = 6\times10^{-5}$ cm. The distance between the holes is 1 mm and from the holes to the screen 3 m. Find the position of the first three light bands.

16.6. In an experiment with Fresnel's mirrors the distance between the virtual images of a light source is 0.5 mm and the distance to a screen is 5 m. Interference bands in green light were obtained at a distance of 5 mm from each other. Find the wavelength of the green light.

16.7. In Young's experiment a thin glass plate is placed in the path of one of the interfering rays. This causes the central light band to shift into a position which was initially occupied by the fifth light band (not counting the central one). The ray falls onto the plate perpendicularly. The refractive index of the plate is 1.5. The wavelength is 6×10^{-7} m. What is the thickness of the plate?

16.8. In Young's experiment a steel plate 2 cm thick is placed in the path of one of the interfering rays perpendicular to the ray. How much can the refractive indices differ from each other at various spots on the plate for the change in the path difference due to this heterogeneity not to exceed 1 micron?

16.9. White light falls at an angle of 45° onto a soap film (n=1.33). At what minimum thickness of the film will the reflected rays be coloured yellow ($\lambda = 6\times10^{-5}$ cm)?

16.10. A vertical soap film forms a wedge due to the liquid trickling down. By observing the interference bands in the reflected light of a mercury arc ($\lambda=5{,}461$ Å), we find that the distance between five bands is 2 cm. Find the wedge angle in seconds. The light falls at right angles to the film surface. The refractive index of the soapy water is 1.33.

16.11. A vertical soap film forms a wedge. Interference is observed in reflected light through a red glass ($\lambda=6.31\times10^{-5}$ cm). The distance between adjacent red bands is 3 mm. Then the same film is observed through a dark blue glass ($\lambda=4\times10^{-5}$ cm). Find the distance between adjacent dark blue bands. Assume that the shape of the film does not change and the light falls onto the film normally.

16.12. A beam of light ($\lambda=5.82\times10^{-7}$ m) falls normally onto a glass wedge. The wedge angle is 20″. What is the number of dark interference bands per unit of wedge length? The refractive index of the glass is 1.5.

16.13. A plant used to produce Newton's rings is illuminated by monochromatic light. Observations are done in reflected light. The radii of two adjacent dark rings are 4.0 mm and 4.38 mm, respectively. The radius of curvature of the lens is 6.4 m. Find the ordinal numbers of the rings and the wavelength of the incident light.

16.14. Newton's rings are formed between a flat glass and a lens with a radius of curvature of 8.6 m. Monochromatic light falls normally. It has been found by measurement that the diameter of the fourth dark ring (assuming the central dark spot as the zero ring) is equal to 9 mm. Find the wavelength of the incident light.

16.15. A plant for producing Newton's rings is illuminated by normally incident white light. Find: (1) the radius of the fourth dark-blue ring ($\lambda_1=4\times10^{-5}$ cm), and (2) the radius of the third red ring ($\lambda_2=6.3\times10^{-5}$ cm). The observations are made in transmitted light. The radius of curvature of the lens is 5 m.

16.16. The distance between the fifth and the twenty-fifth Newton's bright rings is 9 mm. The radius of curvature of the lens is 15 m. Find the wavelength of monochromatic light normally incident onto the plant. The observation is made in transmitted light.

16.17. Find the distance between the third and sixteenth Newton's dark rings if the distance between the second and the twentieth dark rings is equal to 4.8 mm. The observation is made in reflected light.

16.18. A plant for producing Newton's rings is illuminated by normally incident light of a mercury arc. The observation is made in transmitted light. What bright ring (counting from the beginning) corresponding to the line $\lambda_1=5{,}791$ Å coincides with the next bright ring which corresponds to the line $\lambda_2=5{,}770$ Å?

16.19. The space between the lens and the glass plate in a plant used to observe Newton's rings is filled with liquid. Find the refractive index of the liquid if the radius of the third bright ring is equal to 3.65 mm. The observation is made in transmitted light. The radius of curvature of the lens is 10 m. The wavelength of light is $5.89\times\times10^{-5}$ cm.

16.20. A plant used to observe Newton's rings is illuminated by normally incident monochromatic light with a wavelength of 0.6 μ. Find the thickness of the air layer between the lens and the glass plate where the fourth dark ring is observed in reflected light.

16.21. A plant used to observe Newton's rings in reflected light is illuminated by normally incident monochromatic light with $\lambda=5\times\times10^3$ Å. The space between the lens and the glass plate is filled with water. Find the thickness of the water layer between the lens and the glass plate where the third bright ring is observed.

16.22. A plant used to observe Newton's rings in reflected light is illuminated by normally incident monochromatic light. After the space between the lens and the glass plate is filled with liquid, the radii of the dark rings diminished to 0.8 of the original ones. Find the reflective index of the liquid.

16.23. In an experiment with a Michelson interferometer, the mirror had to be shifted over a distance of 0.161 mm to displace the interference pattern by 500 bands. Find the wavelength of the incident light.

16.24. An evacuated tube with a length of $l=14$ cm is placed into one of the arms of a Michelson interferometer to measure the refractive index of ammonia. The ends of the tube are shut with plane-parallel glasses. When the tube is filled with ammonia, the interference pattern for the wavelength $\lambda=0.59$ μ is displaced by 180 bands. Find the refractive index of ammonia.

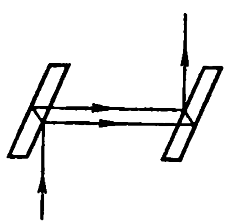


Fig. 63

16.25. An evacuated tube 10 cm long is placed in the way of one of the rays issuing from a Jamin interferometer (Fig. 63). When the tube is filled with chlorine, the interference pattern is displaced by 131 bands. In this experiment, the wavelength of monochromatic light is 5.9×10^{-5} cm. Find the refractive index of chlorine.

16.26. A beam of white light falls at right angles onto a glass plate with a thickness of $d=0.4$ μ. The refractive index of the glass $n=1.5$. What wavelengths lying within the limits of the visible spectrum (from 4×10^{-4} to 7×10^{-4} mm) are amplified in the reflected beam?

16.27. A thin film with a refractive index of $n_2=1.2$ (a coating film) is applied to the surface of a lens ($n_1=1.5$). At what minimum thickness of this film will the reflected light be attenuated most in the middle of the visible spectrum?

16.28. Light from a monochromatic source ($\lambda=0.6$ μ) falls at right angles onto a diaphragm with a round hole. The diameter of the hole is 6 mm. A screen is placed behind the diaphragm 3 m away from it.

(1) How many Fresnel zones can be accommodated in the diaphragm hole? (2) Will the centre of the diffraction pattern be dark or bright on the screen?

16.29. Calculate the radii of the first five Fresnel zones if the distance from the source of light to the wave surface is 1 m, the distance from the wave surface to the point of observation is also 1 m, and $\lambda = 5\times10^{-7}$ m.

16.30. Calculate the radii of the first five Fresnel zones for a plane wave. The distance from the wave surface to the point of observation is 1 m. The wavelength $\lambda=5\times10^{-7}$ m.

16.31. A diffraction pattern is observed at a distance of l from a point source of monochromatic light ($\lambda=6\times10^{-5}$ cm). A circular opaque barrier 1 cm in diameter is placed at a distance of $0.5l$ from the source. What will the distance l be if the barrier closes only the Fresnel central zone?

16.32. A diffraction pattern is observed at a distance of 4 m from a point source of monochromatic light ($\lambda=5\times10^{-7}$ m). A diaphragm with a round aperture is placed halfway between the screen and the source of light. At what radius of the aperture will the centre of the diffraction rings observed on the screen be the darkest?

16.33. A parallel beam of monochromatic light ($\lambda=6\times10^{-7}$ m) falls normally onto a diaphragm with a round aperture. A diffraction pattern is observed on the screen. At what maximum distance between the diaphragm and the screen will a dark spot still be observed at the centre of the diffraction pattern? The aperture diameter is 1.96 mm.

16.34. A parallel beam of monochromatic light with a wavelength of $\lambda=5{,}890$ A falls normally onto a slit 2 μ wide. Find the angles in whose direction the minima of light will be observed.

16.35. A parallel beam of monochromatic light with a wavelength of $\lambda=5\times10^{-5}$ cm falls normally onto a slit 2×10^{-3} cm wide. Find the width of the slit image on a screen removed by $l=1$ m from the slit. Take the width of the image to be the distance between the first diffraction minima located at both sides of the principal maximum of illumination.

16.36. A parallel beam of monochromatic light with a wavelength of λ falls normally onto a slit. The width of the slit is 6λ. At what angle will the third diffraction minimum of light be observed?

16.37. What is the constant of a diffraction grating if a telescope is set at an angle of 30° to the collimator axis to see a red line ($\lambda=7\times10^{-7}$ m) in the second-order spectrum? What number of lines are there on 1 cm of the grating length? The light falls normally onto the grating.

16.38. How many lines are there on 1 mm of a diffraction grating if a green line of mercury ($\lambda=5{,}461$ Å) is observed in the first-order spectrum at an angle of 19°8′?

16.39. A beam of light falls normally onto a diffraction grating. The diffraction angle for the sodium line ($\lambda = 5{,}890$ Å) in the first-order spectrum was found to equal 17° 8′. A certain line produces a diffraction angle of 24° 12′ in the second-order spectrum. Find the wavelength of this line and the number of lines per millimetre of the grating.

16.40. A beam of light from a discharge tube falls normally onto a diffraction grating. What should the constant of the diffraction grating be for the maxima of the two lines $\lambda_1 = 6{,}563$ Å and $\lambda_2 = 4{,}102$ Å to coincide in the direction $\varphi = 41°$?

16.41. A beam of light falls normally onto a diffraction grating. When a goniometer is turned through an angle of φ, the line $\lambda = 4.4 \times 10^{-4}$ mm in the third-order spectrum appears in the field of vision. Will any other spectral lines corresponding to the wavelengths within the visible spectrum (from 4×10^{-4} to 7×10^{-4} mm) be seen at the same angle φ?

16.42. A beam of light from a discharge tube filled with helium falls normally onto a diffraction grating. Onto what line in the third-order spectrum will the red line of helium ($\lambda = 6.7 \times 10^{-5}$ cm) of the second-order spectrum be superimposed?

16.43. Light from a discharge tube filled with helium falls normally onto a diffraction grating. First a telescope is adjusted to see the violet lines ($\lambda = 3.89 \times 10^{-5}$ cm) at both sides of the central band in the first-order spectrum. The readings on the dial to the right from zero are 27°33′ and 36° 27′, respectively. Then the telescope is adjusted to the red lines at both sides of the central band in the first-order spectrum. The readings on the dial to the right from zero show 23° 54′ and 40°6′. Find the wavelength of the red line of the helium spectrum.

16.44. Find the maximum order of a spectrum for the yellow line of sodium $\lambda = 5{,}890$ Å if the constant of the diffraction grating is 2 μ.

16.45. A beam of monochromatic light falls normally onto a diffraction grating. A maximum of the third order is observed at an angle of 36° 48′ to the normal. Find the constant of the grating expressed in the wavelengths of the incident light.

16.46. How many maxima are produced by the diffraction grating of the previous problem?

16.47. The telescope of a goniometer with a diffraction grating is placed at an angle of 20° to a collimator axis, and the red line of the helium spectrum ($\lambda_1 = 6{,}680$ Å) is visible in its field of vision. What is the constant of the diffraction grating if a dark-blue line ($\lambda_2 = 4{,}470$ Å) of a higher order can be seen at the same angle? The maximum order of a spectrum which can be observed with this grating is 5. The light falls upon the grating normally.

16.48. What is the constant of a diffraction grating if it can resolve in the first order the lines of the potassium spectrum $\lambda_1 = 4{,}044$ Å and $\lambda_2 = 4{,}047$ Å? The grating is 3 cm wide.

16.49. What is the constant of a diffraction grating 2.5 cm wide for the sodium doublet $\lambda_1=5{,}890$ Å and $\lambda_2=5{,}896$ Å to be resolved in the first order?

16.50. The constant of a diffraction grating 2.5 cm wide is 2 μ. What wavelength difference can be resolved by this grating in the region of the yellow rays ($\lambda=6\times10^{-5}$ cm) in a second-order spectrum?

16.51. Determine the angular dispersion of a diffraction grating for $\lambda=5{,}890$ Å in a first-order spectrum. The grating constant is $2.5\times\times10^{-4}$ cm.

16.52. The angular dispersion of a diffraction grating for $\lambda=6{,}680$ Å in a first-order spectrum is 2.02×10^5 rad/m. Find the period of the diffraction grating.

16.53. Find the linear dispersion (mm/Å) of the diffraction grating of the previous problem if the focal length of the lens which projects the spectrum onto a screen is 40 cm.

16.54. At what distance from each other will two lines of a mercury arc ($\lambda_1=5{,}770$ Å and $\lambda_2=5{,}791$ Å) be arranged on a screen in a first-order spectrum obtained by means of a diffraction grating with the period 2×10^{-4} cm? The focal length of the lens projecting the spectrum onto the screen is 0.6 m.

16.55. A beam of light falls normally onto a diffraction grating. A red line ($\lambda=6{,}300$ Å) is visible in the third-order spectrum at an angle of $\varphi=60°$. (1) What spectral line is visible at the same angle in the fourth-order spectrum? (2) What number of lines are there on the diffraction grating per mm of length? (3) What is the angular dispersion of this grating for the line $\lambda=6{,}300$ Å in the third-order spectrum?

16.56. For what wavelength does a diffraction grating with the constant $d=5$ μ have an angular dispersion of $D=6.3\times10^5$ rad/m in the third-order spectrum?

16.57. Find the focal length of a lens which so projects onto a screen a spectrum obtained with the aid of a diffraction grating that the distance between two potassium lines 4,044 Å and 4,047 Å is 0.1 mm in the first-order spectrum. The constant of the diffraction grating is 2 μ.

16.58. Determine the angle of complete polarization when light is reflected from glass with a refractive index of 1.57.

16.59. The limit angle of total internal reflection for a certain substance is 45°. What is the angle of complete polarization for this substance?

16.60. At what angle to the horizon should the Sun be for its rays reflected from the surface of a lake to be polarized the most completely?

16.61. What is the refractive index of glass if a beam reflected from it is completely polarized at an angle of refraction of 30°?

16.62. A beam of light passes through a liquid poured into a glass ($n=1.5$) vessel and is reflected from the bottom. The reflected beam is completely polarized when it falls onto the bottom of the vessel at an angle of 42°37′. Find: (1) the refractive index of the liquid, (2) the angle at which a beam of light passing in this liquid should fall onto the bottom of the vessel to obtain total internal reflection.

16.63. A beam of plane-polarized light whose wavelength is 5,890 A in vacuum falls onto a plate of Iceland spar at right angles to its optical axis. Find the wavelengths of an ordinary and an extraordinary rays in the crystal if the refractive indices of Iceland spar for an ordinary and an extraordinary rays are $n_o=1.66$ and $n_e=1.49$, respectively.

16.64. What is the angle between the principal planes of a polarizer and an analyzer if the intensity of natural light after passing through the polarizer and the analyzer reduces to one-fourth of the original value? Disregard the absorption of the light.

16.65. Natural light passes through a polarizer and an analyzer so placed that the angle between their principal planes is α. Both the polarizer and the analyzer absorb and reflect 8 per cent of the incident light. It was found that the intensity of the beam issuing from the analyzer is 9 per cent of that of the natural light falling onto the polarizer. Find the angle α.

16.66. Determine the reflection factor of natural light falling upon glass ($n=1.54$) at the angle of complete polarization. Find the degree of polarization of the rays that have passed into the glass. Disregard the absorption of the light.

16.67. A ray of natural light passes through a plane-parallel glass plate ($n=1.54$) falling on it at the angle of complete polarization. Find the degree of polarization of the rays that passed through the plate.

16.68. Determine: (1) the reflection factor and the degree of polarization of the reflected rays when natural light falls upon glass ($n=1.5$) at an angle of 45°, (2) the degree of polarization of the refracted rays.

17. Elements of the Theory of Relativity

The length l' of a body moving with the velocity v with respect to a certain reference system is related to the length l_0 of a body immobile in this system by the expression

$$l' = l_0\sqrt{1-\beta^2}$$

where $\beta = \frac{v}{c}$; and c is the velocity of propagation of light.

The time interval $\Delta\tau'$ in a system moving with the velocity v with respect to an observer is related to the time interval $\Delta\tau_0$ in a system immobile with respect to the observer by the ratio

$$\Delta\tau' = \frac{\Delta\tau_0}{\sqrt{1-\beta^2}}$$

The mass m of a body depends on the velocity of its motion according to the equation

$$m = \frac{m_0}{\sqrt{1-\beta^2}}$$

where m_0 is the rest mass of this body.

The kinetic energy of a body is related to the velocity by the equation

$$E_k = m_0c^2\left(\frac{1}{\sqrt{1-\beta^2}} - 1\right)$$

The change in the mass of a system by Δm corresponds to a change in the energy of the system by

$$\Delta E = c^2\Delta m$$

17.1. At what relative velocity of motion will the relativistic contraction in the length of a moving body be 25 per cent?

17.2. What should the velocity of a moving body be for its longitudinal dimensions to be halved?

17.3. The mesons of cosmic rays reach the Earth with various velocities. Find the relativistic contraction in the dimensions of a meson having a velocity equal to 95 per cent of the velocity of light.

17.4. How many times will the life of an unstable particle (as shown by the watch of a stationary observer) increase if the particle begins to move with a velocity equal to 99 per cent of that of light?

17.5. A meson in cosmic rays travels with a velocity equal to 95 per cent of that of light. What time interval on the watch of an observer on Earth corresponds to one second of the intrinsic time of the meson?

17.6. How much will the mass of an α-particle increase when it is accelerated from its initial velocity equal to zero to a velocity equal to 0.9 of that of light?

17.7. Find the relationship between the charge of an electron and its mass for the following velocities: (1) $v \ll c$, (2) 2×10^{10} cm/s, (3) 2.2×10^{10} cm/s, (4) 2.4×10^{10} cm/s, (5) 2.6×10^{10} cm/s, (6) 2.8×10^{10} cm/s. Compile a table and draw a diagram showing how m and $\frac{e}{m}$ depend on the ratio $\frac{v}{c}$ for these velocities.

17.8. At what velocity is the mass of a moving electron double its rest mass?

17.9. What energy can particles be accelerated to in a cyclotron for the relative increase in the mass of the particles not to exceed 5 per cent? Solve the problem for (1) electrons, (2) protons, (3) deuterons.

17.10. What accelerating difference of potentials must an electron travel through for its velocity to become equal to 95 per cent of that of light?

17.11. What accelerating difference of potentials must a proton travel through for its longitudinal dimensions to become half the initial ones?

17.12. Find the velocity of a meson if its total energy is 10 times greater than its rest energy.

17.13. What part of the velocity of light should the velocity of a particle be for its kinetic energy to equal the rest energy?

17.14. A proton synchrotron produces a beam of protons with a kinetic energy of 10,000 MeV. What part of the velocity of light does the velocity of the protons in this beam form?

17.15. What is the relativistic contraction in the dimensions of a proton in the conditions of the previous problem?

17.16. Electrons flying out of a cyclotron have a kinetic energy of 0.67 MeV. What part of the velocity of light does the velocity of these electrons form?

17.17. Compile for electrons and protons a table showing how their kinetic energy E_k depends on the velocity (in fractions of the velocity of light) for the following values of β: (1) 0.1, (2) 0.5, (3) 0.6, (4) 0.7, (5) 0.8, (6) 0.9, (7) 0.95, and (8) 0.999.

17.18. The mass of a moving electron is double its rest mass. Find the kinetic energy of this electron.

17.19. What change in mass does a change in energy of one calorie correspond to?

17.20. Find the change in energy which corresponds to a change in mass of one atomic unit.

17.21. Find the change in energy which corresponds to a change in mass equal to the rest mass of an electron.

17.22. Find the loss in mass upon the formation of one kilomole of water if the relevant reaction is

$$2H_2+O_2=2H_2O+5.75\times 10^5\ \text{kJ}$$

17.23. During the fission of a nucleus of uranium $_{92}U^{235}$, energy approximately equal to 200 MeV is released. Find the change in mass during the fission of one kilomole of uranium.

17.24. The Sun radiates energy equal to 6.5×10^{21} kW-h every minute. Considering the radiation of the Sun to be constant, find the time in which the mass of the Sun will be halved.

18. Thermal Radiation

The emittance (emissive power) of a black body, i. e., the energy radiated in 1 second by a unit surface of the body, is determined by the Stefan-Boltzmann formula

$$M_e = \sigma T^4$$

where T = temperature in degrees Kelvin
σ = Stefan-Boltzmann constant

$$\sigma = 5.67 \times 10^{-8} \ \mathrm{W/m^2 \cdot deg^4}$$

If a radiating body is not absolutely black, then $M_e' = k\sigma T^4$, where k is always less than unity. The emittance M_e is related to the energy density $M_{e\lambda}'$ (emittance in terms of wavelength) of a black body by the formula

$$M_e' = \int_0^\infty M_{e\lambda} d\lambda$$

According to the Wien displacement law, the product of the absolute temperature of a black body and the wavelength at which the energy density of the body is maximum is constant, i. e.,

$$\lambda_{max} T = C_1 = 2.9 \times 10^{-3} \ \mathrm{m \cdot deg}$$

The maximum energy density of a black body increases in proportion to the 5th power of the absolute temperature (Wien's second law)

$$(M_{e\lambda})_{max} = C_2 T^5$$

where $C_2 = 1.29 \times 10^{-5} \ \mathrm{W/m^3 \cdot deg^5}$.

18.1. Find the temperature of a furnace if 8.28 cal per second is radiated from a hole in it 6.1 cm² in size. Consider the radiation close to that of a black body.

18.2. What amount of energy is radiated by the Sun in one minute? Consider its radiation close to that of a black body. The temperature of the Sun's surface is 5800° K.

18.3. What amount of energy is radiated by one square centimetre of solidifying lead per second? The ratio between the emittances of the lead surface and a black body is 0.6 for this temperature.

18.4. The radiant power of a black body is 34 kW. Find the temperature of the body if its surface is 0.6 m².

18.5. A glowing metal surface with an area of 10 cm² radiates 4×10^4 J per minute. The temperature of the surface is 2500° K. Find: (1) the radiation of this surface if it were absolutely black, (2) the ratio between the emittances of this surface and of a black body at the given temperature.

18.6. The diameter of a tungsten filament in an electric lamp is 0.3 mm and its length is 5 cm. When the lamp is connected to a 127-volt circuit, a current of 0.31 A flows through it. Find the temperature of the lamp. Assume that after equilibrium is reached all the heat evolved in the filament is lost due to radiation. The ratio between the emittances of the tungsten and of a black body is 0.31 at this temperature.

18.7. The temperature of a tungsten filament in a 25-watt electric lamp is 2450° K. The ratio between the emittances of the lamp and of a black body is 0.3 at this temperature. Find the magnitude of the radiating surface of the filament.

18.8. Find the solar constant, i. e., the quantity of radiant energy sent by the Sun every minute through an area of 1 cm^2 perpendicular to the solar rays and at the same distance from the Sun as the Earth. The temperature of the Sun's surface is 5800° K. Consider the radiation of the Sun close to that of a black body.

18.9. Assuming that the atmosphere absorbs 10 per cent of the radiant energy emitted by the Sun, find the power received from the Sun by a horizontal plot of land with an area of 0.5 ha. The Sun's altitude above the horizon is 30°. Consider the radiation of the Sun close to that of a black body.

18.10. Knowing the solar constant for the Earth (see Problem 18.8), find it for Mars.

18.11. Find the quantity of energy radiated from 1 cm^2 of a surface in one second by a black body if the maximum energy density corresponds to a wavelength of 4,840 Å.

18.12. The radiation power of a black body is 10 kW. Find the area of the radiating surface of the body if the wavelength corresponding to the maximum energy density is 7×10^{-5} cm.

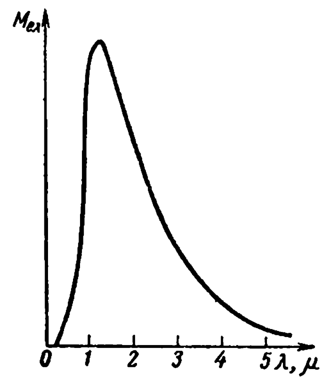


Fig. 64

18.13. In what regions of the spectrum do the wavelengths corresponding to the maximum energy density lie if the light source is (1) the filament of an electric lamp (T=3000° K), (2) the Sun's surface (T=6000° K), and (3) an atomic bomb in which a temperature of about 10 million degrees develops at the moment of explosion? Consider the radiation close to that of a black body.

18.14. Figure 64 contains a curve showing the distribution of the energy density of a black body at a certain temperature. What temperature does this curve relate to? Using Fig. 64, also find the percentage of the radiant energy falling to the share of the visible spectrum at this temperature.

18.15. When a black body is heated, the wavelength corresponding to the maximum energy density changes from 0.69 to 0.5 μ. How many times does the emittance of the body increase in this case?

18.16. What wavelength does the maximum energy density of a black body with the temperature of a human body, i. e., $t=37°$ C, correspond to?

18.17. Upon heating, the temperature of a black body changes from 1000 to 3000° K. (1) How many times does its emittance increase? (2) How many times does the wavelength which the maximum energy density corresponds to increase? (3) How many times does the maximum energy density of the body increase?

18.18. A black body has a temperature of $T_1=2900°$ K. When the body cools, the wavelength which the maximum energy density corresponds to changes by $\Delta\lambda=9\ \mu$ What temperature T_2 is the body cooled to?

18.19. The surface of a body is heated to 1000° K. Next one half of this surface is heated by 100° and the other cooled by 100°. How many times will the emittance of the body surface change?

18.20. What power must be supplied to a black metal ball with a radius of 2 cm to keep its temperature 27° above the ambient temperature, which is 20° C? Assume that heat is lost only as a result of radiation.

18.21. A black ball is cooled from 27° C to 20° C. How much will the wavelength corresponding to the maximum energy density change?

18.22. (1) Find how much the mass of the Sun will change during a year due to radiation. (2) Assuming the radiation of the Sun to be constant, find the time during which its mass will be halved. The temperature of the Sun's surface is 5800° K.

Chapter 6

ATOMIC AND NUCLEAR PHYSICS

UNITS OF RADIOACTIVITY AND IONIZING RADIATION

The basic and some derived units for measuring radioactivity and ionizing radiation in conformity with GOST 8848-63 are given in Table 17.

TABLE 17

Quantity and symbol	Formula	Unit	Symbol of unit	Dimension of quantity
		Basic Units		
Length l	—	metre	m	l
Mass m	—	kilogramme	kg	m
Time t	—	second	s	t
Current intensity I	—	ampere	A	I
		Derived Units		
Activity of isotope in radioactive source (n)	$n=\frac{dN}{dt}$	disintegration per second	d/s	t^{-1}
Radiation intensity I_e	$I_e=\frac{E}{S}$	watt per square metre	W/m²	mt^{-3}
Absorbed radiation dose D_a	$D_a=\frac{E}{m}$	joule per kilogramme	J/kg	l^2t^{-2}
Power of absorbed radiation dose P_a	$P_a=\frac{D_a}{t}$	watt per kilogramme	W/kg	l^2t^{-3}
Exposure dose of X- and gamma radiation	$D_e=\frac{q}{m}$	coulomb per kilogramme	C/kg	$m^{-1}tI$
Power of exposure dose of X- and gamma radiation	$P_e=\frac{D_e}{t}$	ampere per kilogramme	A/kg	$m^{-1}I$

Note. The definitions of the units for measuring the absorbed radiation dose and the exposure dose of X- and gamma radiation are as follows.

Joule per kilogramme is the absorbed radiation dose measured by an energy of 1 J of ionized radiation of any kind transferred to a mass of 1 kg of the irradiated substance.

Coulomb per kilogramme is the exposure dose of X- and gamma radiation at which a conjugated corpuscular emission per kg of dry atmospheric air produces ions in air carrying an electric charge of 1 C of each sign.

This standard also permits the use of the non-system units given in Table 18.

TABLE 18

Quantity	Unit and conversion factor to SI units
Activity of isotope in radioactive source	1 curie (c) $=3.7\times10^{10}$ d/s
Absorbed radiation dose	1 rad $=10^{-2}$ J/kg
Exposure dose of X- and gamma radiation	1 roentgen (r) $=2.57976\times10^{-4}$ C/kg

Note. The unit of the exposure dose of X- and gamma radiation, coulomb per kilogramme, and also the non-system unit roentgen can be used to measure radiation with a quantum energy not exceeding 5×10^{-13} J (approximately 3 MeV).

EXAMPLES OF SOLUTIONS

Example 1. Air in standard conditions is irradiated by X-rays. The radiation dose is 1 r. Find the number of ion pairs formed by this radiation in 1 cm³ of the air.

Solution. The ions produced in the mass m of air by an exposure dose D_e of X-radiation transfer a charge of

$$q=D_e m \qquad (1)$$

The mass m and the volume V of the air are related by the formula

$$m=V\frac{p\mu}{RT} \qquad (2)$$

where p = air pressure
T = air temperature
μ = mass of one kilomole
R = gas constant.

The sought number of ion pairs is

$$N=\frac{q}{e} \qquad (3)$$

where e is the charge of each ion. From Eqs. (1), (2) and (3) we have

$$N=\frac{D_e V p\mu}{eRT} \qquad (4)$$

According to the initial condition, $D_e=1$ r $=2.58\times10^{-4}$ C/kg, $V=1$ cm$^3=10^{-6}$ m^3, $p=760$ mm Hg $\cong10^5$ N/m^2, $\mu=29$ kg/kmole, $R=8.31\times10^3$ J/kmole·deg, $T=273$°K and $e=1.6\times10^{-19}$ C. Upon inserting these data into Eq. (4), we get $N\approx2.1\times10^9$ ion pairs.

Example 2. The half-life of an artificially obtained radioactive isotope of calcium $_{20}Ca^{45}$ is equal to 164 days. Find the activity of 1 μg of this preparation.

Solution. The number of atoms of a radioactive substance ΔN disintegrating during the time Δt is determined by the formula $|\Delta N| = \frac{\log_e 2}{T} N\Delta t$, where T is the half-life of the isotope and N is the number of its atoms in the given mass. The number of atoms N is related to the mass m of the preparation by the expression $N = \frac{m}{A} N_A$, where N_A is Avogadro's number and A is the mass of one kg-atom. According to the conditions, $T = 164 \times 24 \times 3{,}600$ s, $m = 10^{-9}$ kg, $N_A = 6.02 \times 10^{26}$ 1/kg-atom, $A = 45$ kg/kg-atom. Upon inserting these data, we obtain the number of disintegrations per second: $\frac{\Delta N}{\Delta t} = 6.53 \times 10^{8}$ d/s $= 1.77 \times 10^{-2}$ c $= 17.7$ mc.

19. Quantum Nature of Light and Wave Properties of Particles

The energy of a quantum of light (photon) is found from the formula

$$\varepsilon = h\nu$$

where $h = 6.625 \times 10^{-34}$ J·s is Planck's constant, and ν is the oscillation frequency.

The momentum of a photon is

$$\bar{p}_{ph} = \frac{h\nu}{c}$$

and its mass

$$m = \frac{h\nu}{c^2}$$

where c is the velocity of light in a vacuum.

The relationship between the energy of a photon causing the photoemissive effect and the maximum kinetic energy of the emitted electrons is described by the Einstein formula

$$h\nu = W + \frac{mv^2}{2}$$

where W = work of exit of electron from metal
m = mass of an electron.

If $v = 0$, then $h\nu_0 = W$, where ν_0 is the frequency corresponding to the photoelectric threshold of the photoeffect.

The pressure of light is

$$p=\frac{E}{c}(1+\rho)$$

where E = quantity of energy incident upon a unit surface in unit time
ρ = light reflection coefficient.

The change in the wavelength of X-rays in Compton scattering can be determined from the formula

$$\Delta\lambda=\frac{h}{mc}(1-\cos\varphi)$$

where φ = scattering angle
m = mass of an electron.

A beam of elementary particles has the properties of a plane wave which propagates in the direction of movement of these particles. The wavelength λ which corresponds to this beam can be found from de Broglie's relation

$$\lambda=\frac{h}{mv}=\frac{h}{\sqrt{2E_k m}}$$

where v = velocity of the particles
m = mass of the particles
E_k = their kinetic energy.

If the velocity of the particles v is commensurable with the velocity of light c, the previous formula may be written as

$$\lambda=\frac{h}{m_0 v}\sqrt{1-\beta^2}=\frac{h}{\sqrt{2E_k m_0+\frac{E_k^2}{c^2}}}$$

where $\beta = v/c$
m_0 = rest mass of a particle.

19.1. Find the mass of a photon of (1) red light rays ($\lambda=7\times10^{-5}$ cm), (2) X-rays ($\lambda=0.25$ Å), and (3) gamma rays ($\lambda=1.24\times10^{-2}$ Å).

19.2. Determine the energy, mass and momentum of a photon if the wavelength corresponding to it is 0.016 Å.

19.3. A mercury arc is rated at 125 W. How many light quanta are emitted every second in radiation with wavelengths of (1) 6,123 Å, (2) 5,791 Å, (3) 5,461 Å, (4) 4,047 Å, (5) 3,655 Å, and (6) 2,537 Å? The intensity of these lines is, respectively: (1) 2%, (2) 4%, (3) 4%, (4) 2.9%, (5) 2.5%, and (6) 4% of the intensity of the mercury arc. Assume that 80% of power is spent for radiation.

19.4. What velocity must an electron travel with for its kinetic energy to equal the energy of a photon with a wavelength of $\lambda=5{,}200$ Å?

19.5. What velocity must an electron travel with for its momentum to equal that of a photon with a wavelength of $\lambda=5{,}200$ Å?

19.6. What energy must a photon have for its mass to equal the rest mass of an electron?

19.7. The momentum transferred by a monochromatic beam of photons through an area of $S=2$ cm^2 during the time $t=0.5$ min is equal to $\bar{p}_{ph}=3\times10^{-4}$ g·cm/s. Find the energy incident on a unit of area in a unit of time for this beam.

19.8. At what temperature will the kinetic energy of a molecule of a biatomic gas be equal to the energy of a photon with a wavelength of $\lambda=5.89\times10^{-4}$ mm?

19.9. Since it is difficult to measure doses of X- and gamma radiation in roentgens at high energies, GOST 8848-63 allows the use of a roentgen as a unit of dose for radiation with a quantum energy up to 3 MeV. Find the limit wavelength of X-radiation for which the roentgen can be used as a unit of measurement.

19.10. Find the mass of a photon whose momentum is equal to that of a hydrogen molecule at a temperature of 20° C. Assume the velocity of the molecule to be equal to the mean quadratic velocity.

19.11. The Russian scientist A. G. Stoletov was the first to establish the basic laws of the photoeffect. He formulated one of the results of his experiments as follows: "A discharging action is a property of rays with the highest refractivity whose length is less than 295×10^{-6} mm". Determine the work of emission of an electron from the metal used by Stoletov for his experiment.

19.12. Find the photoelectric threshold of the photoeffect for lithium, sodium, potassium and cesium.

19.13. The photoelectric threshold of the photoeffect for a certain metal is 2,750 Å. What is the minimum energy of a photon producing the photoeffect?

19.14. The photoelectric threshold of the photoeffect for a certain metal is 2,750 Å. Find: (1) the work of emission of an electron from this metal, (2) the maximum velocity of the electrons ejected from the metal by light with a wavelength of 1,800 Å, (3) the maximum kinetic energy of these electrons.

19.15. Find the frequency of the light which ejects from a metal surface electrons fully retarded by a reverse potential of 3 V. The photoeffect begins in this metal at a frequency of incident light of 6×10^{14} s^{-1}. Find the work of emission of an electron from this metal.

19.16. Find the retarding potential for photoelectrons emitted when potassium is illuminated by light with a wavelength of 3,300 Å.

19.17. In a photoeffect from a platinum surface, the retarding potential is 0.8 V. Find: (1) the wavelength of the radiation used, (2) the maximum wavelength at which the photoeffect is still possible.

19.18. Light quanta with an energy of $\varepsilon=4.9$ eV eject photoelectrons from metal with the work of emission $W=4.5$ eV. Find the maximum impulse transmitted to the surface of the metal when each electron flies out.

19.19. Determine Planck's constant h if the photoelectrons ejected from the surface of a certain metal by light with a frequency of 2.2×10^{15} s^{-1} are fully retarded by a reverse potential of 6.6 V, and those ejected by light with a frequency of 4.6×10^{15} s^{-1}—by a potential of 16.5 V.

19.20. A vacuum photocell consists of a central cathode (a tungsten ball) and an anode (the internal surface of a flask coated inside with silver). The contact potential difference between the electrodes, equal to $U_0=0.6$ V, accelerates the emitting electrons. The photocell is illuminated by light with a wavelength of $\lambda=2.3\times 10^{-7}$ m. (1) What retarding potential difference should be applied between the electrodes for the photocurrent to drop to zero? (2) What velocity will be imparted to the photoelectrons when they reach the anode if no external potential difference is applied between the cathode and anode?

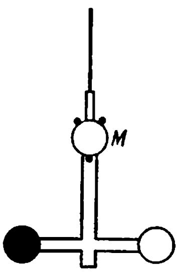


Fig. 65

19.21. A retarding potential difference of 1 V is applied between the electrodes of the photocell of the previous problem. At what limit wavelength λ of light incident onto the cathode will the photoeffect begin?

19.22. Figure 65 shows part of the device with which the Russian scientist P. N. Lebedev conducted his experiments for measuring the pressure of light. A glass crosspiece suspended on a thin thread is enclosed in an evacuated vessel and carries on its ends two light disks made of platinum foil. One of the disks is painted black, and the other is left bright. The pressure of light can be found by directing light onto one of the disks and measuring the angle through which the thread turns (with the aid of mirror M). Find: (1) the pressure of light, (2) the energy falling from an arc lamp in one second onto 1 cm^2 of the surface of the disks if when illuminating the bright disk, the deflection of the mirror light spot is 76 mm on a scale 1,200 mm from the mirror. The diameter of the disks is 5 mm. The distance from a disk centre to the axis of rotation is 9.2 mm. The reflection coefficient for the bright circle is 0.5. The constant k of the thread twisting moment ($M=k\alpha$) is equal to 2.2×10^{-4} dyn·cm/rad.

19.23. In one of Lebedev's experiments, the twisting angle of the thread was 10′ when light fell onto the black disk ($\rho=0$). Find: (1) the pressure of light, (2) the power of the incident light. Take the data necessary from the previous problem.

19.24. In one of Lebedev's experiments, the power of monochromatic light ($\lambda=5.6\times10^{-5}$ cm) falling on the disks was 0.5 J/min. Find: (1) the number of photons falling onto 1 cm^2 of the surface of the wings a second, (2) the impulse of the force imparted to 1 cm^2 of the surface of the disks a second. Find the value of the impulse for (a) $\rho=0$, (b) $\rho=0.5$, and (c) $\rho=1$. Take the data necessary from Problem 19.22.

19.25. The Russian astronomer F. A. Bredikhin attributed the shape of comet trains to the pressure of Sun rays. Find: (1) the pressure of the light of Sun rays on a black body placed at the same distance from the Sun as the Earth, (2) the mass of a particle in a comet train at this distance when the force of light pressure exerted on it is balanced by the force of attraction of the particle by the Sun. The area of the particle which reflects all the incident rays is 0.5×10^{-8} cm^2. The solar constant is 8.21 J/min$\cdot cm^2$.

19.26. Find the pressure of light exerted on the walls of a 100-watt electric lamp. The lamp bulb is a spherical vessel with a radius of 5 cm. The walls of the lamp reflect 4 per cent and let through 6 per cent of the incident light. Assume that all the consumed power is spent on radiation.

19.27. Light energy amounting to 63 J falls every minute upon a surface with an area of 100 cm^2. Find the pressure of the light when the surface (1) reflects all the rays, and (2) absorbs all the incident rays.

19.28. A monochromatic beam of light ($\lambda=4{,}900$ Å) normally incident upon a surface produces a pressure of 5×10^{-7} kgf/m^2 on it. How many light quanta fall every second upon a unit area of this surface? The reflection coefficient of light $\rho=0.25$.

19.29. X-rays with a wavelength of $\lambda_0=0.708$ Å undergo Compton scattering on paraffin. Find the wavelength of the X-rays scattered in the directions (1) $\frac{\pi}{2}$, (2) π.

19.30. What is the wavelength of X-radiation if upon Compton scattering of this radiation by graphite at an angle of 60° the wavelength of the scattered radiation is 2.54×10^{-9} cm?

19.31. X-rays with a wavelength of $\lambda_0=0.2$ Å undergo Compton scattering at an angle of 90°. Find: (1) the change in the wavelength of the X-rays in scattering, (2) the energy of a recoil electron, (3) the momentum of a recoil electron.

19.32. In the Compton phenomenon, the energy of an incident photon is equally distributed between the scattered photon and the recoil electron. The scattering angle is $\frac{\pi}{2}$. Find the energy and the momentum of the scattered photon.

19.33. The energy of X-rays is 0.6 MeV. Find the energy of a recoil electron if the wavelength of the X-rays changes by 20 per cent after Compton scattering.

19.34. Find the de Broglie wavelength for electrons which passed through a potential difference of (1) 1 V, and (2) 100 V.

19.35. Solve the previous problem for a beam of protons.

19.36. Find the de Broglie wavelength for (1) an electron flying with a velocity of 10^8 cm/s, (2) a hydrogen atom moving with a velocity equal to the mean quadratic velocity at a temperature of 300° K, (3) a ball with a mass of 1 g moving with a velocity of 1 cm/s.

19.37. Find the de Broglie wavelength for an electron with a kinetic energy of (1) 10 keV, (2) 1 MeV.

19.38. A charged particle accelerated by a potential difference of 200 V has a de Broglie wavelength equal to 0.0202 Å. Find the mass of this particle if its charge is numerically equal to the charge of an electron.

19.39. Compile a table of de Broglie wavelengths for an electron depending on the velocity v for the following velocities: (1) 2×10^8 m/s, (2) 2.2×10^8 m/s, (3) 2.4×10^8 m/s, (4) 2.6×10^8 m/s, (5) 2.8×10^8 m/s.

19.40. An α-particle moves along a circle with a radius of 0.83 cm in a homogeneous magnetic field having an intensity of 250 Oe. Find the de Broglie wavelength for this α-particle.

19.41. Find the de Broglie wavelength for a hydrogen atom traveling at a temperature of 20° C with the most probable velocity.

20. Bohr's Atom. X-Rays

According to Bohr's first postulate, an electron can move around a nucleus only along definite orbits whose radii satisfy the ratio

$$mv_k r_k = k\frac{h}{2\pi}$$

where m = mass of the electron
v_k = velocity of the electron on the k-th orbit
r_k = radius of this orbit
h = Planck's constant
k = any integer (quantum number).

According to Bohr's second postulate, the radiation frequency which corresponds to transition of an electron from one orbit to another is determined from the formula

$$h\nu = E_n - E_k$$

where k and n are the numbers of the orbits ($n>k$), and E_k and E_n are the corresponding values of the electron energy.

The formula used to find the frequency ν or wavelength λ which correspond to the lines of the hydrogen spectrum has the form

$$\nu = \frac{c}{\lambda} = Rc\left(\frac{1}{k^2} - \frac{1}{n^2}\right)$$

where k and n = numbers of the orbits
c = velocity of light in a vacuum
R = Rydberg constant equal to

$$R=\frac{e^4 m}{8\varepsilon_0^2 h^3 c}=1.097\times 10^7 \text{ m}^{-1}$$

here e = charge of an electron
m = mass of an electron
h = Planck's constant
ε_0 = dielectric constant.

The formula used to determine the frequencies ν or wavelengths λ for hydrogen-like ions is as follows:

$$\nu=\frac{c}{\lambda}=RcZ^2\left(\frac{1}{k^2}-\frac{1}{n^2}\right)$$

where Z is the atomic number of the element.

In the diffraction of X-rays, Bragg's law applies:

$$2d\sin\varphi=n\lambda$$

where d = interplanar spacing of crystal lattice planes
φ = angle between the X-ray beam and the crystal surface
n = integer.

The short-wave boundary ν_0 of a continuous X-ray spectrum can be found from the expression

$$h\nu_0=eU$$

where U is the difference of potentials applied to an X-ray tube.

The frequencies and wavelengths of characteristic X-rays can be found from Moseley's formula

$$\nu=\frac{c}{\lambda}=Rc(Z-b)^2\left(\frac{1}{k^2}-\frac{1}{n^2}\right)$$

where Z = atomic number of the element which the anticathode is made of
b = screening constant.

The latter formula can be rewritten as

$$\sqrt{\nu}=a(Z-b), \text{ where } a=\sqrt{Rc\left(\frac{1}{k^2}-\frac{1}{n^2}\right)}$$

The intensity of a beam of X-rays which have passed through a plate with a thickness of x is determined from the formula

$$I=I_0 e^{-\mu x}$$

where I_0 = intensity of the beam falling upon the plate
μ = linear absorption coefficient.

The absorption coefficient μ depends on the wavelength of the X-rays and the density of the material. The mass absorption coefficient μ_m is related to the linear coefficient μ by the expression $\mu_m=\frac{\mu}{\rho}$, where ρ is the density of the material.

The absorption of X-rays by various materials can be characterized by the so-called half-value layer, i. e., by the thickness of a plate which halves the intensity of the incident rays.

20.1. Find: (1) the radii of the first three Bohr electron orbits in a hydrogen atom, (2) the velocity of an electron in them.

20.2. Find the numerical value of the kinetic, potential and total energy of an electron in the first Bohr orbit.

20.3. Calculate the kinetic energy of an electron in the n-th orbit of a hydrogen atom. Solve the problem for $n=1$, 2, 3 and ∞.

20.4. Find: (1) the period of revolution of an electron in the first Bohr orbit of a hydrogen atom, (2) the angular velocity of the electron.

20.5. Find the minimum and maximum wavelengths of hydrogen spectral lines in the visible region of the spectrum.

20.6. (1) Find the maximum wavelength in the ultraviolet series of a hydrogen spectrum. (2) What minimum velocity should electrons have for this line to appear when hydrogen atoms are excited by impacts of electrons?

20.7. Determine the ionization potential of a hydrogen atom.

20.8. Determine the first excitation potential of a hydrogen atom.

20.9. (1) What minimum energy (in electron-volts) must electrons have for all the lines of all the series of the hydrogen spectrum to appear when the hydrogen atoms are excited by impacts of these electrons? (2) What is the minimum velocity of these electrons?

20.10. Within what limits should the energy of the bombarding electrons be for the hydrogen spectrum to have only one spectral line when hydrogen atoms are excited by impacts of these electrons?

20.11. What minimum energy (in electron-volts) should electrons have for the hydrogen spectrum to consist of three spectral lines when hydrogen atoms are excited by impacts of these electrons? Find the wavelengths of these lines.

20.12. Within what limits should the wavelengths of monochromatic light be for three spectral lines to appear when hydrogen atoms are excited by quanta of this light?

20.13. How much will the kinetic energy of an electron in a hydrogen atom change if the atom emits a photon with a wavelength of $\lambda=$ $=4{,}860$ Å?

20.14. Within what limits should the wavelengths of monochromatic light be for the electron orbital radius to increase 9 times when hydrogen atoms are excited by quanta of this light?

20.15. A beam of light from a discharge tube filled with atomic hydrogen falls normally upon a diffraction grating. The constant of the grating is 5×10^{-4} cm. What transition of the electron does the spectral line observed with the aid of this grating in the fifth-order spectrum at an angle of 41° correspond to?

20.16. Find the de Broglie wavelength for an electron traveling along the first Bohr orbit in a hydrogen atom.

20.17. Find: (1) the radius of the first Bohr electron orbit for singly ionized helium, (2) the velocity of the electron in it.

20.18. Find the first excitation potential of (1) singly ionized helium, (2) doubly ionized lithium.

20.19. Find the ionization potential of (1) singly ionized helium, and (2) doubly ionized lithium.

20.20. Find the wavelength of a photon which corresponds to the transfer of an electron from the second Bohr orbit to the first one in a singly ionized helium atom.

20.21. Solve the previous problem for a doubly ionized lithium atom.

20.22. The D-line of sodium is radiated by such a transition of an electron from one atomic orbit to another when the energy of an atom decreases by 3.37×10^{-19} J. Determine the wavelength of the sodium D-line.

20.23. Figure 66 schematically shows a device used to determine the resonance potential of sodium. The tube contains sodium vapours. Electrodes G and A have the same potential. At what minimum accelerating potential difference between the cathode C and the grid G is a spectral line with a wavelength of 5,890 Å observed?

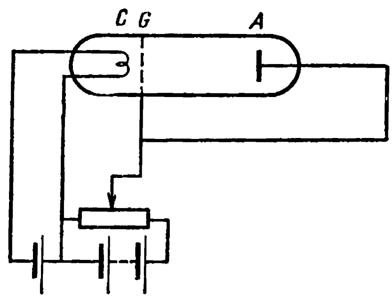


Fig. 66

20.24. An electron passing through a potential difference of 4.9 V collides with a mercury atom and transfers it to the first excited state. What is the wavelength of a photon corresponding to the transition of the mercury atom to its normal state?

20.25. Figure 67 shows an experimental arrangement for observing the diffraction of X-rays. As crystal C rotates, only that ray whose wavelength satisfies Bragg's equation will be reflected onto photographic plate P. What is the minimum angle between the crystal plane and a beam of X-rays at which X-rays with a wavelength of 0.2 Å are reflected? The constant of the crystal lattice is 3.03 Å.

20.26. Find the lattice constant of rock salt, knowing the mass of one kilomole of rock salt and its density ($\rho=2.2$ g/cm^3). Rock salt crystals have a simple cubic structure.

20.27. When Planck's constant h is determined experimentally by means of X-rays, a crystal is placed at a certain angle θ and the potential difference applied to the X-ray tube is increased until the line corresponding to this angle appears. Find Planck's constant from the following data: a rock salt crystal is placed at an angle of 14°, the potential difference at which the line corresponding to this angle appears for the first time is 9,100 V, and the crystal lattice constant is 2.81 Å.

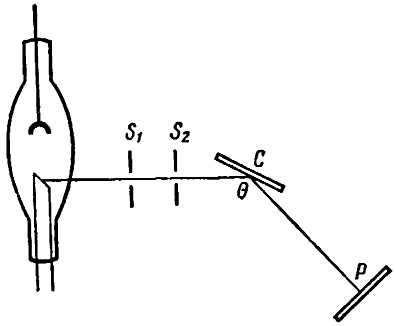


Fig. 67

20.28. A potential difference of 60 kV is applied to the electrodes of an X-ray tube. The minimum wavelength of the X-rays obtained from this tube is 0.206 A. Find Planck's constant from these data.

20.29. Find the short-wave boundary of a continuous X-ray spectrum when an X-ray tube receives a potential difference of (1) 30 kV, (2) 40 kV, and (3) 50 kV.

20.30. Find the short-wave boundary of a continuous X-ray spectrum if a reduction of the voltage applied to an X-ray tube by 23 kV doubles the wavelength.

20.31. The wavelength of gamma radiation of radium C is equal to 0.016 Å. What potential difference should be applied to an X-ray tube to obtain X-rays with this wavelength?

20.32. What minimum voltage should be applied to an X-ray tube to obtain all the lines of the K-series if the anticathode is made of (1) copper, (2) silver, (3) tungsten, and (4) platinum?

20.33. Assuming that Moseley's formula describes with sufficient accuracy the relationship between the frequency of characteristic X-rays and the atomic number of the element which the anticathode is made of, find the maximum wavelength of the K-series of X-rays produced by a tube with an anticathode made of (1) iron, (2) copper, (3) molybdenum, (4) silver, (5) tantalum, (6) tungsten, and (7) platinum. For the K-series the screening constant is unity.

20.34. Find the screening constant for the L-series of X-rays if it is known that X-rays with a wavelength of $\lambda=1.43$ Å are emitted when an electron in a tungsten atom is transferred from the M-layer to the L-layer.

20.35. When an electron is transferred in an atom from the L-layer to the K-layer, the emitted X-rays have a wavelength of 0.788 Å. What is this atom? The screening constant is equal to unity for the K-series.

20.36. Air in a certain volume V is irradiated by X-rays. The radiation dose is 4.5 r. Find the part of the atoms in the volume which will be ionized by this radiation.

20.37. An X-ray tube produces a dose of 2.58×10^{-5} A/kg at a certain distance. How many ion pairs does the tube produce per second in one gramme of air at this distance?

20.38. Air in standard conditions in an ionization chamber with a volume of 6 cm^3 is irradiated by X-rays. The dose of the X-rays is 0.48 mr/h. Find the ionizing saturation current.

20.39. Find for aluminium the thickness of the half-value layer for X-rays of a certain wavelength if the mass absorption coefficient of aluminium for this wavelength is 5.3 m^2/kg.

20.40. How many times will the intensity of X-rays with a wavelength of 0.2 Å diminish when they pass through a layer of iron 0.15 mm thick? The mass absorption coefficient of iron for this wavelength is 1.1 m^2/kg.

20.41. Find the thickness of the half-value layer for iron in the conditions of the previous problem.

20.42. The following table gives for some materials the thickness of the half-value layer for X-rays whose energy is equal to 1 MeV. (1) Find the linear and the mass absorption coefficients of these materials for the given energy of the X-rays. (2) Find the wavelength of the X-rays for which these data are obtained.

Material	Water	Aluminium	Iron	Lead
x, cm	10.2	4.5	1.56	0.87

20.43. How many half-value layers must be used to reduce the intensity of X-rays to 1/80th?

21. Radioactivity

The number of atoms of a radioactive material disintegrating during the time dt is proportional to the number of atoms present and can be determined from the expression

$$\frac{dN}{dt} = -\lambda N$$

where λ is the radioactive disintegration constant. Upon integrating, we obtain

$$N = N_1 e^{-\lambda t}$$

where N_1 = number of atoms present at the moment of time $t=0$
N = number of atoms after the time t has elapsed.

The half-life T and the disintegration constant λ are related by the expression

$$T = \frac{\log_e 2}{\lambda}$$

The reciprocal of the disintegration constant $\tau = \frac{1}{\lambda}$ is known as the mean lifetime of a radioactive atom.

If a certain amount of a radioactive preparation A is placed into a closed vessel, and its disintegration produces radioactive preparation B, the quantity of the material B in the vessel after the time t can be determined from the formula

$$N_B = N_{1A} \frac{\lambda_A}{\lambda_B - \lambda_A} (e^{-\lambda_A t} - e^{-\lambda_B t})$$

where N_{1A} = quantity of preparation A at $t=0$
λ_A and λ_B = disintegration constants of preparations A and B, respectively.

If the half-life of preparation A is much greater than that of B, the last formula will become

$$N_B = N_{1A} \frac{\lambda_A}{\lambda_B} (1 - e^{-\lambda_B t})$$

In radioactive equilibrium $\frac{N_A}{N_B} = \frac{\lambda_B}{\lambda_A}$.

The specific activity of a radioactive preparation is determined by the number of disintegration events a second per unit of mass of the disintegrating material.

21.1. Given one million polonium atoms, how many disintegrate in twenty-four hours?

21.2. Given one million radon atoms, how many disintegrate in twenty-four hours?

21.3. Find the number of disintegrations in 1 g of radium a second.

21.4. Find the mass of a radon whose activity is 1 curie.

21.5. Find the quantity of polonium $_{84}Po^{210}$ whose activity is 3.7×10^{10} d/s.

21.6. Find the disintegration constant of radon if the number of its atoms diminishes by 18.2% during twenty-four hours.

21.7. Find the specific activity of (1) uranium $_{92}U^{235}$, and (2) radon $_{86}Rn^{222}$.

21.8. Even in the absence of a radioactive preparation, Geiger-Müller ionization counters have a certain background which may be caused by cosmic radiation or radioactive contamination. What amount of radon does a background producing 1 pulse of the counter in 5 seconds correspond to?

21.9. An ionization counter is used to investigate the disintegration rate of a certain radioactive preparation. At the initial moment of time the counter gives 75 pulses in 10 seconds. What number of pulses will be given by the counter in 10 seconds after the time $\frac{t}{2}$ s elapses? Assume that $t \gg 10$ seconds.

21.10. A certain radioactive preparation has a disintegration constant of $\lambda = 1.44 \times 10^{-3}$ h^{-1}. In what time will 75 per cent of the initial number of atoms disintegrate?

21.11. Natural uranium is a mixture of three isotopes: ${}_{92}U^{234}$, ${}_{92}U^{235}$ and ${}_{92}U^{238}$. The content of uranium ${}_{92}U^{234}$ is negligible (0.006 per cent), uranium ${}_{92}U^{235}$ accounts for 0.71 per cent, and the remainder (99.28 per cent) is uranium ${}_{92}U^{238}$. The half-lives of these isotopes are 2.5×10^5 years, 7.1×10^8 years and 4.5×10^9 years, respectively. Calculate the share of radioactivity in per cent introduced by each isotope into the total radioactivity of natural uranium.

21.12. The kinetic energy of an α-particle which flies out of the nucleus of a radium atom in radioactive disintegration is 4.78 MeV. Find: (1) the velocity of the α-particle, (2) the total energy evolved during the escape of the α-particle.

21.13. What amount of heat is evolved by one curie of radon (1) in one hour, (2) during the mean lifetime? The kinetic energy of an α-particle escaping from the radon is 5.5 MeV.

21.14. One gramme of uranium ${}_{92}U^{238}$ in equilibrium with its disintegration products emits a power of 1.07×10^{-7} W. Find the total amount of heat liberated by one gramme-atom of uranium during the mean lifetime of the uranium atoms.

21.15. What is the activity of the radon formed from 1 g of radium during one hour?

21.16. The disintegration of 1 g of radium forms a certain amount of helium in a year which under standard conditions occupies a volume of 0.043 cm^3. Find Avogadro's number from these data.

21.17. A preparation containing 1.5 g of radium is put into a closed vessel (ampoule). How much radon will accumulate in the vessel after the time $t = \frac{T}{2}$ has elapsed, where T is the half-life of radon?

21.18. A certain amount of radium is placed into a closed vessel. (1) In what time will the number of the radon atoms N in this vessel differ by 10 per cent from the number of these atoms N' corresponding to radioactive equilibrium of the radium with the radon in the vessel?

(2) Plot a curve showing how $\frac{N}{N'}$ depends on time within the interval $0 \leqslant t \leqslant 6T$. Take the half-life of radon T as the unit of time on the axis of abscissas.

21.19. A certain amount of radon N' is placed in an empty vessel. (1) Plot a curve showing the change in the quantity of radon $\frac{N}{N'}$ in the vessel with time for the period $0 \leqslant t \leqslant 20$ days at intervals of 2 days. For radon $\lambda = 0.181$ days^{-1}. (2) Find the half-life from the curve $\frac{N}{N'} = f(t)$.

21.20. The following table shows the results of measuring the dependence of the activity a of a certain radioactive element on the time t.

t, h	0	3	6	9	12	15
a, mc	21.6	12.6	7.6	4.2	2.4	1.8

Find the half-life of this element.

21.21. Radon with an activity of 400 mc is placed into an ampoule. In what time after the ampoule is filled will the radon disintegrate at a rate of 2.22×10^9 d/s?

21.22. Since the lead contained in uranium ore is the final decay product of the uranium series, the age of the ore can be found from the relationship between the amount of uranium in the ore and the amount of lead in it. Determine the age of uranium ore if 320 g of lead $_{82}Pb^{206}$ are contained in this ore per kg of uranium $_{92}U^{238}$.

21.23. Knowing the half-lives of radium and uranium, find the number of uranium atoms per atom of radium in natural uranium ore.

Note. Remember that the radioactivity of natural uranium is due mainly to the isotope $_{92}U^{238}$.

21.24. From what minimum amount of ore containing 42 per cent of pure uranium can 1 g of radium be obtained?

21.25. Alpha-particles escape from a radium preparation at the rate of 1.5×10^4 km/s and collide with a fluorescent screen. Assuming that the screen consumes 0.25 W per candela find the light intensity of the screen if it receives all the α-particles emitted by 1 μg of radium.

21.26. What part of the initial quantity of a radioactive isotope disintegrates during the lifetime of this isotope?

21.27. Find the activity of 1 μg of polonium $_{84}Po^{210}$.

21.28. Find the specific activity of the artificial radioactive isotope $_{38}Sr^{90}$.

21.29. 30 mg of the non-radioactive isotope $_{20}Ca^{40}$ are mixed with 10 mg of the radioactive isotope $_{20}Ca^{45}$. How much will the specific activity of the preparation decrease?

21.30. What amount of the radioactive isotope $_{83}Bi^{210}$ should be added to 5 mg of the non-radioactive isotope $_{83}Bi^{209}$ for the ratio of the number of disintegrated atoms to that of the undisintegrated ones to be 50 per cent in 10 days? The disintegration constant of $_{83}Bi^{210}$ is $\lambda=0.14$ days^{-1}.

21.31. What isotope will be produced from $_{90}Th^{232}$ after four α-decays and two β-decays?

21.32. What isotope will be produced from $_{92}U^{238}$ after three α-decays and two β-decays?

21.33. What isotope will be produced from $_{92}U^{239}$ after two β-decays and one α-decay?

21.34. What isotope will be produced from the radioactive isotope $_3Li^8$ after one β-decay and one α-decay?

21.35. What isotope will be produced from the radioactive isotope of antimony $_{51}Sb^{133}$ after four β-decays?

21.36. The kinetic energy of an α-particle escaping from the nucleus of a polonium atom $_{84}Po^{214}$ is 7.68 MeV in radioactive decay. Find: (1) the velocity of the α-particle, (2) the total energy emitted during the escape of the α-particle, (3) the number of ion pairs formed by the α-particle assuming that the energy $E_0=34$ eV is required to produce one pair of ions in air, (4) the saturation current in an ionization chamber produced by all the α-particles emitted by 1 microcurie of polonium.

22. Nuclear Reactions

The nuclear binding energy of any isotope can be determined from the ratio

$$\Delta E=c^2\,\Delta M$$

where ΔM is the difference between the mass of the particles forming the nucleus and the mass of the nucleus itself. Obviously

$$\Delta M=ZM_p+(M-Z)\,M_n-M_{nu} \qquad (1)$$

where Z = atomic number of the isotope
M = mass number
M_p = mass of a proton
M_n = mass of a neutron
M_{nu} = mass of the nucleus of the isotope.

Since $M_{nu}=M_A-Zm$, where M_A is the mass of the isotope and m is the mass of an electron, the previous equation may be replaced by

$$\Delta M=ZM_{_1H^1}+(M-Z)\,M_H-M_A \qquad (2)$$

where $M_{_1H^1}$ is the mass of the hydrogen isotope $_1H^1$ and M_A is the mass of the given isotope.

The change of energy in a nuclear reaction is determined from the expression

$$\Delta E = c^2 (\sum M_1 - \sum M_2) \tag{3}$$

where $\sum M_1$ and $\sum M_2$ are the sums of the masses of the particles before and after the reaction, respectively.

If $\sum M_1 > \sum M_2$, the reaction takes place with the evolution of energy, and if $\sum M_1 < \sum M_2$, with the absorption of energy. Let us note that we may introduce into the latter formula the mass of the isotopes instead of that of the nuclei, as in calculating the nuclear binding energy, since the corrections for the mass of the shell electrons have opposite signs and may therefore be cancelled.

22.1. Find the number of protons and neutrons in the nuclei of three magnesium isotopes: (1) $_{12}Mg^{24}$, (2) $_{12}Mg^{25}$, and (3) $_{12}Mg^{26}$.

22.2. Find the nuclear binding energy of the lithium isotope $_3Li^7$.

22.3. Find the nuclear binding energy of a helium atom $_2He^4$.

22.4. Find the nuclear binding energy of an aluminium atom $_{13}Al^{27}$.

22.5. Find the nuclear binding energies of (1) $_1H^3$ and (2) $_2He^3$. Which of these nuclei is more stable?

22.6. Find the binding energy per nucleon in the nucleus of an oxygen atom $_8O^{16}$.

22.7. Find the binding energy of a deuterium nucleus $_1H^2$.

22.8. Find the binding energy E_0 per nucleon in the nuclei (1) $_3Li^7$, (2) $_7N^{14}$, (3) $_{13}Al^{27}$, (4) $_{20}Ca^{40}$, (5) $_{29}Cu^{63}$, (6) $_{48}Cd^{113}$, (7) $_{80}Hg^{200}$, and (8) $_{92}U^{238}$. Plot the relation $E_0 = f(M)$, where M is the mass number.

22.9. Find the energy released during the nuclear reaction

$$_3Li^7 + {_1H^1} \rightarrow {_2He^4} + {_2He^4}$$

22.10. Find the energy absorbed during the reaction

$$_7N^{14} + {_2He^4} \rightarrow {_1H^1} + {_8O^{17}}$$

22.11. Find the energy evolved during the nuclear reactions:

$$(1)\ _1H^2 + {_1H^2} \rightarrow {_1H^1} + {_1H^3}$$
$$(2)\ _1H^2 + {_1H^2} \rightarrow {_2He^3} + {_0n^1}$$

22.12. Find the energy evolved during the following thermonuclear reactions:

$$(1)\ _1H^2 + {_2He^3} \rightarrow {_1H^1} + {_2He^4}$$
$$(2)\ _3Li^6 + {_1H^2} \rightarrow {_2He^4} + {_2He^4}$$
$$(3)\ _3Li^6 + {_1H^1} \rightarrow {_2He^3} + {_2He^4}$$

22.13. What amount of water can be heated from 0° C to its boiling point if we use all the heat released during the reaction $_3Li^7$ (p, α) with full disintegration of one gramme of lithium?

22.14. Insert the missing symbols in the following nuclear reactions:

(1) $_{13}Al^{27}(n, \alpha)x$
(2) $_9F^{19}(p, x)_8O^{16}$
(3) $_{25}Mn^{55}(x, n)_{26}Fe^{55}$
(4) $_{13}Al^{27}(\alpha, p)x$
(5) $_7N^{14}(n, x)_6C^{14}$
(6) $x(p, \alpha)_{11}Na^{23}$

22.15. Find the energy released during the reaction

$$_3Li^7 + {}_1H^2 \rightarrow {}_4Be^8 + {}_0n^1$$

22.16. Find the energy released during the reaction

$$_4Be^9 + {}_1H^2 \rightarrow {}_5B^{10} + {}_0n^1$$

22.17. Bombardment of the nitrogen isotope $_7N^{14}$ by neutrons produces the carbon isotope $_6C^{14}$, which is β-radioactive. Write the equations of both reactions.

22.18. Bombardment of the aluminium isotope $_{13}Al^{27}$ by α-particles produces the radioactive phosphorus isotope $_{15}P^{30}$ which then disintegrates releasing a positron. Write the equations of both reactions. Find the specific activity of the isotope obtained if its half-life is 130 seconds.

22.19. Bombardment of the isotope $_{11}Na^{23}$ by deuterons produces the β-radioactive isotope $_{11}Na^{24}$. A β-particle counter is installed near the preparation containing the radioactive $_{11}Na^{24}$. During the first measurement the counter showed 170 pulses a minute, and on the following day 56 a minute. Write the equations of both reactions. Find the half-life of the isotope $_{11}Na^{24}$.

22.20. (1) What energy will be released if during the reaction

$$_{13}Al^{27} + {}_2He^4 \rightarrow {}_{14}Si^{30} + {}_1H^1$$

all the nuclei in one gramme of aluminium are transformed? (2) What energy must be spent for this transformation if when a nucleus of aluminium is bombarded by α-particles with an energy of 8 MeV only one of 2×10^6 particles causes the transformation?

22.21. Bombardment of the lithium isotope $_3Li^6$ by deuterons produces two α-particles with the release of energy equal to 22.3 MeV. Knowing the masses of a deuteron and an α-particle, find the mass of the lithium isotope $_3Li^6$.

22.22. Assuming that the source of the energy of solar radiation is the energy of the formation of helium from hydrogen according to the following cyclic reaction:

$$
\begin{aligned}
&{}_6C^{12} + {}_1H^1 \rightarrow {}_7N^{13} \rightarrow {}_6C^{13} + {}_{+1}e^0\\
&{}_6C^{13} + {}_1H^1 \rightarrow {}_7N^{14}\\
&{}_7N^{14} + {}_1H^1 \rightarrow {}_8O^{15} \rightarrow {}_7N^{15} + {}_{+1}e^0\\
&{}_7N^{15} + {}_1H^1 \rightarrow {}_6C^{12} + {}_2He^4
\end{aligned}
$$

find how many tons of hydrogen must be converted every second into helium. The solar constant is 1.96 cal/cm^2·min. Assuming that hydrogen forms 35 per cent of the Sun's mass, calculate in how many years this hydrogen will be used up if the radiation of the Sun is constant.

22.23. The reaction of decomposing a deuteron by gamma-rays is as follows:

$$ {}_1H^2 + h\nu \rightarrow {}_1H^1 + {}_0n^1 $$

Find the mass of the neutron from the following data: the energy of γ-quanta is 2.66 MeV and the energy of the escaping protons as measured by the ionization they produce is 0.22 MeV. Consider the energy of the neutron to be equal to that of the proton. The masses of the deuteron and the proton are known.

22.24. Insert the missing symbols in the following nuclear reactions caused by photons:

(1) ${}_{13}Al^{27}\ (\gamma,\ x)_{12}Mg^{26}$
(2) ${}_{13}Al^{27}\ (\gamma,\ n)\ x$
(3) ${}_{29}Cu^{63}\ (\gamma,\ x)_{29}Cu^{62}$
(4) $x\ (\gamma,\ n)_{74}W^{181}$

22.25. The yield of a reaction of formation of radioactive isotopes may be characterized either by k_1—the ratio between the number of events of nuclear transformation and the number of bombarding particles, or by k_2—the ratio between the activity of the product obtained to the number of particles bombarding the target. Find the relationship between the quantities k_1 and k_2.

22.26. Bombardment of ${}_3Li^7$ by protons produces the radioactive beryllium isotope ${}_4Be^7$ with a half-life of 4.67×10^6 s. Find the value of k_1 (see the previous problem) for this reaction if the bombarding protons with a total charge of 1 μA·h cause an activity of the obtained preparation equal to 176 μc.

22.27. The nuclear reaction ${}_{26}Fe^{56}$ $(p,\ n)$ produces a radioactive isotope of cobalt with a half-life of 80 days. Find the yield of this reaction k_1 (see Problem 22.25) if the activity of the isotope ${}_{27}Co^{56}$ is $5.2\times$ $\times10^7$ d/s after a ${}_{26}Fe^{56}$ target is irradiated for two hours with a proton current of 10 μA.

22.28. A tube containing a powder of beryllium $_4Be^9$ and gaseous radon is used as a source of neutrons. Neutrons are produced when α-particles of the radon react with the beryllium. (1) Write the reaction of neutron formation. (2) Find the amount of radon originally introduced into the source if it produces 1.2×10^6 neutrons per second after five days. The yield of this reaction is $\frac{1}{4{,}000}$, i. e., only one α-particle out of 4,000 induces the reaction.

22.29. The tube described in the previous problem is a source of neutrons. Find the number of neutrons produced in one second by α-particles from 1 curie of radon as they impinge upon the beryllium powder. Assume that only one α-particle of 4,000 induces the reaction.

22.30. The reaction of formation of the radioactive carbon isotope $_6C^{11}$ can be written as $_5B^{10}$ (d, n), where d is the symbol of a deuteron—the nucleus of deuterium $_1H^2$. The half-life of the isotope $_6C^{11}$ is 20 min. (1) What amount of energy is evolved during this reaction? (2) Find the yield of the reaction k_2 if $k_1=10^{-8}$ (see the initial condition of Problem 22.25).

22.31. In the reaction $_7N^{14}$ (α, p) the kinetic energy of an α-particle $E_\alpha=7.7$ MeV. Find the angle to the direction of motion of the α-particle at which a proton escapes if its kinetic energy $E_{pr}=8.5$ MeV.

22.32. Bombardment of the lithium isotope $_3Li^6$ by deuterons forms two α-particles which fly away symmetrically at an angle of φ to the direction of the velocity of the bombarding deuterons. (1) Find the kinetic energy of the α-particles formed if the energy of the bombarding deuterons is 0.2 MeV. (2) Find the angle φ.

22.33. The helium isotope $_2He^3$ is produced by bombarding tritium nuclei $_1H^3$ with protons. (1) Write the equation of the nuclear reaction. (2) Find the energy evolved in this reaction. (3) Find the "threshold" of the nuclear reaction, i. e., the minimum value of the kinetic energy of a bombarding particle at which this reaction occurs.

Note. Remember that the relative velocity of the particles produced by the reaction is zero at the threshold value of the kinetic energy of the bombarding particle.

22.34. Find the threshold of the nuclear reaction $_7N^{14}$ (α, p).

22.35. Find the threshold of the nuclear reaction $_3Li^7$ (p, n).

22.36. An artificial nitrogen isotope $_7N^{13}$ is produced by bombarding the nuclei of carbon $_6C^{12}$ by deuterons. (1) Write the equation of the nuclear reaction. (2) Find the quantity of heat absorbed during this reaction. (3) Find the threshold of this reaction. (4) Find the total kinetic energy of the reaction products at the threshold value of the kinetic energy of the deuterons. Assume the carbon nuclei to be immobile.

22.37. The reaction $_5B^{10}$ (n, α) takes place when boron is bombarded by neutrons whose velocity is very small ("thermal" neutrons).

(1) Find the energy evolved in this reaction. (2) Find the velocity and kinetic energy of an α-particle, assuming the boron nucleus to be immobile and neglecting the velocities of the neutrons.

22.38. Bombardment of the lithium isotope $_3Li^7$ by protons produces two α-particles. The energy of each α-particle when formed is 9.15 MeV. What is the energy of the bombarding protons?

22.39. Find the minimum energy of a γ-quantum sufficient for the reaction of decomposition of a deuteron by γ-rays

$$_1H^2 + h\nu \rightarrow {_1H^1} + {_0n^1}$$

22.40. Find the minimum energy of a γ-quantum sufficient for the reaction $_{12}Mg^{24}$ (γ, n).

22.41. What amount of energy in kilowatt-hours can be obtained from the fission of one gramme of uranium $_{92}U^{235}$ if each fission produces an energy of about 200 MeV?

22.42. What amount of uranium $_{92}U^{235}$ is consumed every day at an atomic power plant rated at 5,000 kW? The efficiency is 17 per cent. Assume that an energy of 200 MeV is liberated in each event of decay.

22.43. When a hydrogen bomb explodes, a thermonuclear reaction of helium formation from deuterium and tritium occurs. (1) Write the equation of the nuclear reaction. (2) Find the energy liberated in this reaction. (3) What quantity of energy can be produced in the formation of one gramme of helium?

23. Elementary Particles. Particle Accelerators

The problems in this section are solved on the basis of the laws considered in the previous sections, i. e., the collision of particles, the motion of particles in electric and magnetic fields, etc. Some problems must be solved with the aid of the formulas of the theory of relativity.

23.1. In nuclear physics, the number of charged particles bombarding a target is characterized by the total charge in microampere-hours (μA$\cdot$h). Find the number of charged particles which 1 μA$\cdot$h corresponds to. Solve the problem for (1) electrons, and (2) α-particles.

23.2. The kinetic energy of a neutron diminishes 1.4 times when it collides elastically and centrally with the stationary nucleus of a moderating material. Find the mass of the nuclei of the moderating material.

23.3. What part of the initial velocity of a neutron is formed by its velocity after an elastic central impact with an immobile nucleus of the isotope $_{11}Na^{23}$?

23.4. To obtain slow neutrons, they are passed through a material containing hydrogen (paraffin, for example). Find the maximum part of its kinetic energy which a neutron with a mass of m_0 can transmit to (1) a proton (mass m_0), and (2) a lead atom nucleus (mass $m=207\ m_0$). The major part of the transferred energy corresponds to an elastic central collision.

23.5. Find in the previous problem the distribution of energy between a neutron and a proton if the impact is not central and upon each collision the neutron is deflected, on an average, by 45°.

23.6. A neutron with an energy of 4.6 MeV collides with protons and is retarded. Assuming that upon each collision the neutron is deflected, on an average, by 45°, find the number of collisions which will reduce its energy to 0.23 eV.

23.7. A flux of charged particles flies into a homogeneous magnetic field having an induction of 3 Wb/m^2. The velocity of the particles is 1.52×10^7 m/s and is perpendicular to the direction of the force lines of the field. Find the charge of each particle if the force acting on it is 1.46×10^{-11} N.

23.8. A charged particle flies into a homogeneous magnetic field having an induction of 0.5 T and travels along a circle with a radius of 10 cm. The velocity of the particle is 2.4×10^6 m/s. Find the ratio between the charge and the mass of this particle.

23.9. An electron is accelerated by a potential difference of 180 kV. (1) Find the mass, velocity, kinetic energy and the e/m ratio for this electron, taking into account the corrections of the theory of relativity. (2) Find the velocity of this electron, disregarding the relativistic correction.

23.10. The energy of fast mesons in cosmic rays is approximately equal to 3,000 MeV, and the rest energy of such a meson is 100 MeV. What distance in the atmosphere can be traveled by a meson during its lifetime as registered by a laboratory clock? The intrinsic life of the meson $\tau_0=2\times10^{-6}$ s.

23.11. A meson of cosmic rays has a kinetic energy of $E=7\ M_0c^2$, where M_0 is the rest mass of the meson. How many times is the intrinsic life of this meson smaller than its life counted in a coordinate system in a laboratory?

23.12. A positron and an electron combine to form two photons. (1) Find the energy of each photon if the kinetic energy of the electron and the positron was negligibly small before their collision. (2) Find the wavelength of these photons.

23.13. When an electron and a positron are formed from a photon, the energy of the photon was 2.62 MeV. What is the total kinetic energy of the positron and the electron at the moment of formation?

23.14. An electron and a positron formed by a quantum with an energy of 5.7 MeV form paths with a radius of curvature of 3 cm in a cloud chamber placed in a magnetic field. Find the induction of the magnetic field.

23.15. When an immobile neutral π-meson disintegrates it forms two identical photons. Find the energy of each photon. The rest mass of the π-meson $M=264.2\ m_0$, where m_0 is the rest mass of an electron.

23.16. A neutron and an antineutron combine forming two photons. Find the energy of each photon assuming that the initial energy of the particles is negligibly small.

23.17. A K^0-meson disintegrates into two charged π-mesons. The mass of each meson is 1.77 times greater than its rest mass. Assuming that the K^0-meson was initially at rest and its rest mass was 965 m_0, where m_0 is the rest mass of an electron, find: (1) the rest mass of the π-mesons formed, (2) the velocity of the π-mesons at the moment of their formation.

23.18. (1) Deduce a formula relating the induction of the magnetic field of a cyclotron and the frequency of the potential difference applied to dees. (2) Find the frequency of the potential difference applied to the dees for (a) deuterons, (b) protons, and (c) α-particles. The induction of the magnetic field is 12.6 kGs.

23.19. (1) Deduce a formula relating the energy of particles flying out from a cyclotron and the maximum radius of curvature of the path of these particles. (2) Find the energy of (a) deuterons, (b) protons, and (c) α-particles flying out from a cyclotron if the maximum radius of curvature $R=48.3$ cm. The frequency of the potential difference applied to the dees is 12 MHz.

23.20. In a cyclotron with a maximum radius of curvature of the trajectory of particles $R=0.35$ m, the frequency of the potential difference applied to the dees is $\nu=1.38\times10^7$ Hz. Find for operation with protons: (1) the induction of the magnetic field necessary for synchronous functioning of the cyclotron, (2) the maximum energy of the escaping protons.

23.21. Solve the previous problem for work with (1) deuterons, and (2) α-particles.

23.22. The magnitude of the ion current generated in a cyclotron working with α-particles is 15 μA. How many times is this cyclotron more productive than one gramme of radium?

23.23. The maximum radius of curvature of the trajectory of particles in a cyclotron is $R=0.5$ m and the induction of the magnetic field $B=10^4$ Gs. What constant potential difference would protons have to pass through to obtain the same acceleration as in the given cyclotron?

23.24. A cyclotron produces deuterons with an energy equal to 7 MeV. The induction of the applied magnetic field is 15,000 Gs. Find the maximum radius of curvature of the trajectory of the deuteron.

23.25. A variable potential difference of $U=75$ kV with a frequency of $\nu=10$ MHz is applied between the dees of a cyclotron with a radius of 50 cm. Find: (1) the induction of the magnetic field of the cyclotron, (2) the velocity and energy of the particles escaping from the cyclotron, (3) the number of revolutions completed by a charged particle before it leaves the cyclotron. Solve the problem for deuterons, protons and α-particles.

23.26. What energy can α-particles be accelerated to in a cyclotron if the relative increase in the mass of a particle $k=\frac{m-m_0}{m_0}$ should not exceed 5 per cent?

23.27. The energy of deuterons accelerated by a synchrotron is 200 MeV. Find for these deuterons: (1) the ratio $\frac{M}{M_0}$, where M is the mass of a moving deuteron and M_0 is its rest mass, (2) their velocity.

23.28. In a synchrocyclotron, an increase in the mass of a particle as its velocity grows is compensated for by an increase in the period of the accelerating field. In a synchrocyclotron which accelerates protons, the frequency of the voltage applied to the dees changes from 25 MHz to 18.9 MHz for each accelerating cycle. Find for this synchrocyclotron: (1) the induction of the magnetic field, and (2) the kinetic energy of escaping protons.

23.29. A synchrocyclotron was used to investigate protons accelerated to an energy of 660 MeV, and α-particles accelerated to 840 MeV. To compensate for an increase in the mass, the period of the accelerating field in the synchrocyclotron was changed. How many times was this period changed (for each accelerating cycle) in operation with (1) protons, (2) α-particles?

ANSWERS AND SOLUTIONS

Chapter 1

PHYSICAL FUNDAMENTALS OF MECHANICS

1. Kinematics

1.1. The average velocity of the car is determined from the formula $\bar{v}=\frac{l}{t}$, where $l=l_1+l_2=v_1t_1+v_2t_2$. According to the conditions of the problem, $t_1=t_2=\frac{t}{2}$. Thus, $\bar{v}=\frac{v_1\frac{t}{2}+v_2\frac{t}{2}}{t}=\frac{v_1+v_2}{2}=60$ km/h.

1.2. $\bar{v}=\frac{l}{t}$, where $t=t_1+t_2=\frac{l_1}{v_1}+\frac{l_2}{v_2}$. According to the given conditions, $l_1=l_2=\frac{l}{2}$. Thus, $\bar{v}=\frac{l}{\frac{l}{2v_1}+\frac{l}{2v_2}}=\frac{2v_1v_2}{v_1+v_2}=53.3$ km/h.

1.3. (1) 12.3 km/h, (2) 0.83 m/s.

1.4. (1) 3 m/s, (2) 1 m/s, (3) 2.24 m/s.

1.5. (1) The airplane must fly southwest at an angle of $\varphi=3°52'$ to the meridian with the velocity $v=798$ km/h, (2) northwest, $\varphi=3°52'$, $v=798$ km/h, (3) westward, $v=746$ km/h, (4) eastward, $v=854$ km/h.

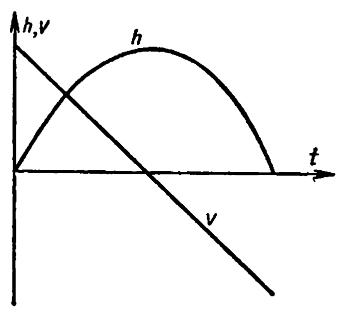


Fig. 68

1.6. (1) 30 min, (2) 30.2 min, (3) 26.8 min.

1.7. (1) $v=0.60$ m/s, (2) $t=250$ seconds.

1.8. (1) $v_0=14.7$ m/s, (2) $h=11$ m.

1.9. (1) $t=2.9$ seconds, (2) $h_1=4h=40$ m.

1.10. (1) 8.4 seconds, (2) 7.3 seconds, (3) 7.8 seconds.

1.11. The relationship between the height h, velocity v and time t for a body thrown vertically upward is depicted in Fig. 68.

1.12. (1) The distance traveled by the body during the first 0.1 second of motion is $h_1=\frac{gt_1^2}{2}=0.049$ m. (2) The entire distance will be traveled by the body during $t=\sqrt{\frac{2h}{g}}=2$ seconds. During the last 0.1 second of motion the body will cover the distance $h_3=h-h_2$, where h_2 is the distance covered by the body during $t_2=(2-0.1)=1.9$ seconds. Since $h_2=\frac{gt_2^2}{2}=17.7$ m, the distance sought $h_3=19.6-17.7=1.9$ m.

1.13. (1) The first metre of the distance will be covered by the body during the time $t_1=\sqrt{\frac{2h_1}{g}}=0.45$ second, (2) the total duration of falling $t=\sqrt{\frac{2h}{g}}=2$ seconds. The last metre of the distance will be traveled by the body during the time $t_3=t-t_2$, where t_2 is the time required to cover the distance $h_2=(19.6-1)\text{ m}=18.6$ m. Since $t_2=\sqrt{\frac{2h_2}{g}}=1.95$ seconds, the time sought $t_3=(2-1.95)=0.05$ second.

1.14. (1) $h=57$ m, (2) $t=3.4$ seconds.

1.15. The distance traveled by the body A is $h_1=v_1t-\frac{gt^2}{2}$ and that traveled by the body B is $h_2=\frac{gt^2}{2}$. The distance between the bodies $x=h-(h_1+h_2)$. Since $h_1+h_2=v_1t$, the relation sought is $x=h-v_1t$. The bodies will meet at $x=0$, i. e., at the moment of time $t=\frac{h}{v_1}$

1.16. (1) $a=0.13\text{ m/s}^2$, (2) $t=3.6$ min.

1.17. The following two equations of motion are true in uniformly variable motion

$$s=v_0t+\frac{at^2}{2} \qquad (1)$$

and

$$v=v_0+at \qquad (2)$$

From the conditions of the problem $v=0$. Hence from equation (2)

$$a=-\frac{v_0}{t} \qquad (3)$$

By inserting Eq. (3) into (1), we find that

$$s=\frac{v_0t}{2} \qquad (4)$$

Substitution of the numerical data in (3) and (4) gives

$$a=-0.5\text{ m/s}^2 \text{ and } s=100\text{ m}$$

1.18. (1) $a=-0.055\text{ m/s}^2$, (2) $s=566$ m.

1.19. $t=30$ seconds, $s=225$ m.

1.20. $t=\frac{v_0''-v_0'}{a_1+a_2}$. Since the time t must always be greater than 0, the problem has a solution if $v_0''>v_0'$.

1.21. $a=\frac{v_0''-v_0'}{\Delta t}$. In our case $a=1\text{ m/s}^2$.

1.22. (1) $v=(2-6t+12t^2)$ m/s, $a=(-6+24t)$ m/s^2, (2) $s=24$ m, $v=38$ m/s and $a=42$ m/s^2.

The distance s, velocity v, and acceleration a of the body versus the time t are shown in Fig. 69.

1.23. $\bar{v}=7$ m/s, $\bar{a}=4$ m/s^2. The distance, velocity and acceleration of the body versus the time are depicted in Fig. 70.

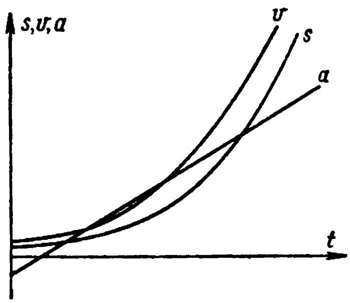


Fig. 69

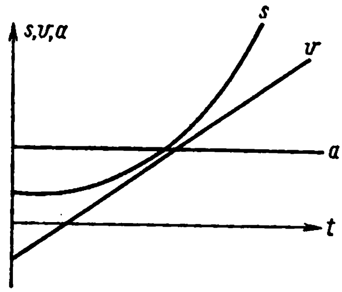


Fig. 70

1.24. $\bar{v}_1=3$ m/s, $\bar{v}_2=5$ m/s, $\bar{v}_3=7$ m/s, $\bar{a}_1=\bar{a}_2=\bar{a}_3=2$ m/s^2.

1.25. (1) In 12 seconds, (2) $\bar{a}=0.64$ m/s^2.

1.26. The motion of the stone thrown horizontally can be resolved into two components: a horizontal one s_x and a vertical one s_y (see Fig. 71). Using the law of the independency of motion, we have $s_y=H=\frac{gt^2}{2}$, $s_x=v_0t$, where t is the duration of motion. Hence: (1) $t=\sqrt{\frac{2s_y}{g}}=2.26$ seconds, (2) $s_x=v_0t=33.9$ m, (3) $v_y=gt=22.1$ m/s and $v=\sqrt{v_x^2+v_y^2}=26.7$ m/s, (4) $\sin\varphi=\frac{v_y}{v}=$ $=0.827$, $\varphi=55°48'$.

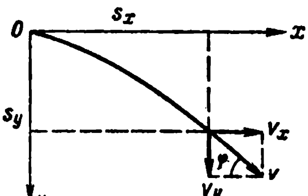


Fig. 71

1.27. (1) $h=1.22$ m, (2) $v_0=10$ m/s, (3) $v=$ $=11.1$ m/s, (4) $\varphi=26°12'$.

1.28. (1) $v_0=11.1$ m/s, (2) $\varphi=68°12'$.

1.29. $v_0=4.4$ m/s.

1.30. Since the horizontal component of the velocity of the stone is constant, the horizontal component of the acceleration is zero. For this reason the full acceleration of the stone is constantly directed vertically downward and is equal to the gravity acceleration. Thus, $a=g=\sqrt{a_t^2+a_n^2}$. Figure 72 shows that $\cos\varphi=\frac{v_x}{v}=\frac{a_n}{a}=\frac{a_n}{g}$, and $\sin\varphi=\frac{v_y}{v}=\frac{a_t}{a}=\frac{a_t}{g}$ Hence $a_t=g\frac{v_y}{v}=\frac{g^2t}{\sqrt{v_x^2+g^2t^2}}$ and $a_n=g\frac{v_x}{v}=\frac{gv_x}{\sqrt{v_x^2+g^2t^2}}$. Substitution of numerical values in these formulas gives $a_t=5.4$ m/s^2 and $a_n=8.2$ m/s^2.

1.31. $R = 305$ m.

1.32. (1) Let us find the maximum height $s_{y \cdot max}$ which the body will rise to if thrown with the velocity v_0 at an angle of α to the horizon. We have (see Fig. 73):

$$v_y = v_0 \sin\alpha - gt \qquad (1)$$

and

$$s_y = v_0 t \sin\alpha - \frac{gt^2}{2} \qquad (2)$$

At the top point $v_y = 0$ and from Eq. (1) we obtain $v_0 \sin\alpha = gt_1$. Hence, when the ball rises, $t_1 = \frac{v_0 \sin\alpha}{g}$. By substituting this value for t_1 in Eq. (2), we get $s_{y \cdot max} = \frac{v_0^2 \sin^2\alpha}{2g} = 2.1$ m.

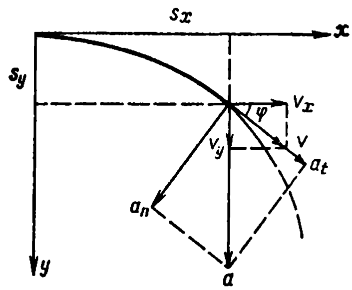


Fig. 72

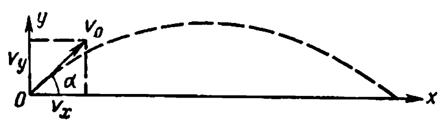


Fig. 73

(2) Let us find the distance $s_{x \cdot max}$ covered by the body projected at an angle of α to the horizon. We have (see Fig. 73):

$$v_x = v_0 \cos\alpha \qquad (3)$$

$$s_x = v_x t = v_0 t \cos\alpha \qquad (4)$$

The body will drop onto the ground after the time $t_2 = 2t_1 = \frac{2v_0 \sin\alpha}{g}$. By substituting this value for t_2 in Eq. (4), we get

$$s_{x \cdot max} = \frac{v_0^2}{g} \sin 2\alpha = 10.0 \text{ m}$$

(3) $t_2 = 2t_1 = \frac{2v_0 \sin\alpha}{g} = 1.3$ seconds.

1.33. Since $s_x = \frac{v_0^2 \sin 2\alpha}{g}$, then $\frac{s_1}{s_2} = \frac{g_2}{g_1}$, whence $s_2 = s_1 \frac{g_1}{g_2}$, where g_1 and g_2 are the gravity accelerations in Leningrad and Tashkent, respectively. The insertion of numerical data gives $s_2 = 16.23$ m.

1.34. 5.9 m. **1.35.** $h = 7.4$ m.

1.36. Let us find the time during which the body will rise to the top point of its trajectory $t_1 = \frac{v_0 \sin\alpha}{g} = 0.75$ second. Hence it can be seen that in $t = 1.25$ seconds the body will be descending. Now, the problem can be formulated as follows—the body is thrown horizontally with the velocity $v_0' = v_0 \cos\alpha = 12.7$ m/s. Find the tangential and normal accelerations in $t' = (1.25 - 0.75) = 0.5$ second after motion begins. Thus we obtain a problem similar to Problem 1.30.

Upon solving it in the same way, we get

$$a_t = g\frac{v_y}{v} = \frac{g^2 t'}{\sqrt{(v_0')^2 + g^2 (t')^2}} = 3.52 \text{ m/s}^2; \quad a_n = \frac{g v_0'}{v} = 9.15 \text{ m/s}^2$$

Check whether the total acceleration of the body directed downwards is equal to the gravity acceleration g (see the solution of Problem 1.30).

1.37. $R = 6.3$ m. **1.38.** $v_0 = 9.4$ m/s, $\alpha = 54°44'$.

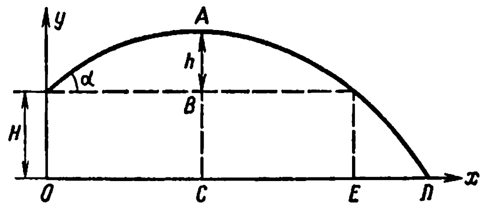


Fig. 74

1.39. The motion of the body thrown from the height H at an angle of α to the horizon can be divided into two stages: the motion of the body to the highest point A (see Fig. 74) and the motion of the body thrown from the point A horizontally with the velocity $v_x = v_0 \cos\alpha$. The height AC which the body rises to is $AC = s_y = H + h = H + \dfrac{v_0^2 \sin^2\alpha}{2g}$.

(1) The time during which the stone will be in motion $t = t_1 + t_2$, where $t_1 = \dfrac{v_0 \sin\alpha}{g}$ is the time required for the stone to reach the height h, and $t_2 = \sqrt{\dfrac{2s_y}{g}}$ is the time during which the stone falls. Upon inserting the numerical data, we obtain $s_y = 27.9$ m, $t_1 = 0.77$ s, $t_2 = 2.39$ s. Hence $t = 3.16$ s.

(2) The distance from the tower base to where the stone will drop to the ground $OD = OC + CD$; here $OC = \dfrac{OE}{2} = \dfrac{v_0^2 \sin 2\alpha}{2g} = 9.96 \text{ m} \cong 10$ m, $CD = v_x t_2 = v_0 t_2 \cos\alpha = 31.1$ m, whence $OD = 41.1$ m.

(3) $v = \sqrt{v_x^2 + v_y^2}$, where $v_x = v_0 \cos\alpha = 13.0$ m/s, $v_y = g t_2 = 23.4$ m/s, whence $v = 26.7$ m/s.

(4) The angle formed by the trajectory of the stone with the horizon at the point of fall can be found from the formula $v_y = v_x \tan\varphi$, whence $\tan\varphi = \dfrac{v_y}{v_x} = 1.8$ and $\varphi = 61°$.

1.40. (1) The ball will strike the wall when it ascends, (2) $y = 2.1$ m, (3) the ball will reach the wall with a velocity whose components are equal to $v_x = v_0 \cos\alpha = 7.07$ m/s and $v_y = v_0 \sin\alpha - gt = 2.91$ m/s, respectively. Hence $v = \sqrt{v_x^2 + v_y^2} = 7.6$ m/s.

1.41. (1) 7.26×10^{-5} rad/s, (2) 14.5×10^{-5} rad/s, (3) 1.74×10^{-3} rad/s, (4) 1.19×10^{-3} rad/s, (5) 7.8 km/s.

1.42. $v = 231$ m/s. **1.43.** $v = 1{,}660$ km/h. **1.44.** $v \geqslant 400$ m/s.

1.45. $r = 8.33$ cm.

1.46. The following two equations of motion apply to uniform variable rotation

$$\varphi=\omega_0 t+\frac{\alpha t^2}{2} \qquad (1)$$

and

$$\omega=\omega_0+\alpha t \qquad (2)$$

According to the given condition, $\omega_0=0$, and equations (1) and (2) will take the form

$$\varphi=\frac{\alpha t^2}{2} \qquad (3)$$

and

$$\omega=\alpha t \qquad (4)$$

Upon solving equations (3) and (4) simultaneously and bearing in mind that $\varphi=2\pi N$, we finally obtain $\alpha=\frac{\omega^2}{4\pi N}=3.2$ rad/s^2.

1.47. $\alpha=1.26$ rad/s^2, $N=360$ rev.

1.48. $\alpha=-0.21$ rad/s^2, $N=240$ rev.

1.49. 10 seconds. **1.50.** (1) In 6.3 seconds, (2) 9.4 rev.

1.51. According to the given condition, $a_t=$const. If t is calculated from the beginning of motion, then

$$a_t=\frac{v}{t} \qquad (1)$$

Further

$$a_n=\frac{v^2}{r} \qquad (2)$$

From Eqs. (1) and (2) we have

$$t=\frac{1}{a_t}\sqrt{a_n r} \qquad (3)$$

(1) If $a_n=a_t$, we have from Eq. (3) $t=\sqrt{\frac{r}{a_t}}=2$ s.

(2) If $a_n=2a_t$, then $t=\sqrt{\frac{2r}{a_t}}=2.8$ s.

1.52. $a_t=\frac{v^2}{4\pi Nr}=0.1$ m/s^2. **1.53.** $a_n=\frac{v^4t^2}{16\pi^2N^2r^3}=0.01$ m/s^2.

1.54. $\omega=4.4\times10^{16}$ rad/s, $a_n=9.7\times10^{22}$ m/s^2.

1.55. (1) In uniformly variable rotational motion the angular velocity ω is related to the time t by the equation $\omega=\omega_0+\alpha t$. According to the initial condition, $\omega_0=0$ and hence $\omega=\alpha t$, i. e., ω grows in proportion to time. At the end of the first second $\omega=3.14$ rad/s.

(2) Since $v=\omega r$, the linear velocity is also proportional to the time. At the end of the first second $v=0.314$ m/s.

(3) The tangential acceleration $a_t=\alpha r$ does not depend on t, i. e., it is constant during motion. In our case $a_t=0.314$ m/s^2.

(4) The normal acceleration $a_n=\omega^2r=\alpha^2t^2r$, i. e., it grows in proportion to the square of the time: when $t=1$ s we have $a_n=0.986$ m/s^2.

(5) The total acceleration increases with time according to the law $a=\sqrt{a_t^2+a_n^2}=a_t\sqrt{1+\alpha^2t^4}$. When $t=1$ s, we have $a=1.03$ m/s^2

(6) We have $\sin\gamma=\frac{a_t}{a}=\frac{1}{\sqrt{1+\alpha^2t^4}}$, where γ is the angle between the direction of the total acceleration and the radius of the wheel. At the initial moment of time, i.e., when $t=0$, we have $a=a_t$ and the total acceleration is directed tangentially. When $t=\infty$, we have $a=a_n$ (since $a_t=\text{const}$ a_n is proportional to the square of the time), i.e., the total acceleration is directed normally. At the end of the first second $\sin\gamma=\frac{a_t}{a_n}=\frac{0.314}{1.03}=0.305$, i.e., $\gamma=17°46'$.

1.56. $a_n=4.50$ m/s^2, $a_t=0.06$ m/s^2.

1.57. $v=4$ m/s, $a_t=2$ m/s^2, $a_n=2$ m/s^2, $a=2.83$ m/s^2.

1.58. $\alpha=0.43$ rad/s^2.

1.59. $r=\frac{a}{\alpha\sqrt{1+\alpha^2t^4}}=6.1$ m.

1.60. (1) $\omega=14$ rad/s, (2) $v=1.4$ m/s, (3) $\alpha=12$ rad/s^2, (4) $a_t=1.2$ m/s^2, $a_n=19.6$ m/s^2.

1.61. $\Delta a_t=0.3$ m/s^2.

1.62. The angle is determined from the equality $\tan\gamma=\frac{a_t}{a_n}$, where a_t is the tangential and a_n the normal acceleration. But $a_t=\frac{dv}{dt}$ and $a_n=\frac{v^2}{r}$. Therefore in the conditions of our problem $\tan\gamma=\frac{(3+2t)\,r}{(3t+t^2)^2}$. By inserting in this formula the values of $t=0$, 1, 2, 3, 4 and 5 seconds, we obtain: (1) $t=0$, $\tan\gamma=\infty$, i.e., $\gamma=90°$ and the total acceleration is directed tangentially, (2) $t=1$ second, $\tan\gamma=3.13$ and $\gamma=72°17'$, (3) $t=2$ seconds, $\tan\gamma=0.7$ and $\gamma=35°0'$, (4) $t=3$ seconds, $\tan\gamma=0.278$ and $\gamma=15°32'$, (5) $t=4$ seconds, $\tan\gamma=0.14$ and $\gamma=7°58'$, (6) $t=5$ seconds, $\tan\gamma=0.081$ and $\gamma=4°38'$. When $t=\infty$, $\tan\gamma=0$, i.e., $\gamma=0$ and the total acceleration is directed normally.

1.63. $r=1.2$ m. **1.64.** $\frac{a_n}{a_t}=0.58$.

2. Dynamics

2.1. A descending balloon is acted upon by the lifting force F_1 (upwards), the force of the air resistance F_2 (upwards) and the weight of the balloon F_3 (downwards). Since the balloon moves uniformly, the resultant force according to Newton's first law is zero, i.e.,

$$F_1+F_2=F_3 \tag{1}$$

When the ballast is dropped and the balloon begins to ascend, we have, instead of equation (1),

$$F_1=F_2+(F_3-F_x) \tag{2}$$

By solving Eqs. (1) and (2) simultaneously, we get $F_x=2(F_3-F_1)$. In our case $F_3=1{,}600$ kgf $=1{,}600\times9.81$ N, and $F_1=1{,}200$ kgf $=1{,}200\times9.81$ N. Hence $F_x=7.85\times10^3$ N $=800$ kgf.

2.2. (1) The load being lifted is acted upon by its weight G directed downwards and the thread tension T directed upwards. Using Newton's second law as applied to a load being lifted, we find $ma=T-G$, and the tension of the thread will be

$$T=ma+G=m(a+g) \tag{1}$$

In our case $m=1$ kg, $a=5$ m/s^2 and $g=9.81$ m/s^2. By inserting these values in Eq. (1), we obtain $T=14.8$ N $=1.51$ kgf.

(2) The load being lowered is acted upon by the weight G (downwards) and the thread tension T (upwards). Therefore $ma = G - T$, whence

$$T = m(g - a) \tag{2}$$

If the load is lowered with the acceleration g (free falling), i. e., if $a = g$, the thread tension will obviously be zero. By inserting the numerical data in Eq. (2), we obtain $T = 4.8$ N $= 0.49$ kgf.

2.3. $a = 1.25$ m/s^2.

2.4. (1) $a = 4.9$ m/s^2 (the lift is going up), (2) $a = 2.45$ m/s^2 (the lift is going down).

2.5. $a_2 = 13.8$ m/s^2.

2.6. The problem can be solved either with the aid of Newton's second law, or of the law of conservation of energy.

(1) According to Newton's second law

$$F = ma \tag{1}$$

where F is the braking force, m the mass of the automobile, and a its acceleration (negative in our case). Since the automobile is uniformly retarded, we can easily obtain from the kinematics of uniformly variable motion:

$$a = \frac{2s}{t^2} \tag{2}$$

and

$$v_0 = \frac{2s}{t} \tag{3}$$

(See the solution to Problem 1.17.) By substituting for a in Eq. (1) its value from Eq. (2), we get

$$F = \frac{2sm}{t^2} \tag{4}$$

In our case $s = 25$ m, $m = 1{,}020$ kg and $t = 5$ s. By inserting these data in Eqs. (3) and (4), we get $v_0 = 10$ m/s $= 36$ km/h and $F = 2{,}040$ N $= 208$ kgf.

(2) When the automobile is braked, its kinetic energy is transferred into work against the braking force, i. e.,

$$\frac{mv_0^2}{2} = Fs \tag{5}$$

But from the equations of kinematics

$$v_0 = \frac{2s}{t} \tag{3}$$

By substituting for v_0 in Eq. (5) its value from Eq. (3), we obtain as before

$$F = \frac{2sm}{t^2} \tag{4}$$

2.7. $F = 2.77 \times 10^4$ N.

2.8. (1) $\bar{F} = 3{,}000$ N, (2) $\bar{F} = 3.0 \times 10^4$ N, (3) $\bar{F} = 3.0 \times 10^5$ N.

2.9. The force which should be applied to the car is spent to overcome friction and to impart acceleration to it, i. e., $F = F_{fr} + F_{ac}$. But $F_{fr} = fG$, where G is the weight of the car and f the coefficient of friction, and $F_{ac} = ma = \frac{G}{g}a$. Thus, $F = fG + \frac{G}{g}a$ Since the car moves with uniform acceleration, $s = \frac{at^2}{2}$. Hence $a = \frac{2s}{t^2}$ and then, finally, $F = fG + \frac{2Gs}{gt^2}$ In our case $f = 0.05$,

$G=16$ tonf$=16\times9.81\times10^3$ N, $s=11$ m, $g=9.81$ m/s^2 and $t=30$ s. Upon inserting these data, we obtain $F=8{,}200$ N.

2.10. $v_0=11.75$ m/s.

2.11. (1) $F=6{,}000$ N, (2) in 50 seconds, (3) $s=375$ m.

2.12. According to Newton's second law, $F=ma$, but $a=\frac{dv}{dt}$. In our case $v=\frac{ds}{dt}=-B+2Ct-3Dt^2$, and, consequently, $a=\frac{dv}{dt}=2C-6Dt$. Hence

$$F=ma=m(2C-6Dt)=0.5(10-6t)\ \text{N} \qquad (1)$$

Equation (1) gives the relation between the force F and the time t. At the end of the first second $F=2$ N.

2.13. $m=4.9$ kg. **2.14.** $F=-0.123$ N.

2.15. $F\Delta t=5.6\times10^{-23}$ N·s.

2.16. According to Newton's second law, $F\Delta t=m\Delta v$, where Δv is the vectoral difference. Assuming the direction of the external normal N to the wall (Fig. 75)

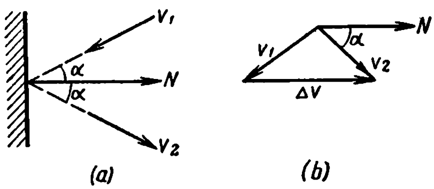


Fig. 75

positive, we get $\Delta v=v_2\cos\alpha-(-v_1\cos\alpha)=v_2\cos\alpha+v_1\cos\alpha$. According to the initial condition, $v_1=v_2=v$ and hence $\Delta v=2v\cos\alpha$. Thus, $F\Delta t=2mv\cos\alpha=2.8\times10^{-23}$ N·s.

2.17. 0.51 s.

2.18. $F=86$ N.

Note. During the time t the wall is struck by the mass of water in a cylinder with a length of $l=vt$ and a cross section of A, i. e., $m=\rho Avt$, where ρ is the density of the water.

2.19. (1) $v_{max}=21.6$ km/h, (2) $t=73$ s, (3) $a=-0.098$ m/s^2, (4) $s=218$ m.

2.20. (1) $F_1=980$ N, (2) $F_2=3{,}000$ N.

2.21. $\alpha=14°$.

Note. The resultant of the forces of gravity and inertia should be perpendicular to the surface of the liquid.

2.22. $\alpha=6°30'$. **2.23.** $f=0.15$.

2.24. Let us denote the weight of a unit of rope length by G_0. Hence the weight of the hanging part of the rope $G_1=G_0\times0.25l$. This weight is balanced by the force of friction acting on the part of the rope which lies on the table, $F_{fr}=fG_0 0.75l$. Thus, $G_0 0.25l=fG_0 0.75l$, whence $f=\frac{0.25}{0.75}=0.33$.

2.25. (1) The power developed by the engine of the automobile running up the grade is spent to overcome the force of friction and the component of the force of gravity parallel to its path (Fig. 76): $F=F_{fr}+F_1$, where $F_{fr}=fF_2=fG\cos\alpha$, and $F_1=G\sin\alpha$. Thus, the tractive effort is

$$F=G(f\cos\alpha+\sin\alpha) \qquad (1)$$

Upon inserting the numerical data and bearing in mind that $\sin\alpha \cong 0.04$ and $\cos\alpha \cong 1$, we obtain $F = 1{,}370$ N.

(2) When the automobile runs downhill $F = G(f\cos\alpha - \sin\alpha) = 590$ N. If the force of friction is less than the component of the force of gravity parallel to the route, i. e., if $Gf\cos\alpha < G\sin\alpha$, then $F < 0$. In this case, a retarding force must be applied for the automobile to run uniformly downhill. If no such force is applied, the automobile will run downhill with the acceleration $a = g(\sin\alpha - f\cos\alpha)$.

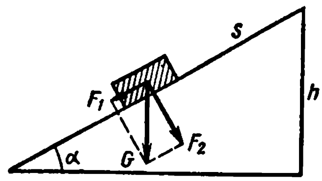


Fig. 76

2.26. $F = 2{,}370$ N.

2.27. (1) $f \leqslant 0.07$, (2) $a = 0.39$ m/s², (3) $t = 22.7$ s, (4) $v = 8.85$ m/s.

2.28. $f = \tan\alpha - \dfrac{v^2}{2gs\cos\alpha} = 0.2$.

2.29. $f = 0.5$.

2.30. (1) The force $G_1 - G_2$ imparts to the two weights the acceleration

$$a = \frac{G_1 - G_2}{m_1 + m_2} = \frac{g(m_1 - m_2)}{m_1 + m_2} \tag{1}$$

Upon inserting the numerical data, we obtain $a = 3.27$ m/s².

(2) The equations for the movement of the weights G_1 and G_2 can be written thus

$$m_1 a = m_1 g - T_1 \tag{2}$$

$$m_2 a = T_2 - m_2 g \tag{3}$$

(see the solution to Problem 2.2). From Equations (1), (2) and (3) it is easy to obtain that

$$T_1 = T_2 = \frac{2m_1 m_2 g}{m_1 + m_2} = 13.0 \text{ N}.$$

2.31. (1) $a = \dfrac{g(m_1 - fm_2)}{m_1 + m_2} = 4.4$ m/s²

(2) $T_1 = T_2 = \dfrac{m_1 m_2 g(1 + f)}{m_1 + m_2} = 5.4$ N

2.32. (1) $a = \dfrac{(m_1 - m_2\sin\alpha)g}{m_1 + m_2} = 2.45$ m/s²

(2) $T_1 = T_2 = \dfrac{m_1 m_2 g(1 + \sin\alpha)}{m_1 + m_2} = 7.35$ N

2.33. (1) $a = \dfrac{[m_1 - m_2(\sin\alpha + f\cos\alpha)]g}{m_1 + m_2} = 2.02$ m/s²

(2) $T_1 = T_2 = \dfrac{m_1 m_2 g[1 + (\sin\alpha + f\cos\alpha)]}{m_1 + m_2} = 7.77$ N

2.34. (1) $a = \dfrac{(m_1\sin\beta - m_2\sin\alpha)g}{m_1 + m_2} = 1.02$ m/s²

(2) $T_1 = T_2 = \dfrac{m_1 m_2 g(\sin\alpha + \sin\beta)}{m_1 + m_2} = 5.9$ N

2.35. (1) $a=\frac{[m_1(\sin\beta-f\cos\beta)-m_2(\sin\alpha+f\cos\alpha)]g}{m_1+m_2}=0.244$ m/s²

(2) $T_1=T_2=\frac{m_1m_2g[\sin\alpha+\sin\beta+f(\cos\alpha-\cos\beta)]}{m_1+m_2}=6.0$ N

2.36. The work W is performed to increase the potential energy of the load and accelerate it, i. e., $W=mgh+mah=mh(g+a)$, whence $a=\frac{W-mgh}{mh}$. In our case $W=8$ kgf-m $=8\times9.81$ J, the mass $m=2$ kg, and $h=1$ m. Upon inserting these data, we obtain $a=29.4$ m/s².

2.37. Ten times. 2.38. (1) $W_1=21.0$ J, (2) $W_2=64.0$ J.

2.39. $\Delta\bar{p}=-3.5$ kg·m/s. 2.40. $f=0.01$.

2.41. (1) $W=2.25\times10^6$ J, (2) $s=375$ m. 2.42. $v\leqslant50$ km/h.

2.43. $f=0.05$. 2.44. $W=35.6$ J. 2.45. $m=0.06$ kg.

2.46. With a mean engine power of P and an average speed of v the work performed by the engine over the distance s is $W=\frac{Pt}{\eta}=\frac{Ps}{\eta v}$, where η is the engine efficiency. The amount of petrol required to do this work is $m=\frac{W}{q}=\frac{Ps}{q\eta v}$, where q is the heating value of the petrol. In our case $P=15$ hp $=15\times736$ W, $s=10^5$ m, $q=4.6\times10^7$ J/kg, $\eta=0.22$ and $v=30$ km/h $=8.35$ m/s. Upon inserting these data, we obtain $m=13$ kg.

2.47. $\eta=0.22$.

2.48. The change with time of the kinetic, potential and total energy of the stone thrown vertically upward is illustrated in Fig. 77.

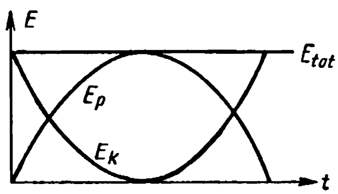


Fig. 77

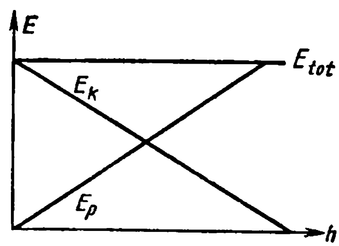


Fig. 78

2.49. The change with distance of the kinetic, potential and total energy of the stone thrown vertically upward is illustrated in Fig. 78.

2.50. $E_k=E_p=98.1$ J.

2.51. $E_k=32.2$ J, and $E_p=39.4$ J.

2.52. (1) $E'_k=6.6$ J, $E'_p=15.9$ J, $E'_{tot}=22.5$ J,

(2) $E''_k=5.7$ J, $E''_p=16.8$ J, $E''_{tot}=22.5$ J

Let us note that according to the law of conservation of energy $E'_{tot}=E''_{tot}=22.5$ J.

2.53. $t=1.5$ seconds, $s_x=19.1$ m. 2.54. $a_t=0.1$ m/s².

2.55. The potential energy of the body sliding down the inclined plane transforms into kinetic energy and work against the friction forces, i. e.,

$mgh = \frac{mv^2}{2} + F_{fr}l$. But $h = l\sin\alpha$ and $F_{fr} = fmg\cos\alpha$, where f is the coefficient of friction and α is the angle of inclination of the plane.

(1) $E_k = \frac{mv^2}{2} = mgh - F_{fr}l = mgl(\sin\alpha - f\cos\alpha)$. In our case $\sin\alpha = \frac{h}{l} =$ 0.1, i. e., $\alpha = 5°44'$. Therefore $\cos\alpha = 0.995$. Upon inserting the numerical data, we obtain $E_k = 4.9$ J.

(2) $v = \sqrt{\frac{2E_k}{m}} = 3.1$ m/s.

(3) The kinetic energy of the body at the base of the plane is transformed into work against the friction forces along the horizontal portion of the route, i. e.,

$$E_k = F_{fr}s = fmgs, \text{ whence } s = \frac{E_k}{fmg} = 10 \text{ m}.$$

2.56. $f = 0.07$. **2.57.** (1) $f = 0.22$, (2) $Q = 5.7$ J.
2.58. (1) $W = 7\times10^6$ J, (2) $P = 29.4$ kW.
2.59. The power developed by the vehicle engine is determined from the formula

$$P = Fv = fGv$$

(1) Over a level road

$$P = fGv = 6.9 \text{ kW}$$

(2) When the vehicle runs uphill it has to overcome the force of friction and the component of the force of gravity parallel to the route (see the solution to Problem 2.25), i.e., $F = G(f\cos\alpha + \sin\alpha)$. Consequently, $P = Gv(f\cos\alpha + \sin\alpha)$. In our case $\sin\alpha = 0.05$. Since α is small, it may be assumed that $\cos\alpha \cong 1$, and thus $P = 11.8$ kW.

(3) When the vehicle runs downhill, the power developed by the engine will be $P = Gv(f\cos\alpha - \sin\alpha) = 1.98$ kW.

2.60. For the vehicle to run downhill with its engine shut off at a constant speed, the force of friction should be equal to the component of the force of gravity parallel to its route, i.e., $fmg\cos\alpha = mg\sin\alpha$, whence $f = \tan\alpha$. The power developed by the engine when the vehicle runs uphill can be found from the formula $P = Fv = Gv(f\cos\alpha + \sin\alpha)$. By inserting $f = \tan\alpha$ in this formula, we obtain $P = Gv\,2\sin\alpha = 11.8$ kW.

2.61 (1) When the flat car is standing, the initial velocity of the shell relative to the Earth is obviously equal to its velocity relative to the cannon. From the law of conservation of momentum we have

$$(m_1 + m_2 + m_3)v_1 = m_3v_0 + (m_1 + m_2)v_x \qquad (1)$$

where m_1 is the mass of the flat car, m_2 the mass of the cannon, and m_3 the mass of the shell. In our case $v_1 = 0$. Equation (1) can thus be written as

$$v_x = -\frac{m_3v_0}{m_1 + m_2} = -3.33 \text{ m/s} = -12 \text{ km/h}$$

The minus sign shows that if the direction of motion of the shell is positive, i.e., $v_0 > 0$, then $v_x < 0$ and the flat car begins to move opposite to the direction of the shell.

(2) If the shell is fired in the direction of motion of the car, the initial velocity of the shell with respect to the Earth is $v_2 = v_0 + v_1$ and then the law

of conservation of momentum gives

$$(m_1+m_2+m_3)\,v_1=m_3\,(v_0+v_1)+(m_1+m_2)\,v_x \qquad (2)$$

whence

$$v_x=\frac{(m_1+m_2+m_3)\,v_1-m_3\,(v_0+v_1)}{m_1+m_2}=6\text{ km/h}$$

Let us note that $v_x>0$, i.e., the car continues to move in the same direction, but with a reduced velocity.

(3) If the shell is fired in the direction opposite to the motion of the car and $v_0>0$, we have $v_1<0$. Equation (2) now becomes

$$-(m_1+m_2+m_3)\,v_1=m_3\,(v_0-v_1)+(m_1+m_2)\,v_x$$

or

$$v_x=\frac{-(m_1+m_2+m_3)\,v_1-m_3\,(v_0-v_1)}{m_1+m_2}=-30\text{ km/h}$$

Let us note that v_x and v_1 are directed identically ($v_x<0$ and $v_1<0$) and the car therefore continues to move in the same direction but with a higher velocity.

2.62. $v=0.6$ m/s. **2.63.** (1) $v=5.14$ km/h, (2) $v=1.71$ km/h.

2.64. (1) $v=17.8$ km/h, (2) $v=53.5$ km/h, (3) $v=-17.8$ km/h. The minus sign shows that the car continues to move in the direction opposite to the shell but with a reduced speed.

2.65. $v=-12.5$ m/s.

2.66. (1) 0.67 m/s, (2) 0.83 m/s, (3) 0.5 m/s. **2.67.** $s=0.3$ m.

2.68. $E_k=49$ J. **2.69.** $\Delta t=0.58$ s. **2.70.** $\bar{F}=20.0$ N.

2.71. (1) 284 m, (2) 71 m, (3) 1,770 m. **2.72.** $E_k=1.5\times10^5$ J.

2.73. (1) $v_1=v_2=1.8$ m/s, (2) $v_1=0.6$ m/s and $v_2=2.6$ m/s.

2.74. $\dfrac{m_1}{m_2}=\dfrac{1}{3}$.

2.75. Before the collision the first body had the kinetic energy $E_1=\dfrac{m_1v_1^2}{2}$. After the inelastic impact both the bodies began to move with a common velocity $v_2=\dfrac{m_1v_1}{m_1+m_2}$. The kinetic energy of the two bodies after the impact became $E_2=\dfrac{(m_1+m_2)\,v_2^2}{2}=\dfrac{m_1^2v_1^2}{2\,(m_1+m_2)}$. The difference E_1-E_2 is equal to the amount of heat evolved during the collision: $Q=\Delta E=\dfrac{m_1v_1^2}{2}-\dfrac{m_1^2v_1^2}{2\,(m_1+m_2)}$. Upon inserting the numerical data, we obtain $Q=12$ J.

2.76. $E_1=5.62$ J, $E_2=0.62$ J. **2.77.** $E=7.5$ J. **2.78.** 1.25 times.

2.79. (1) $h_1=5\times10^{-3}$ m, $h_2=0.08$ m, (2) $h=2\times10^{-2}$ m.

2.80. $v=550$ m/s. **2.81.** $l=0.64$ m.

2.82. $Q=0.188$ J $=0.045$ cal. **2.83.** $\bar{p}=0.75$ N·s.

2.84. (1) $h=0.5$ m, (2) $Q=1.48$ J.

2.85. A ball falling from a height of h_1 reaches the floor with the velocity v_1 and rebounds from it with the velocity $v_2=kv_1$, where k is the coefficient of restitution. Since $mgh_1=\dfrac{mv_1^2}{2}$ and $mgh_2=\dfrac{mv_2^2}{2}$, then $\dfrac{h_2}{h_1}=\dfrac{v_2^2}{v_1^2}=\dfrac{k^2v_1^2}{v_1^2}=k^2$, i.e., $h_2=k^2h_1$. The interval of time from the moment of the fall to the second impact is $t=t_1+2t_2$, where t_1 is the duration of falling from the height h_1 and t_2 is the duration of falling from the height h_2. Since $t_1=\sqrt{\frac{2h_1}{g}}$ and $t_2=\sqrt{\frac{2h_2}{g}}=$ $=k\sqrt{\frac{2h_1}{g}}$, then $t=\sqrt{\frac{2h_1}{g}}(1+2k)$, whence $k=\frac{t-\sqrt{\frac{2h_1}{g}}}{2\sqrt{\frac{2h_1}{g}}}$. In our case $\sqrt{\frac{2h_1}{g}}=0.45$ s, and $t=1.3$ s. Upon inserting these numerical data, we find that $k=0.94$.

2.86. (1) $h=0.84$ m, (2) $t=1.4$ s. 2.87. $k=0.9$.

2.88. (1) $\bar{p}=0.17$ N·s, (2) $Q=37.2\times10^{-3}$ J.

2.89. The kinetic energy of the first body before the impact $E_1=\frac{m_1v_1^2}{2}$ and that of the second body $E_2=0$. After the impact the kinetic energy of the two bodies $E=\frac{(m_1+m_2)u^2}{2}$, where u is the total velocity of the bodies; $u=$ $=\frac{m_1v_1}{m_1+m_2}$. Therefore $E=\frac{m_1^2v_1^2}{2(m_1+m_2)}$. Hence the kinetic energy transformed during the impact into heat is

$$E_1-E=\frac{m_1v_1^2}{2}-\frac{m_1^2v_1^2}{2(m_1+m_2)}=\frac{m_1v_1^2}{2}\left(1-\frac{m_1}{m_1+m_2}\right)$$

and the ratio sought is

$$\frac{E_1-E}{E_1}=1-\frac{m_1}{m_1+m_2}=\frac{m_2}{m_1+m_2}$$

(1) If $m_1=m_2$, then $\frac{E_1-E}{E_1}=0.5$, (2) if $m_1=9m_2$, then $\frac{E_1-E}{E_1}=0.1$.

2.90. The kinetic energy of the first body before the impact $E_1=\frac{m_1v_1^2}{2}$ and that of the second body $E_2=0$. After the impact the second body acquires the kinetic energy $E_2'=\frac{m_2u_2^2}{2}$, where $u_2=\frac{2m_1v_1}{m_1+m_2}$. Thus, the first body imparts the kinetic energy $E_2'=\frac{m_2}{2}\left(\frac{2m_1v_1}{m_1+m_2}\right)^2$ to the second body The ratio sought is $\frac{E_2}{E_1}=\frac{4m_1m_2}{(m_1+m_2)^2}$.

(1) When $m_1=m_2$, the ratio $\frac{E_2'}{E_1}=1$, (2) when $m_1=9m_2$ we have $\frac{E_2'}{E_1}=0.36$.

2.91. (1) $\frac{m_1}{m_2}=5$, (2) $E_2=\frac{5}{9}$ kJ. 2.92. 1.4 times.

2.93. (1) $\frac{\Delta v}{v}=\frac{2}{13}$, (2) $\frac{\Delta v}{v}=\frac{2}{236}$.

2.94. $x=\frac{\frac{mv^2}{r}}{mg}=\frac{v^2}{rg}=\frac{\omega^2r}{g}=0.34$ per cent.

2.95. 1 h 25 min. **2.96.** $F=245$ N. **2.97.** (1) $v=2.43$ m/s, (2) at the highest point $T=0$, at the lowest point $T=39.2$ N.

2.98. $\nu=\frac{1}{2\pi}\sqrt{\frac{9g}{l}}=2.1$ rev/s. **2.99.** $m=0.5$ kg.

2.100. $\nu=59$ rev/min. **2.101.** $T=1.96$ N. **2.102.** $f=0.2$.

2.103. (1) $r_1=1{,}600$ m, (2) $r_2=711$ m. **2.104.** $\alpha=22°$.

2.105. $\alpha=1°$. **2.106.** (1) $\nu_1=2$ rev/s, (2) $\nu_2=1.5$ rev/s. **2.107.** $v=47$ km/h.

2.108. At the moment the weight passes through the position of equilibrium, the tension of the thread $T=mg+\frac{mv^2}{l}$. Besides, $mgh=\frac{mv^2}{2}$, whence $v=\sqrt{2gh}$. But (Fig. 79) $h=l-l\cos\alpha=l(1-\cos\alpha)$. Hence $\frac{mv^2}{l}=\frac{m}{l}2gh=\frac{m}{l}2gl(1-\cos\alpha)=2mg(1-\cos\alpha)$ and $T=mg[1+2(1-\cos\alpha)]=12.4$ N.

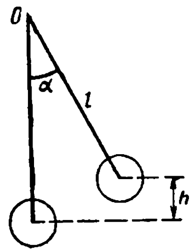


Fig. 79

2.109. (1) $\alpha=45°34'$, (2) $T=632$ N, (3) $v=6$ m/s.

2.110. (1) $\alpha=60°$, (2) 2.3 times.

2.111. $T=3G$. **2.112.** $\alpha=60°$. **2.113.** $h=2$ m.

2.114. The lateral pressure of the water

$$p=\frac{F}{ld} \tag{1}$$

where F is the centrifugal force, d the pipe diameter and l the length of the part of the pipe acted upon by the pressure. Further

$$F=\frac{mv^2}{r} \tag{2}$$

where

$$m=\rho lA \tag{3}$$

is the mass of the water in the volume Al (A is the cross-sectional area of the pipe and ρ is the density of the water). The velocity of the water v can be determined from the formula

$$v=\frac{M}{\rho At} \tag{4}$$

where M is the mass of the water flowing through the pipe cross section A during the time t. Upon inserting Eqs. (2), (3) and (4) into Eq. (1), we obtain $p=\frac{M^2}{r\rho dAt^2}$, or, after inserting the numerical data, $p=56.0$ N/m²

2.115. $p=1{,}250$ N/m².

2.116. The work performed during compression of the spring is determined from the formula

$$W=-\int_0^{\varepsilon} F\,d\varepsilon \tag{1}$$

where ε is the deformation in compression. According to the given condition, the force is proportional to the deformation, i. e.,

$$F=-k\varepsilon \tag{2}$$

where k is the coefficient of deformation determined by the stiffness of the spring and equal numerically to the force causing a unit deformation. Upon inserting Eq. (2) into Eq. (1), we obtain $W=\int_0^{\varepsilon} k\varepsilon d\varepsilon=\frac{k\varepsilon^2}{2}$. In our case $k=\frac{29.4}{0.01}$ N/m $=2{,}940$ N/m, and $\varepsilon=0.2$ m. Upon inserting these values we get $W=58.8$ J.

2.117. In static deflection $G=kx_0$, where G is the weight of the load. Hence $k=\frac{G}{x_0}$. When this load is dropped from the height h we have $G(h+x)=\frac{kx^2}{2}=\frac{Gx^2}{2x_0}$ or $x^2-2x_0x-2x_0h=0$. Solution of this equation gives $x=x_0\pm\sqrt{x_0^2+2x_0h}$.

(1) If $h=0$, then $x=2x_0=4$ cm, (2) if $h=100$ cm, then $x=22.1$ cm.

2.118. $h_1=1.23$ m. **2.119.** 10 graduations. **2.120.** 7.4 kgf.

2.121. $v=3.6$ km/h. **2.122.** $v=22.1$ m/s. **2.123.** $\frac{E_1}{E_2}=\frac{k_2}{k_1}$.

2.124. $l=\frac{k_2L}{k_1+k_2}=6\times10^{-2}$ m, i. e., the load should be attached at a distance of 6 cm from the first spring.

2.125. $F=\frac{m\Delta x}{(\Delta t)^2}=13.7$ N.

2.126. The tension of the cord (see Fig. 80) is equal to $T=\frac{G}{\cos\alpha}=5.7$ N. The tension T stretches the cord by Δl, and $T=k\Delta l$. Therefore $\Delta l=\frac{T}{k}=9.5\times10^{-3}$ m. It can be seen from Fig. 80 that

$$\frac{l}{R}=\frac{T}{F} \qquad (1)$$

But

$$F=T\sin\alpha=\frac{mv^2}{R}=4\pi^2\nu^2mR \qquad (2)$$

From (1) and (2) we have $l=\frac{T}{4\pi^2\nu^2m}=7.25\times10^{-2}$ m. Thus, the length of the stretched rubber cord is $l=72.5\times10^{-3}$ m and its length before stretching $l_0=l-\Delta l=63\times10^{-3}$ m $=6.3$ cm.

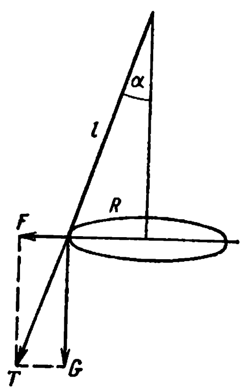


Fig. 80

2.127. $l=10.8$ cm.

Note. The potential energy of the lifted load is transformed into the work of stretching the cord and the kinetic energy of the load.

2.128. The ball floats in equilibrium if its weight is equalized by the Archimedean force, i. e., if $G=F_{Arch}$, or

$$mg=\rho_0v_0g \qquad (1)$$

where v_0 is the volume of the ball segment with the height h submerged in the water in equilibrium, ρ_0 the water density, and m the mass of the ball. Obviously $H+h=r$, i. e., the radius of the ball. If the ball is submerged into the water to a depth of x, the Archimedean force will exceed the weight of the ball, and the resulting force pushing the ball out of the water will be equal to

$$F_x=F'_{Arch}-mg \qquad (2)$$

The work must be performed against the force F_x. Obviously,

$$F'_{Arch}=\rho_0 v_1 g \quad (3)$$

where v_1 is the volume of the ball segment with the height $(h+x)$. From Eqs. (1), (2) and (3) we have $F_x=\rho_0 v_1 g-\rho_0 v_0 g=\rho_0 g(v_1-v_0)=\rho_0 g v_x$, where v_x is the volume of the layer of the ball with the height x. The volume of a spherical segment with a height of l, as is well known, is equal to $v=\frac{1}{3}\pi l^2(3r-l)$ where r is the radius of the ball. Therefore the volume of the layer of the ball $v_x=v_1-v_0=\frac{1}{3}\pi(x+h)^2[3r-(x+h)]-\frac{1}{3}\pi h^2(3r-h)$. Hence

$$F_x=\rho_0 g v_x=\frac{\rho_0 g\pi}{3}[3r(x+h)^2-(x+h)^3-h^2(3r-h)] \quad (4)$$

The work which must be performed against this force when the ball is submerged to its diametral plane will be

$$W=\int_0^H F_x dx \quad (5)$$

After inserting Eq. (4) into Eq. (5), integrating and remembering that $H+h=r$, we obtain, upon inserting the numerical data, $W=0.74$ J.

2.129. $W=0.17$ J (see the solution to Problem 2.128).

2.130. $W=0.84$ J (see the solution to Problem 2.128).

2.131. $W=\frac{Agh^2(\rho_0-\rho_1)^2}{2\rho_0}=7.84$ J. Here ρ_0 is the density of water and ρ_1 that of ice.

2.132. $F=1.86\times10^{-44}$ N. **2.133.** $E=-3.8\times10^{-10}$ J.

2.134. $\gamma=\frac{3g}{4\pi\rho R}=6.7\times10^{-11}$ m^3/kg·s^2.

2.135.

Planet	ρ, kg/m^3	Planet	ρ, kg/m^3
Mercury	5,500	Jupiter	1,320
Venus	4,800	Saturn	710
Earth	5,500	Uranus	1,260
Mars	3,900	Neptune	1,600

2.136. The rocket will be attracted by the Earth and the Moon with the same force at a distance of 3.4×10^5 km from the Earth's surface.

2.137. $g_{Moon}=0.165g_{Earth}$.

2.138. The period of oscillations of the mathematical pendulum will increase 2.46 times.

2.139. The force of attraction between the body and the Earth is equal to $F=\gamma\frac{mM}{R^2}$, where m is the mass of the body, M the mass of the Earth, and R the distance between them. Near the Earth's surface R is equal to the Earth's radius and $F=mg$. Hence

$$F=mg=\gamma\frac{mM}{R^2} \tag{1}$$

When the body moves around the Earth in a circular orbit, the force of gravity is centripetal Thus,

$$F=\frac{mv^2}{R} \tag{2}$$

Therefore the velocity sought

$$v=\sqrt{\frac{\gamma M}{R}}=\sqrt{gR}=7.9\times10^3 \text{ m/s}=7.9 \text{ km/s}$$

2.140. For the body to depart from the Earth, its kinetic energy should be sufficient to overcome the potential energy of the forces of gravity, i. e.,

$$\frac{mv^2}{2}\geqslant\gamma\frac{mM}{R} \tag{1}$$

But near the Earth's surface $\frac{\gamma M}{R^2}=g$ [see equation (1) of the solution to the previous problem]. Therefore, $\frac{mv^2}{2}\geqslant mgR$, whence the velocity sought is $v\geqslant\sqrt{2gR}$. Upon inserting the numerical data, we obtain $v\geqslant 11.2$ km/s.

2.141.

Planet	v_1, km/s	v_2, km/s	Planet	v_1, km/s	v_2, km/s
Mercury	3.0	4.25	Jupiter	42.6	60.4
Venus	7.2	10.2	Saturn	25.7	36.4
Earth	7.9	11.2	Uranus	15.2	21.5
Mars	3.57	5.05	Neptune	16.6	23.5

2.142. $v=30$ km/s.

2.143.

h, km	v, km/s	T
0	7.91	1 h 25 min
200	7.79	1 h 28 min
7,000	5.46	4 h 16 min

2.144. (1) $T=\sqrt{\frac{3\pi}{\gamma\rho}}$, where ρ is the density of the central body and γ is the gravitational constant.

(2)

Planet	T, hours	Planet	T, hours
Mercury	1.41	Jupiter	2.86
Venus	1.50	Saturn	3.90
Earth	1.41	Uranus	2.94
Mars	1.66	Neptune	2.61

2.145. $a_n = 9.20\ \text{m/s}^2$. **2.146.** $T_1 = 7.8$ hours, $T_2 = 31.2$ hours.
2.147. At a distance of 35,800 km from the Earth's surface.
2.148. $v = 1.7$ km/s, $T = 1$ h 50 min.
2.149. $v_1 = 1.7$ km/s, $v_2 = 2.4$ km/s.
2.150. On the Earth's surface

$$F = mg = \gamma \frac{mM}{R^2} \tag{1}$$

where R is the Earth's radius. At the altitude h above the surface

$$mg_1 = \gamma \frac{mM}{(R+h)^2} \tag{2}$$

From Eqs. (1) and (2) we obtain

$$\frac{g_1}{g} = \frac{R^2}{(R+h)^2} \tag{3}$$

Equation (3) gives the relationship between $\frac{g_1}{g}$ and the altitude h. Let us denote $\frac{g_1}{g} = n$. Hence, from Eq. (3) we have $h^2 + 2Rh + \left(R^2 - \frac{R^2}{n}\right) = 0$. Upon solving this quadratic equation, we find that $h = -R \pm \frac{R}{\sqrt{n}}$. Since h must be greater than zero, take the solution with the plus sign, i. e., $h = -R + \frac{R}{\sqrt{n}}$ In this case h will always be positive, since $n < 1$. Upon inserting $n = 0.25$, we find that $h = R$, i. e., $g_1 = 0.25\,g$ at an altitude equal to the Earth's radius Let us note that if $h \ll R$, equation (3) may be written as

$$\frac{g_1}{g} = \frac{R^2}{(R+h)^2} \cong 1 - \frac{2h}{R}$$

2.151. At a distance of 13,600 km from the Earth's surface.
2.152. 2 times.
2.153. Assume that the mass of the body at the distance h from the Earth's surface and at the distance r from its centre is m. According to the assumption made in the Problem, we can write $F_1 = mg_1 = \gamma \frac{mM_1}{r^2}$, where M_1 is the mass of a sphere with the radius r and a density equal to that of the Earth Since $M_1 = \frac{4}{3}\pi r^3 \rho$, where ρ is the Earth's density, then $mg_1 = \gamma m \frac{4}{3}\pi r \rho$. On the

Earth's surface $F=mg=\gamma\frac{mM}{R^2}=\gamma m\frac{4}{3}\pi R\rho$. Hence the relation between $\frac{g_1}{g}$ and the depth h will be $\frac{g_1}{g}=\frac{r}{R}=\frac{R-h}{R}$, whence $h=R\left(1-\frac{g_1}{g}\right)$. If $\frac{g_1}{g}=0.25$, then $h=0.75R$.

2.154. $h=2H$.

2.155. According to Kepler's third law

$$\frac{T_1^2}{T_2^2}=\frac{R_1^3}{R_2^3} \tag{1}$$

Since we are interested in the period of revolution of a planet of the solar system, it will be good to take the Earth as a planet with known values of T_2 and R_2. (Let us note that if Kepler's law is applied to artificial satellites of the Earth, it will be natural to take the Moon as a satellite with known values of T_2 and R_2.) In our case $T_2=12$ months, $R_2=1.5\times10^8$ km. According to the given condition, $R_1=1.5\times10^8\text{ km}+24\times10^6\text{ km}=1.74\times10^8$ km. From Eq. (1)

$T_1=T_2\sqrt{\left(\frac{R_1}{R_2}\right)^3}=15\text{ months}=450\text{ days}.$

2.156. $v=27.6$ km/s, $T=450$ days.

2.157. (1) $R_2=1.46\times10^4$ km, (2) $T_2=104$ min.

2.158. $T=88$ min.

2.159 Let us take the ring element dl (Fig. 81) The attraction between this element and the mass m at point A will be $dF=\gamma\frac{m\rho\pi r^2 dl}{x^2}$ The force dF is

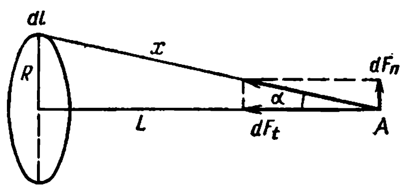


Fig. 81

directed along the line x which connects the ring element dl to the mass m. Obviously all the forces dF should be summed geometrically to find the force of attraction by the entire ring. The force dF can be resolved into two components dF_n and dF_t. The components dF_n from each two diametral elements are mutually cancelled and hence $F=\int dF_t$. But $dF_t=dF\cos\alpha=dF\frac{L}{x}$ and

$$F=\int\frac{L}{x}dF=\frac{\gamma m\rho\pi r^2 L}{x^3}\int_0^{2\pi R}dl=\frac{\gamma m\rho\pi r^2 L 2\pi R}{x^3} \tag{1}$$

But $x=\sqrt{R^2+L^2}$ and finally

$$F=\frac{2\pi^2\gamma m\rho r^2 RL}{(R^2+L^2)^{3/2}} \tag{2}$$

2.160 It can be seen from formula (2) of the previous problem that if $L=0$, then $F=0$. It is easy to see that the function F first grows and then begins to diminish, with an increase in L. Let us find the maximum of the function F.

Upon expressing the variable quantities x and L through the angle α, we have $x=\frac{R}{\sin\alpha}$, $L=x\cos\alpha=\frac{R}{\sin\alpha}\cos\alpha$. Hence formula (2) of the previous problem will become

$$F=\frac{2\pi^2\gamma m\rho r^3\cos\alpha\sin^2\alpha}{R}=C\cos\alpha\sin^2\alpha$$

To find the maximum of the function F let us take the derivative $\frac{dF}{d\alpha}$ and equate it to zero. We have $\frac{dF}{d\alpha}=C(2\cos^2\alpha\sin\alpha-\sin^3\alpha)=0$, whence $\tan^2\alpha=2$. Consequently, the distance L at which the force F is maximum is equal to $L=\frac{R}{\sin\alpha}\cos\alpha=\frac{R}{\tan\alpha}=\frac{R}{\sqrt{2}}$. (1) Figure 82 shows the nature of the relationship $F=f(L)$ (it is convenient to plot L cm along the axis of abscissas and $F\times10^{11}$ N along the axis of ordinates), (2) $L_{max}=14.1$ cm, (3) $F_{max}=4.33\times10^{-11}$ N.

2.161. 1.3. times

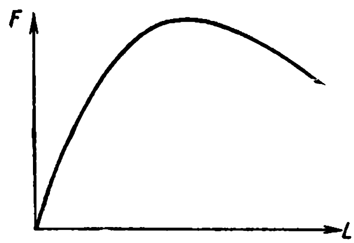


Fig. 82

3. Rotational Motion of Solids

3.1. (1) 9.7×10^{37} kg·m², (2) 7×10^{33} kg·m²/s.

3.2. (1) $I_1=63.5\times10^{-3}$ kg·m², (2) $I_2=62.5\times10^{-3}$ kg·m², (3) $\delta=1.6$ per cent.

3.3. The resulting moment of the forces under the action of which the disk rotates is

$$M=Fr-M_{fr} \qquad (1)$$

According to the basic law of dynamics, this moment is related to the angular acceleration of the body by the equation

$$M=I\alpha \qquad (2)$$

where

$$I=\frac{mr^2}{2} \qquad (3)$$

is the moment of inertia of the disk. From Eqs. (1), (2) and (3) it is easy to find the mass of the disk $m=\frac{2(Fr-M_{fr})}{\alpha r^2}$. In our case $F=98.1$ N, $r=0.2$ m, $\alpha=100$ rad/s², and $M_{fr}=0.5$ kgf-m $=0.5\times9.81$ N·m. Upon inserting these values, we obtain $m=7.36$ kg. Thus, the weight of the disk $G=7.36$ kgf $=72$ N.

3.4. $\alpha=2.35$ rad/s². **3.5.** $F=4.0$ N. **3.6.** $M=100$ N·m.

3.7. (1) $\alpha=7.8$ rad/s², (2) in 1 min 20 s.

3.8. $\nu=23.4$ rev/s. **3.9.** (1) $M=513$ N·m, (2) $N=600$ rev.

3.10. The weight G_1 moves down under the action of two forces: G_1—its own weight (directed downwards) and T_1—the thread tension (directed upwards). Therefore, for the weight G_1 we have

$$m_1a=m_1g-T_1 \qquad (1)$$

The weight G_2 moves upwards with the same acceleration a under the action of the following forces: G_2—its own weight (down) and T_2—the thread tension (up). Therefore, for the weight G_2

$$m_2a=T_2-m_2g \qquad (2)$$

The thread will be stretched on both sides of the pulley differently, and the difference in the tensions T_1-T_2 will create the moment rotating the pulley. By using the basic law of dynamics, we obtain

$$(T_1-T_2)\,r=I\alpha=I\frac{a}{r} \tag{3}$$

where

$$I=\frac{Mr^2}{2} \tag{4}$$

In formula (4), M is the mass of the pulley.

(1) Upon solving Eqs. (1), (2), (3) and (4) simultaneously, we find

$$a=\frac{G_1-G_2}{m_1+m_2+\frac{I}{r^2}}=\frac{G_1-G_2}{m_1+m_2+\frac{M}{2}} \tag{5}$$

Substitution of numerical values gives $a=2.8$ m/s². If in equation (5) we assume $M=0$, i. e., the mass of the pulley is neglected, we shall obtain the solution to Problem 2.30.

(2) By inserting Eq. (5) into Eqs. (1) and (2), we obtain, respectively,

$$T_1=\frac{G_1\left(2m_2+\frac{I}{r^2}\right)}{m_1+m_2+\frac{I}{r^2}} \tag{6}$$

and

$$T_2=\frac{G_2\left(2m_1+\frac{I}{r^2}\right)}{m_1+m_2+\frac{I}{r^2}} \tag{7}$$

If we assume that $I=0$ $(M=0)$ in Eqs. (6) and (7), we shall again get the solution to Problem 2.30. Upon inserting the numerical values, we obtain $T_1=14.0$ N and $T_2=12.6$ N.

3.11. The problem can be solved in two ways: (1) by using the basic law of dynamics of rotational motion (see the solution to Problem 3.10), and (2) by using the law of conservation of energy. The solution in the first way is to be done independently; the answer is $a=\frac{2mg}{M+2m}=3$ m/s². Upon using the second way, let us reason as follows; when the load is lowered, its potential energy diminishes and passes into the kinetic energy of the load and the kinetic energy of rotation of the drum. Thus,

$$mgh=\frac{mv^2}{2}+\frac{I\omega^2}{2} \tag{1}$$

where I is the moment of inertia of the drum. But since $I=\frac{Mr^2}{2}$ and $\omega=\frac{v}{r}$, where r is the radius of the drum, equation (1) may be written as

$$mgh=\frac{mv^2}{2}+\frac{Mv^2}{2\times 2}=\frac{v^2}{2}\left(m+\frac{M}{2}\right) \tag{2}$$

Since the load is lowered under the action of a constant force, the motion of the load is uniformly accelerated and therefore

$$h=\frac{at^2}{2} \tag{3}$$

and

$$v=at \tag{4}$$

Upon inserting Eqs. (3) and (4) into (2), it is easy to obtain

$$a=\frac{2mg}{M+2m}=3\ \text{m/s}^2$$

3.12. $I=9.5\ \text{kg}\cdot\text{m}^2$.

3.13. (1) In 1.1 seconds, (2) $E_k=0.81$ J, (3) $T=4.1$ N.

3.14. $T_1-T_2=\frac{1}{r}(I\alpha-M_{fr})=1{,}080$ N.

3.15. (1) $a=3.53\ \text{m/s}^2$, (2) $T_1=6.3$ N, $T_2=4.5$ N.

Check whether Problem 2.31 can be solved by using the formulas giving the solution to this problem.

3.16. The kinetic energy of the disk consists of the kinetic energies of translational motion and rotational motion, i. e.,

$$E_k=\frac{mv^2}{2}+\frac{I\omega^2}{2} \tag{1}$$

But since $I=\frac{mr^2}{2}$ and $\omega=\frac{v}{r}$, where m is the mass of the disk and r its radius, equation (1) takes the form $E_k=\frac{3mv^2}{4}$ Substitution of the numerical values gives $E_k=24.0$ J.

3.17. $E_k=0.1$ J. **3.18.** $E_2=29.4$ J. **3.19.** $Q=2.51\times10^{-3}$ J.

3.20. $\delta=\frac{E_1-E_2}{E_2}=40\%$. Here $E_1=E_{tr}+E_{rot}$; $E_2=E_{tr}$.

3.21. $W=355$ J. **3.22.** 3.8 kg·m²/s. **3.23.** $E=253$ J.

3.24. 4.1 m. **3.25.** $h=2R+\frac{R}{2}\left(1+\frac{m_1}{m}\right)=7.56$ m.

3.26. $W=3.2\pi^3r^5\rho\nu^2=34.1$ J. Here ρ is the density of the copper.

3.27. When a body slides down an inclined plane its potential energy is transformed into kinetic energy. Thus,

$$mgh=\frac{mv^2}{2}+\frac{I\omega^2}{2} \tag{1}$$

where I is the moment of inertia of the body and m is its mass. But

$$h=l\sin\alpha \tag{2}$$

$$\omega=\frac{v}{r} \tag{3}$$

Upon inserting Eqs. (2) and (3) into Eq. (1), we obtain

$$mgl\sin\alpha=\frac{v^2}{2}\left(m+\frac{I}{r^2}\right) \tag{4}$$

Since the bodies move under the action of a constant force, their motion is uniformly accelerated, and therefore

$$l=\frac{at^2}{2} \tag{5}$$

and

$$v=at \tag{6}$$

Upon solving Eqs. (4), (5) and (6) simultaneously, we obtain

$$a=\frac{mg\sin\alpha}{m+\frac{I}{r^2}} \tag{7}$$

By inserting the expressions for the moment of inertia of various bodies in equation (7), we find:

(1) for the ball $a=\frac{5}{7}g\sin\alpha=3.50$ m/s²;

(2) for the disk $a=\frac{2}{3}g\sin\alpha=3.27$ m/s²;

(3) for the hoop $a=\frac{1}{2}g\sin\alpha=2.44$ m/s²;

(4) for the body which slides off the inclined plane without friction we have $a=g\sin\alpha=4.9$ m/s².

3.28. $v=\sqrt{\frac{2mgh}{m+\frac{I}{r^2}}}$ · (1) 2.65 m/s, (2) 2.56 m/s, (3) 2.21 m/s, (4) 3.13 m/s.

3.29. (1) The translational velocity of the cylinders at the base of the inclined plane is determined from the formula

$$v=\sqrt{\frac{2mgh}{m+\frac{I}{r^2}}} \tag{1}$$

(see the previous problem). The aluminium cylinder, whose moment of inertia is smaller than that of the lead cylinder, will attain a higher velocity at the base of the inclined plane and will roll down it faster.

(2) The moment of inertia of the aluminium (solid) cylinder is

$$I_1=\frac{mr^2}{2} \tag{2}$$

The moment of inertia of the lead (hollow) cylinder is $I_2=m\frac{r^2+r_1^2}{2}$. Let us find the internal radius r_1 of the lead cylinder. According to the initial condition, the masses of the two cylinders are the same, i. e., $\rho_1 l\pi r^2=\rho_2 l\pi(r^2-r_1^2)$, where l is the length of the cylinders, ρ_1 the density of aluminium and ρ_2 that of lead. Hence $r_1^2=r^2\frac{\rho_2-\rho_1}{\rho_2}$. In this case the moment of inertia of the lead cylinder

$$I_2=\frac{mr^2}{2}\frac{2\rho_2-\rho_1}{\rho_2} \tag{3}$$

Substitution of the numerical values (see the tables in the Appendix) gives $I_1=9\times10^{-4}$ kg·m², $I_2=15.9\times10^{-4}$ kg·m².

(3) Since the cylinders roll down under the action of a constant force $v=at$ and $l=\frac{h}{\sin\alpha}=\frac{at^2}{2}$. Hence $\frac{h}{\sin\alpha}=\frac{vt}{2}$ and

$$t=\frac{2h}{v\sin\alpha} \tag{4}$$

Upon inserting Eq. (1) into Eq. (4), we finally obtain

$$t=\frac{1}{\sin\alpha}\sqrt{\frac{2h\left(m+\frac{I}{r^2}\right)}{mg}} \qquad (5)$$

Upon inserting Eqs. (2) and (3) into Eq. (5), we obtain $t=\frac{1}{\sin\alpha}\sqrt{\frac{3h}{g}}=0.78$ second for the aluminium cylinder, and $t=\frac{1}{\sin\alpha}\sqrt{\frac{2h\left(1+\frac{2\rho_2-\rho_1}{2\rho_2}\right)}{g}}=0.88$ second for the lead cylinder.

3.30. (1) $\alpha=-0.21$ rad/s², (2) $M_{br}=0.42$ N·m, (3) $W=630$ J, (4) $N=240$ rev.

3.31. (1) $I=0.01$ kg·m², (2) $M_{br}=9.4\times10^{-2}$ N·m.

3.32. (1) $M_{fr}=308$ N·m, (2) $t=100$ seconds. **3.33.** $h=0.865$ m.

3.34. $E_k=\frac{\alpha p t_2^2}{2t_1}=490$ J. **3.35.** $\Delta t=\frac{E_k}{\pi\nu M}=5$ seconds.

3.36. $E_k=\frac{F^2\Delta t^2}{m}=1.92\times10^3\text{ J}=1.92\text{ kJ}$.

3.37. By the angle $\alpha=81°22'$. **3.38.** $v=7.1$ m/s.

3.39. $\omega_1=\omega_2=14$ rad/s; (1) $v_1=1.05$ m/s, (2) $v_2=2.10$ m/s.

3.40. From the law of conservation of angular momentum we have

$$I_1\omega_1=I_2\omega_2 \qquad (1)$$

where I_1 is the moment of inertia of the platform with the man standing on its edge, I_2 the moment of inertia of the platform with the man standing in its centre, ω_1 and ω_2 are the angular velocities of the platform with the man standing on the edge and in the centre, respectively. We have

$$I_1=\frac{m_1r^2}{2}+m_2r^2 \qquad (2)$$

and

$$I_2=\frac{m_1r^2}{2} \qquad (3)$$

where r is the radius of the platform, m_1 the mass of the platform and m_2 the mass of the man. Upon inserting Eqs. (2) and (3) into Eq. (1) and remembering that $\omega=2\pi\nu$, where ν is the number of platform revolutions per second, we obtain

$$\left(\frac{m_1r^2}{2}+m_2r^2\right)2\pi\nu_1=2\pi\nu_2\frac{m_1r^2}{2}$$

whence

$$\nu_2=\nu_1\frac{m_1r^2+2m_2r^2}{m_1r^2}=\nu_1\frac{m_1+2m_2}{m_1}=22\text{ rev/min}$$

3.41. $W=162$ J. **3.42.** $\nu=21$ rev/min. **3.43.** 1.05 times.

3.44. $\nu=0.49$ rev/min. **3.45.** $T=1.16$ s. **3.46.** $T=1.07$ s.

3.47. $l=\frac{T\sqrt{gd}}{\pi}=0.446$ m. **3.48.** $T=1.5$ s.

3.49. The period of small oscillations of a mathematical pendulum is

$$T_1=2\pi\sqrt{\frac{l}{g}} \qquad (1)$$

and that of a physical pendulum $T_2=2\pi\sqrt{\frac{I}{mgl}}$, where I is the moment of inertia of the ball with respect to the axis of revolution, m the mass of the ball and l the distance from the centre of the ball to the point of suspension. In our case $I=\frac{2}{5}mr^2+ml^2=ml^2\left[1+\frac{2}{5}\left(\frac{r}{l}\right)^2\right]=ml^2x$.
Hence

$$T_2=2\pi\sqrt{\frac{lx}{g}} \tag{2}$$

From Eqs. (1) and (2) we have $\frac{T_2}{T_1}=\sqrt{x}$. The error in assuming the suspended ball to be a mathematical pendulum will be

$$\delta=\frac{T_2-T_1}{T_1}=\frac{T_2}{T_1}-1=\sqrt{x}-1$$

Hence $x=\left[1+\frac{2}{5}\left(\frac{r}{l}\right)^2\right]=(1+\delta)^2$ or $\frac{r}{l}=\sqrt{\frac{5}{2}[(1+\delta)^2-1]}$ (3)

In our case $\delta\leqslant 0.01$. Upon inserting it into Eq. (3), we get $\frac{r}{l}\leqslant 0.0224$. Since $r=\frac{D}{2}=0.02$ m, the minimum distance from the centre of the ball to the point of suspension $l=0.089$ m, and the minimum length of the thread $L=l-r=$ $=0.069$ m$=6.9$ cm.

3.50. 1.05 times.

4. Mechanics of Fluids

4.1. $v=0.12$ m/s.

4.2. Let us denote the cross-sectional area of the vessel by A_1 and the velocity with which the water level drops in it by v_1, the cross-sectional area of the hole by A_2 and the velocity with which the water flows out of the hole by v_2. According to Bernoulli's theorem

$$\frac{\rho v_1^2}{2}+\rho gh=\frac{\rho v_2^2}{2}$$

or

$$v_1^2+2gh=v_2^2 \tag{1}$$

With a view to continuity of the stream $v_1A_1=v_2A_2$, or

$$v_2=\frac{v_1A_1}{A_2} \tag{2}$$

By inserting Eq. (2) into Eq. (1) and solving with respect to v_1, we obtain $v_1=\frac{A_2\sqrt{2gh}}{\sqrt{A_1^2-A_2^2}}$ Since $A_1=\frac{\pi D^2}{4}$ and $A_2=\frac{\pi d^2}{4}$, then $v_1=\frac{d^2\sqrt{2gh}}{\sqrt{D^4-d^4}}$ Since $d^4\ll D^4$, then approximately

$$v_1=\frac{d^2}{D^2}\sqrt{2gh} \tag{3}$$

Let us note that if $d=D$, then $v_1=\sqrt{2gh}$. When $h=0.2$ m, $v_1=8\times10^{-4}$ m/s.

4.3. In both cases the jet of water falls onto the table at a distance of 0.4 m from the vessel.

4.4. (1) $v=0$ m/s, (2) $v=1.04$ m/s, (3) $v=1.25$ m/s.

4.5. (1) The velocity with which the water level sinks in the tank $v=\frac{A_2\sqrt{2gy}}{\sqrt{A_1^2-A_2^2}}$ (see the solution to Problem 4.2). Here y is the water level (variable) in the tank. During the time dt the water level in the tank will sink by

$$dy=v\,dt=C\sqrt{y}\,dt \qquad (1)$$

where $C=\frac{A_2\sqrt{2g}}{\sqrt{A_1^2-A_2^2}}$. From (1) $dt=\frac{dy}{C\sqrt{y}}$ whence $t=\frac{1}{C}\int_0^h\frac{dy}{\sqrt{y}}$. It is suggested that integration be carried out to the end to obtain the answer

$$t=\frac{2\sqrt{h}\sqrt{A_1^2-A_2^2}}{A_2\sqrt{2g}}=\sqrt{\frac{2h\left[\left(\frac{A_1}{A_2}\right)^2-1\right]}{g}}=180\text{ s}=3\text{ min}$$

(2) It is easy to see that if the water level were kept constant in the tank at a height of $h=1$ m from the orifice, the time of outflow of the same amount of water would be halved.

4.6. $d=1.4\times10^{-2}$ m. **4.7.** $p=2.5\times10^5$ N/m$^2\cong2.5$ at.

4.8. $v=1.4$ m/s. **4.9.** $\Delta h=1.6\times10^{-3}$ m $=1.6$ mm.

4.10. 3 times. **4.11.** $v=4.1$ m/s. **4.12.** $\eta=2$ N·s/m^2.

4.13. 4 min. **4.14.** $\eta=1.09$ N·s/m^2, $\nu=1.21\times10^{-3}$ m^2/s.

4.15. The velocity with which the level of the castor oil sinks in the vessel depends on the rate of flow of the oil through the capillary tube. The volume of the oil flowing during the time t through the capillary tube is determined from Poiseuille's formula

$$V=\frac{\pi r^4t\Delta p}{8l\eta} \qquad (1)$$

In our case the pressure difference at the ends of the capillary tube is due to the hydrostatic pressure of the layer of the liquid, i. e.,

$$\Delta p=\rho gh \qquad (2)$$

On the other hand

$$V=A_1v_1t=\pi r^2v_1t \qquad (3)$$

where v_1 is the velocity with which the oil flows through the capillary tube. From Eqs. (1), (2) and (3) we have

$$v_1=\frac{r^2\rho gh}{8l\eta} \qquad (4)$$

But since $v_1A_1=vA$, where v is the velocity with which the oil level sinks in the vessel, and A is the cross-sectional area of the vessel, then finally $v=\frac{r^4\rho gh}{8l\eta R^2}$. When $h=26$ cm $=0.26$ m, we have $v=3\times10^{-5}$ m/s.

4.16. $t=1.5$ min. **4.17.** At a distance of 1.1 cm. **4.18.** $D=4.6$ mm.

4.19. In the conditions of the problem the Reynolds number is equal to Re $=1{,}800$, i. e., Re $<3{,}000$ and motion is laminar.

4.20. $D\leqslant0.085$ m.

Chapter 2

MOLECULAR PHYSICS AND THERMODYNAMICS

5. Physical Fundamentals of the Molecular-Kinetic Theory and Thermodynamics

5.1. $T=280°\text{ K}=7°\text{ C}$. **5.2.** $V=7.6\times10^{-3}$ m³.
5.3. $M=1.13$ kg. **5.4.** $T=364°\text{ K}=91°\text{ C}$. **5.5.** $V=3.1\times10^{-2}$ m³.
5.6. $\Delta M=\dfrac{M_1\Delta p}{p_1}=7.5$ kg. **5.7.** $M=0.065$ kg. **5.8.** $M=1{,}200$ kg.
5.9. 1.1 times

5.10. (1) $pV=\dfrac{M}{\mu}RT_1=\dfrac{5\times10^{-4}}{2}\times8.31\times10^3\times273\text{ J}=567\text{ J}$ (1)

(2) $pV=\dfrac{M}{\mu}RT_2=775\text{ J}$ (2)

Take different values of V and use equations (1) and (2) to obtain the corresponding values of p.

5.11. See the solution to the previous problem. **5.12.** $\dfrac{M}{\mu}=0.4$ kmole

5.13. $p_1=1.08\times10^5$ N/m², $p_2=1.16\times10^5$ N/m².

5.14. When the capillary tube is in a horizontal position, each half contains air having a volume $V_0=Ah$ and a pressure p_0, where A is the cross-sectional area of the capillary tube. After the capillary tube is placed vertically, in its upper half the volume of the air $V_1=A(h+\Delta l)$ and the pressure is p_1. According to Boyle-Mariotte's law, $V_0p_0=V_1p_1$, or

$$hp_0=(h+\Delta l)\,p_1 \qquad (1)$$

Similarly for the bottom half of the capillary tube

$$hp_0=(h-\Delta l)\,p \qquad (2)$$

In this case the pressure p_2 in the bottom half of the tube is composed of the pressure p_1 and the pressure of the mercury column p_3, i. e.,

$$p_2=p_1+p_3 \qquad (3)$$

By solving equations (1), (2) and (3) simultaneously, we obtain

$$p_0=\frac{p_3(h-\Delta l)(h+\Delta l)}{2h\Delta l} \qquad (4)$$

In equation (4), p_0 will be expressed in the same units in which the pressure p_3 is measured. Let us express the pressure p_3 in millimetres of mercury column. In our case $p_3=200$ mm Hg, $h=\dfrac{L-l}{2}=0.4$ m and $\Delta l=0.1$ m. By inserting these values in Eq. (4), we get $p_0=375$ mm Hg.

5.15. According to Archimedes' law, the loss in weight of a body submerged in gas is equal to the weight of this gas in the volume of the body. The volume of lead with the mass M is $V_1=\frac{M}{\rho_1}$, where ρ_1 is the density of lead. The air in this volume weighs $m_1g=\frac{\mu pV_1g}{RT}=\frac{\mu pMg}{\rho_1 RT}$. The volume of cork with the mass M is $V_2=\frac{M}{\rho_2}$, where ρ_2 is the density of cork The air in this volume weighs $m_2g=\frac{\mu pMg}{\rho_2 RT}$ The actual weight of the lead $G_1=g(M+m_1)$ and that of the cork $G_2=g(M+m_2)$, and $\Delta G=G_2-G_1=g(m_2-m_1)=\frac{\mu pMg}{RT}\left(\frac{1}{\rho_2}-\frac{1}{\rho_1}\right)=$ $=58.6\ \text{N}\cong 6.0\ \text{kgf}$.

5.16. The resulting lifting force of the balloon is equal to the difference between the weight of the air in the volume of the balloon and the own weight of the balloon (the weight of its envelope and the hydrogen contained inside). Thus, $F=M_2g-(M_1g+x)$, where F is the resulting lifting force, M_2 the mass of the air in the volume of the balloon, M_1 the mass of the hydrogen in the volume of the balloon, and x is the weight of the envelope. According to the initial condition, $F=0$, and therefore $x=g(M_2-M_1)=g\frac{pV}{RT}(\mu_2-\mu_1)=$ $=\frac{4}{3}\pi r^3\frac{pg}{RT}(\mu_2-\mu_1)=0.096\ \text{N}=9.8\times10^{-3}\ \text{kgf}$.

5.17. $\rho=\frac{M}{V}=\frac{p\mu}{RT}=0.083\ \text{kg/m}^3$. 5.18. $\rho=0.081\ \text{kg/m}^3$.

5.19. $\mu=4$ kg/kmole. 5.20. $\rho=1.6\times10^{-14}\ \text{kg/m}^3$. 5.21. 1,400° K.

5.22. The state of a gas is determined from the following equations: before heating

$$p_1V_1=\frac{M}{\mu}RT_1 \qquad (1)$$

and after heating

$$p_2V_2=\frac{M}{\mu}RT_2 \qquad (2)$$

According to the given condition, $p_1=p_2=p$ The quantities sought can be found from equations (1) and (2):

$$(1)\ V_1=\frac{MRT_1}{\mu p}=2.4\times10^{-3}\ \text{m}^3;\ (2)\ T_2=\frac{\mu pV_2}{MR}=1{,}170^\circ\ \text{K};$$

$$(3)\ \rho_1=\frac{\mu p}{RT_1}=4.14\ \text{kg/m}^3;\ (4)\ \rho_2=\frac{\mu p}{RT_2}=1\ \text{kg/m}^3.$$

5.23. $p=1.55\times10^8\ \text{N/m}^2$, $\rho=500\ \text{kg/m}^3$.

5.24. $\rho=\frac{p\mu}{RT}$ When $T=\text{const}$, $\rho=C_1p$, i. e., ρ is directly proportional to p. When $p=\text{const}$, $\rho=\frac{C_2}{T}$ i. e., ρ is inversely proportional to T

5.25. According to Dalton's law, the pressure in the vessel after all the water is evaporated will be $p=p_1+p_2$, where p_1 is the pressure of the oxygen and p_2 the pressure of the water vapours According to Mendeleyev-Clapeyron's equation

$$p_1=\frac{M_1RT}{V\mu_1}=\frac{1.6\times 8.31\times 10^3\times 773}{1\times 32}\ \text{N/m}^2=3.2\times 10^5\ \text{N/m}^2$$

$$p_2=\frac{M_2RT}{V\mu_2}=\frac{0.9\times 8.31\times 10^3\times 773}{1\times 18}\ \text{N/m}^2=3.2\times 10^5\ \text{N/m}^2$$

and the total pressure $p=6.4\times 10^5$ N/m^2.

5.26. According to Dalton's law,

$$p=p_1+p_2 \tag{1}$$

where p_1 and p_2 are the partial pressures. If the temperature is constant $p_1(V_1+V_2)=p_0'V_1$ and $p_2(V_1+V_2)=p_0''V_2$, whence

$$p_1=\frac{p_0'V_1}{V_1+V_2} \tag{2}$$

and

$$p_2=\frac{p_0''V_2}{V_1+V_2} \tag{3}$$

Upon inserting Eqs. (2) and (3) into Eq. (1), we get

$$p=\frac{p_0'V_1+p_0''V_2}{V_1+V_2}=1.4\times 10^5\ \text{N/m}^2$$

5.27. $p=4.15\times 10^5$ N/m^2.

5.28. (1) $\mu=\dfrac{m_1+m_2}{\dfrac{m_1}{\mu_1}+\dfrac{m_2}{\mu_2}}=4.6$ kg/kmole, (2) $V=11.7\times 10^{-3}$ m^3.

5.29. $m=2.5\times 10^{-3}$ kg.

5.30. If the iodine molecules were not dissociated, the pressure in the vessel would be

$$p=\frac{MRT}{\mu V}=\frac{10^{-3}\times 8.31\times 10^3\times 1{,}273}{254\times 0.5\times 10^{-3}}\ \text{N/m}^2=625\ \text{mm Hg}$$

If the degree of dissociation is α, the vessel contains $2\alpha\dfrac{M}{\mu}$ kilomoles of atomic iodine I and $(1-\alpha)\dfrac{M}{\mu}$ kilomoles of molecular iodine I_2. The pressures they create are equal, respectively, to

$$p_1=\frac{2\alpha MRT}{\mu V}\quad\text{and}\quad p_2=\frac{(1-\alpha)MRT}{\mu V}$$

and the pressure of the mixture

$$p_{mix}=p_1+p_2=\frac{MRT}{\mu V}(2\alpha+1-\alpha)=(1+\alpha)\frac{MRT}{\mu V}=p(1+\alpha)$$

i. e.,

$$1+\alpha=\frac{p_{mix}}{p}=\frac{700}{625}=1.12\ \text{and}\ \alpha=0.12$$

5.31. $\dfrac{p_1}{p}=1.25$.

5.32. $\rho=1.2$ kg/m^3, $p_1=0.21\times10^5$ N/m^2, $p_2=0.79\times10^5$ N/m^2
5.33. $\rho=1.98$ kg/m^3.
5.34. (1) $m=1.67\times10^{-27}$ kg, (2) $m=6.65\times10^{-27}$ kg.
5.35. 5.6×10^{-23} N·s. 5.36. 3.3×10^{-23} N·s.
5.37. 2×10^{-23} kg·m/s. 5.38. 3.3×10^{22} 5.39. 7.5×10^{19} cm^{-3}.
5.40. 2×10^{27}. 5.41. 3.4×10^{6} cm^{-3}.

5.42. The gas pressure p in the vessel is related to the number of molecules n in a unit volume of this vessel by the ratio

$$p=nkT=\frac{NkT}{V} \qquad (1)$$

where N is the total number of molecules in the volume V. Since these N molecules form a monomolecular layer on the wall of the vessel,

$$N=\frac{A_1}{A} \qquad (2)$$

where

$$A_1=4\pi r^2 \qquad (3)$$

is the surface of the vessel and A—the cross-sectional area of one molecule. The volume of the vessel is

$$V=\frac{4}{3}\pi r \qquad (4)$$

Upon inserting Eqs. (2), (3) and (4) into Eq. (1), we obtain

$$p=\frac{3kT}{Ar} \qquad (5)$$

or after inserting the numerical values in Eq. (5)

$$p=2.4\ \text{N/m}^2=1.8\times10^{-2}\ \text{mm Hg}$$

5.43. If the degree of dissociation is α, the vessel contains $2\alpha\frac{M}{\mu}$ kilomoles of atomic iodine I and $(1-\alpha)\frac{M}{\mu}$ kilomoles of molecular iodine I_2 The total number of kilomoles in the vessel is equal to $2\alpha\frac{M}{\mu}+(1-\alpha)\frac{M}{\mu}$ and the number of the particles sought will be

$$N=N_A\left[2\alpha\frac{M}{\mu}+(1-\alpha)\frac{M}{\mu}\right]$$

Upon inserting the numerical values, we obtain $N=3.56\times10^{21}$
5.44. $N=4.5\times10^{23}$.
5.45. (1) $V=3.2\times10^{-3}$ m^3, (2) $p_1=7.37\times10^{-4}$ mm Hg, $p_2=2.63\times10^{-4}$ mm Hg, (3) $n=2.6\times10^{13}$ cm^{-3}.
5.46. $\sqrt{\overline{v^2}}=500$ m/s 5.47. $\frac{\sqrt{\overline{v_1^2}}}{\sqrt{\overline{v_2^2}}}=2.65$.
5.48. $\sqrt{\overline{v^2}}=5\times10^5$ m/s. 5.49. $n=4\ 2\times10^{18}$ cm^{-3}
5.50. $p=\frac{1}{3}\rho\overline{v^2}=5\times10^3$ N/m^2. 5.51 1.44×10^7 times

5.52. $m\sqrt{\overline{v^2}}=\sqrt{3kTm}=6.3\times10^{-24}$ kg·m/s

5.53. (1) $\sqrt{\overline{v^2}}=230$ m/s, (2) $N=1.9\times10^{23}$, (3) $\rho=5.0$ kg/m^3.

5.54. $\sqrt{\overline{v^2}}=4.6\times10^{-3}$ m/s. 5.55. $\rho=0.74$ kg/m^3.

5.56. (1) $\sqrt{\overline{v^2}}=1{,}900$ m/s, (2) $\mu=2$ kg/kmole. 5.57. $N=1.88\times10^{22}$.

5.58. The energy of the thermal motion of gas molecules is determined from the formula

$$E=\frac{M}{\mu}\frac{i}{2}RT \qquad (1)$$

For a biatomic gas $i=5$, where $i=3$ relates to translational motion of the molecules, and $i=2$ to rotational motion. Upon inserting the numerical values into Eq. (1), we obtain $E=3.7\times10^3$ J, including $E_{tr}=2.2\times10^3$ J and $E_{rot}=1.5\times10^3$ J.

5.59. $E_k=210$ J. 5.60. $E_{rot}=8.3\times10^4$ J. 5.61. $E=750$ J.

5.62. (1) $M=\frac{2E}{\overline{v^2}}=2.5\times10^{-3}$ kg

(2) $p=\frac{2E}{3V}=1.67\times10^5$ N/m^2

5.63. (1) $T=20{,}000$°K, (2) $T=900$°K.

5.64. $E=\frac{iMp}{2\rho}=5\times10^4$ J. 5.65. $N=1.3\times10^{19}$, $E=0.133$ J.

5.66. (1) $c_v=650$ J/kg·deg, (2) $c_p=910$ J/kg·deg.

5.67. (1) 800 J/kg·deg = 0.19 cal/g·deg,
(2) 1,025 J/kg·deg = 0.245 cal/g·deg,
(3) 970 J/kg·deg = 0.23 cal/g·deg,
(4) 1,040 J/kg·deg = 0.248 cal/g·deg,
(5) 103 J/kg·deg = 0.025 cal/g·deg.

5.68. 1.4. 5.69. $\mu=2$ kg/kmole.

5.70. $c_v=650$ J/kg·deg, $c_v=910$ J/kg·deg.

5.71. $c_v=693$ J/kg·deg, $c_p=970$ J/kg·deg.

5.72. It can be seen from the equation $2H_2+O_2=2H_2O$ that three kilomoles of biatomic gases after reaction produce two kilomoles of a triatomic gas. Therefore $C_v'=3\times\frac{5}{2}R$ and $C_p'=3\times\frac{7}{2}R$ before combustion, and $C_v''=2\times\frac{6}{2}R$ and $C_p''=2\times\frac{8}{2}R$ after combustion. Hence.

$$(1)\ \frac{C_v'}{C_v''}=1.25,\quad (2)\ \frac{C_p'}{C_p''}=1.31$$

5.73. The quantity of heat needed to heat $2\alpha\frac{M}{\mu}$ kilomoles of atomic oxygen and $(1-\alpha)\frac{M}{\mu}$ kilomoles of molecular oxygen at a constant pressure is

$$Q=2\alpha\frac{M}{\mu}C_p'\Delta t+(1-\alpha)\frac{M}{\mu}C_p''\Delta t=\frac{M}{\mu}C_p\Delta t$$

where C_p' and C_p'' are the molecular heat capacities of a monoatomic and a biatomic gas, respectively, and C_p is the molecular heat of the mixture (with $p=\text{const}$). Whence

$$2\alpha C_p'+(1-\alpha)C_p''=C_p$$

or

$$\alpha=\frac{C_p-C_p''}{2C_p'-C_p''} \quad (1)$$

Here, $C_p=c_p\mu=1{,}050\times 32$ J/kmole·deg $=33.6\times 10^3$ J/kmole·deg, $C_p'=20.78\times 10^3$ J/kmole·deg, $C_p''=29.08\times 10^3$ J/kmole·deg. Upon inserting these values in Eq. (1), we obtain $\alpha=0.36$.

5.74. $c_v=90$ J/kg·deg, $c_p=139$ J/kg·deg. **5.75.** $\alpha=23\%$.

5.76. $c_p=685$ J/kg·deg. **5.77.** $\frac{c_p}{c_v}=1.59$. **5.78.** $M=60$ kg.

5.79. (1) The amount of heat received by the gas can be found from the formula $\Delta Q=\frac{M}{\mu}C_p(T_2-T_1)$. To find T_2, let us write the equation of state of the gas before and after heating: $pV_1=\frac{M}{\mu}RT_1$ and $pV_2=\frac{M}{\mu}RT_2$. Hence $T_2=T_1\frac{V_2}{V_1}$. But $V_1=\frac{MRT_1}{\mu p}$ and, therefore

$$T_2=\frac{\mu V_2 p}{MR}=\frac{32\times 10\times 10^{-3}\times 3\times 10^5}{10\times 10^{-3}\times 8.31\times 10^3}\,^\circ\mathrm{K}=1{,}156^\circ\mathrm{K}$$

Thus,

$$T_2-T_1=1{,}156^\circ\mathrm{K}-283^\circ\mathrm{K}=873^\circ\mathrm{K}$$

and

$$\Delta Q=\frac{M}{\mu}C_p\,(T_2-T_1)=\frac{10^{-2}\times 29.08\times 10^3\times 873}{32}\ \mathrm{J}=7.9\times 10^3\ \mathrm{J}$$

(2) The energy of the gas before heating can be found from the formula

$$E_1=\frac{M}{\mu}\frac{i}{2}RT_1 \quad (1)$$

Since oxygen is a biatomic gas, $i=5$. Upon inserting the numerical values into formula (1), we find the energy of the gas before heating: $E_1=1.8\times 10^3$ J. The energy of the gas after heating is

$$E_2=\frac{M}{\mu}\frac{i}{2}RT_2=7.6\times 10^3\ \mathrm{J}$$

5.80. $Q=4.15\times 10^3$ J.

5.81. (1) At a constant pressure $Q=\frac{M}{\mu}C_p\Delta T$ But $pV_1=\frac{M}{\mu}RT_1$ and $pV_2=\frac{M}{\mu}RT_2$, whence $p\Delta V=\frac{M}{\mu}R\Delta T$ or $\frac{M}{\mu}\Delta T=\frac{p\Delta V}{R}$ Hence, $Q=C_p\frac{p\Delta V}{R}=700$ J.

(2) At a constant volume $Q=\frac{M}{\mu}C_v\Delta T$. But $p_1V=\frac{M}{\mu}RT_1$ and $p_2V=\frac{M}{\mu}RT_2$, whence $V\Delta p=\frac{M}{\mu}R\Delta T$ or $\frac{M}{\mu}\Delta T=\frac{V\Delta p}{R}$. Hence $Q=C_v\frac{V\Delta p}{R}=500$ J.

5.82. (1) $T=1{,}500^\circ$K, (2) $V=12.4\times 10^{-3}$ m^3, (3) $Q=12.4$ kJ.

5.83. $Q=545$ J.

5.84. $Q=\frac{M}{\mu}C_x\Delta T$, whence $C_x=\frac{\mu Q}{M\Delta T}=20.8\times 10^3$ J/kmole·deg≅5 cal/mole × × deg. Since oxygen is a biatomic gas, the value of C_x obtained shows that the gas was heated at a constant volume.

5.85. The amount of heat which should be imparted to the air can be determined from the formula

$$\Delta Q=\frac{M}{\mu}C_v\Delta T \qquad (1)$$

To find ΔT, let us write the equation of state of the gas before and after heating. Since $V_1=V_2=V$, then $p_1V=\frac{M}{\mu}RT_1$ and $p_2V=\frac{M}{\mu}RT_2$, whence $V\Delta p=\frac{M}{\mu}R\Delta T$, or

$$\Delta T=\frac{V\Delta p\mu}{MR} \qquad (2)$$

Upon inserting Eq. (2) into Eq. (1), we find

$$Q=C_v\frac{V\Delta p}{R}=\frac{i}{2}V\Delta p \qquad (3)$$

Substitution of the numerical data in Eq. (3) gives $Q=10^4$ J.

5.86. (1) $M=3.7\times 10^{-3}$ kg, (2) $\Delta E=3.3\times 10^{-21}$ J.

5.87 $Q=\frac{\rho VC_v\Delta t}{\mu}=208$ J.

5.88. (1) $T_2=2{,}500°$ K, (2) $Q=C_v\frac{V\Delta p}{R}=16.3$ kJ.

5.89 $i=6$. 5.90. (1) $Q=6.25$ kJ, (2) $T_2=4T_1$, (3) $p_2=4p_1$.

5.91. (1) $Q=102$ J, (2) $\sqrt{\bar{v^2}}=1.57\times 10^3$ m/s, (3) $p_2=1.33\times 10^5$ N/m², (4) $\rho_1=\rho_2=0.164$ kg/m³, (5) $E=4\times 10^2$ J

5.92. $Q=155$ J.

5.93 (1) $\bar{v}=579$ m/s, (2) $\sqrt{\bar{v^2}}=628$ m/s, (3) $v_{pr}=513$ m/s.

5.94. $T=83°$ K $=-190°$ C.

5.95. The molecules are distributed by velocities according to the formula

$$\frac{\Delta N}{N}=\frac{4}{\sqrt{\pi}}e^{-u^2}u^2\Delta u \qquad (1)$$

where u is the relative velocity In our case $v=100$ m/s and $\Delta v=10$ m/s. The most probable velocity $v_{pr}=\sqrt{\frac{2RT}{\mu}}=376$ m/s. Hence $u=\frac{v}{v_{pr}}=\frac{100}{376}$ and $u^2=0.071$, $e^{-u^2}=0.93$ and $\Delta u=\frac{10}{376}$ Then, formula (1) gives us $\frac{\Delta N}{N}=$ $=\frac{4}{\sqrt{\pi}}0.93\times 0.071\times\frac{10}{376}=0.004=0.4\%$. Thus, the number of molecules whose velocities lie within this range is 0.4 per cent of the total number of molecules. This problem can also be solved with the aid of a diagram $\frac{\Delta N}{N\Delta u}=f(u)$ (see Fig. 83) plotted from the data in Table 9 on p. 62. In our case $u=0.27$ The diagram shows that $\frac{\Delta N}{N\Delta u} \cong 0.16$ corresponds to this value of u. Since in our case $\Delta u = 0.027$, then $\frac{\Delta N}{N} = 0.16 \times 0.027 = 0.004 = 0.4$ per cent.

5.96. $\frac{\Delta N}{N} = 2.8\%$ 5.97 $\frac{\Delta N}{N} = 4.5\%$

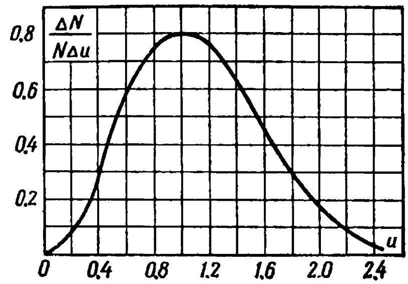


Fig. 83

5.98. $\frac{\Delta N_2}{\Delta N_1} = 1.1$ for any gas at any temperature.

5.99. (1) $v_{pr} = 487$ m/s and $\frac{\Delta N}{N} = 3.4\%$, (2) $v_{pr} = 731$ m/s and $\frac{\Delta N}{N} = 2.2\%$. In this way, as the temperature increases, the maximum of the distribution curve shifts to the right, and its value diminishes.

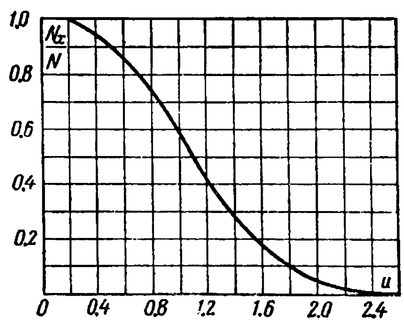


Fig. 84

5.100. Since the range of velocities is great in this problem, Maxwell's formula cannot be used. To solve this problem, proceed as follows: find the numbers of the molecules N_1 and N_2 whose velocities are greater than v_1 and v_2, respectively. Obviously, the number of molecules sought will be $N_x = N_1 - N_2$. The numbers N_1 and N_2 can be determined from a diagram $\frac{N_x}{N} = F(u)$ plotted according to the data in Table 10 on p. 62 (see Fig. 84) In our problem $v_{pr}=\sqrt{\frac{2RT}{\mu}}=500$ m/s. Hence, $u_1=\frac{300}{500}=0.6$ and $u_2=\frac{800}{500}=1.6$. From Fig. 84 for these values of u we find $\frac{N_1}{N}=0.87=87\%$ and $\frac{N_2}{N}=0.17=17\%$, respectively. The data obtained show that 87% of all the molecules have velocities exceeding 300 m/s, and only 17% of the molecules have velocities exceeding 800 m/s. Hence the relative number of molecules whose velocities range from 300 m/s to 800 m/s is $\frac{N_x}{N}=87\%-17\%=70\%$.

5.101. (1) $\frac{N_1}{N}=57\%$, (2) $\frac{N_2}{N}=43\%$. The results of the solution show that the curve of distribution of the molecules by velocities is asymmetrical.

5.102. $N_x=1.9\times10^{22}$.

5.103. For a molecule to have a kinetic energy of translational motion equal to E_0, it should have a velocity of v_0, which satisfies the equation $\frac{mv_0^2}{2}=E_0$ Hence $v_0=\sqrt{\frac{2E_0}{m}}$ Since the most probable velocity $v_{pr}=\sqrt{\frac{2RT}{\mu}}=$ $=\sqrt{\frac{2kT}{m}}$, the relative velocity of this molecule is $u=\frac{v_0}{v_{pr}}=\sqrt{\frac{E_0}{kT}}=1.73$. Let us use the diagram in Fig. 84 to find the relative number of molecules $\frac{N_x}{N}$ whose relative velocity is greater than the velocity $u=1.73$. The diagram shows that $\frac{N_x}{N}=0.12$. Thus, 12% of the oxygen molecules at the given temperature have a kinetic energy which exceeds the energy E_0 The total number of oxygen molecules in the vessel $N=\frac{M}{\mu}N_A=1.5\times10^{23}$. Hence, the number of molecules sought $N_x=0.12N=1.8\times10^{22}$.

5.104. (1) $T=7{,}730°$ K. (2) According to the given condition, $\frac{N_x}{N}=0.5$. The diagram in Fig. 84 shows that $\frac{N_x}{N}=0.5$ and this corresponds to a relative velocity of $u=1.1$. But $u=\sqrt{\frac{E_0}{kT}}$ (see the solution to the previous problem), whence $T=\frac{E_0}{ku^2}=9{,}600°$ K.

5.105. $T=15{,}700°$ K.

5.106. The pressure of the gas p diminishes with the altitude h according to the law $p=p_0e^{-\frac{\mu gh}{RT}}$, where p_0 is the gas pressure at the altitude $h=0$. In our case $\mu=29$ kg/kmole, $h=3.25\times10^3$ m, $R=8.31\times10^3$ J/kmole·deg, $T=278°$ K, hence $\frac{\mu gh}{RT}=0.4$. Since $e^{-0.4}=0.67$, we finally obtain $p=760\times$ $\times0.67$ mm Hg $=510$ mm Hg.

5.107. $h=2.3$ km.

5.108. $p_1=0.354$ atm, $p_2=0.713$ atm, $\Delta p=0.36$ atm.

5.109. 1.7 times. **5.110.** (1) 1.28 kgf, (2) 0.78 kgf.

5.111. (1) $h=5.5$ km, (2) $h=80$ km.

5.112. The barometric formula is

$$p=p_0e^{-\frac{\mu gh}{RT}} \quad (1)$$

The concentration (number of particles in a unit volume) is equal to $n=\frac{p}{kT}$, whence

$$p=nkT \quad (2)$$

By inserting Eq. (2) into Eq. (1), we obtain, respectively, for the altitudes h_1 and h_2: $n_1=n_0e^{-\frac{\mu gh_1}{RT}}$ and $n_2=n_0e^{-\frac{\mu gh_2}{RT}}$, whence

$$\frac{n_1}{n_2}=e^{-\frac{\mu g(h_1-h_2)}{RT}}=e^{-\frac{\mu g(h_2-h_1)}{RT}} \text{ or}$$

$$\log_e\frac{n_1}{n_2}=\frac{\mu g(h_2-h_1)}{RT} \quad (3)$$

Since the mass of a particle $m=\frac{\mu}{N_A}$, formula (3) can be written $\log_e\frac{n_1}{n_2}=$ $=\frac{N_Amg(h_2-h_1)}{RT}$, whence, taking into account the correction for Archimedes' law, we finally obtain

$$N_A=\frac{RT\log_e\frac{n_1}{n_2}}{gV(\rho-\rho')(h_2-h_1)}=6.1\times10^{26}\text{ kmole}^{-1}$$

where ρ is the density of gamboge and ρ' that of the liquid.

5.113. $\bar{l}=8.5\times10^{-4}$ m. 5.114. $\bar{l}=5.6$ km. 5.115. $\bar{l}=9.3\times10^{-8}$ m.

5.116. $\bar{z}=4.9\times10^{5}$ s^{-1}. 5.117. $\bar{z}=2.47\times10^{9}$ s^{-1}.

5.118. $Z=3\times10^{31}$. 5.119. 2.3 times. 5.120. $l=10^{-6}$ m.

5.121 $\bar{l}=\frac{\mu}{\sqrt{2}\,\pi\sigma^2N_A\rho}=1.8\times10^{-6}$ m. 5.122. $\bar{l}=14.2$ cm.

5.123. The mean number of collisions per second of the oxygen molecules can be found from the formula $\bar{z}=\frac{\bar{v}}{\bar{l}_2}$, where $\bar{v}=\sqrt{\frac{8RT}{\pi\mu}}$ and $\bar{l}_2=\bar{l}_1\frac{p_1}{p_2}$. Thus,

$$\bar{z}=\frac{\sqrt{\frac{8RT}{\pi\mu}}}{\bar{l}_1\frac{p_1}{p_2}} \quad (1)$$

According to the initial condition, $\frac{p_1}{p_2}=100$, $\bar{l}_1=9.5\times10^{-8}$ m, and $T=273°$ K. Upon inserting these data into Eq. (1), we obtain $\bar{z}=4.5\times10^{7}$ s^{-1}.

5.124. $\bar{z}=9.6\times10^{9}$ s^{-1}. 5.125. $\bar{l}=2.3\times10^{-8}$ m.

5.126. $\sigma=\sqrt{\frac{\mu}{\sqrt{2}\,N_A\pi\bar{l}\rho}}=3.5\times10^{-10}$ m.

5.127 $\tau=1.6\times10^{-7}$ s

5.128. $\rho=1.6\times10^{-9}$ kg/m^3, $n=3.3\times10^{10}$ cm^{-3}, $l=76.0$ m.

5.129. So that the molecules do not collide with each other, the mean free path should not be smaller than the diameter of the vessel, i. e., $\bar{l} \geqslant D \geqslant \frac{1}{\sqrt{2}\pi\sigma^2 n}$. Hence $n \leqslant \frac{1}{\sqrt{2}\pi\sigma^2 D}$. Upon introducing the numerical values, we obtain $n \leqslant 1.7 \times 10^{13}$ cm^{-3}.

5.130. (1) 7×10^{-3} mm Hg, (2) 7×10^{-4} mm Hg, (3) 7×10^{-5} mm Hg.

5.131. $p \leqslant 3 \times 10^{-3}$ mm Hg. **5.132.** $\rho \leqslant 9.4 \times 10^{-7}$ kg/m^3.

5.133. $\bar{z} = \frac{\bar{v}}{\bar{l}} \sqrt{\frac{8}{3\pi}} = 9.2 \times 10^7$ s^{-1}

5.134. $D = 0.91 \times 10^{-4}$ m^2/s. **5.135.** $D = 8.4 \times 10^{-6}$ m^2/s.

5.136. $D = \frac{1}{3}\bar{v}\bar{l} = \frac{1}{3}\sqrt{\frac{8RT}{\pi\mu}} \frac{kT}{\sqrt{2}\pi\sigma^2 p}$ At a constant pressure $D = CT^{3/2}$

The diffusion coefficient versus the temperature at $p = $ const is shown in Fig. 85.

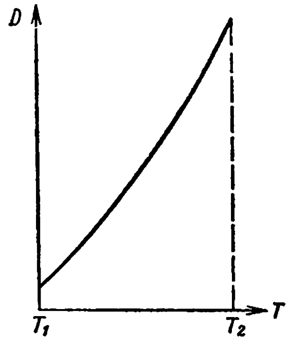


Fig. 85

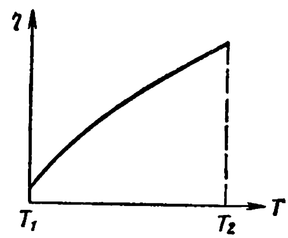


Fig. 86

5.137. $M = 2 \times 10^{-6}$ kg. **5.138.** $M = 9.7 \times 10^{-8}$ kg.

5.139. $\bar{l} = 1.84 \times 10^{-7}$ m. **5.140.** $\eta = 1.78 \times 10^{-5}$ N·s/m^2.

5.141. We have

$$\eta = \frac{1}{3}\bar{v}\bar{l}\rho \qquad (1)$$

where $\bar{v} = \sqrt{\frac{8RT}{\pi\mu}}$ is the arithmetic mean velocity of the molecules, $\bar{l} = \frac{kT}{\sqrt{2}\pi\sigma^2 p}$ is the mean free path and $\rho = \frac{p\mu}{RT}$ is the gas density. Substitution in Eq. (1) gives $\eta = \frac{2k}{3\pi\sigma^2}\sqrt{\frac{\mu T}{R\pi}}$, whence $\sigma^2 = \frac{2k}{3\pi\eta}\sqrt{\frac{\mu T}{R\pi}} = 9 \times 10^{-20}$ m^2 and $\sigma = 3 \times 10^{-10}$ m.

5.142. $\eta = \frac{1}{3}\bar{v}\bar{l}\rho$. Upon inserting here the expressions for $\bar{v}$, $\bar{l}$ and ρ, we find $\eta = C\sqrt{T}$, where C is a certain constant. The coefficient of internal friction η versus the temperature T is shown in Fig. 86.

5.143. $D = 1.48 \times 10^{-5}$ m^2/s, $\eta = 1.85 \times 10^{-5}$ kg/m·s.

5.144. 1.07 times. **5.145.** $n=\frac{N_0\eta}{\mu D}=1.8\times10^{25}$ m^{-3}.

5.146. (1) $\rho=1.6$ kg/m^3, (2) $\bar{l}=8.35\times10^{-8}$ m, (3) $\bar{v}=440$ m/s.

5.147. $v=2.72$ m/s. **5.148.** $F=0.045$ N.

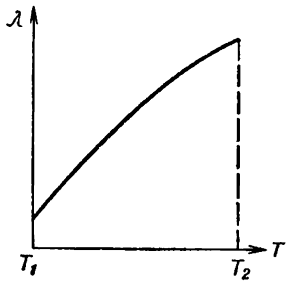


Fig. 87

5.149. $\eta=\frac{F(R-r)}{4\pi^2\nu h\,Rr}=1.8\times10^{-5}$ N·s/m^2.

5.150. $\lambda=0.090$ W/m·deg.

5.151. $\lambda=13.2\times10^{-3}$ W/m·deg $=11.3\times10^{-3}$ kcal/m·h·deg.

5.152. $\lambda=\frac{1}{3}\bar{v}\,\bar{l}c_v\rho$. By inserting the expressions for $\bar{v}$, $\bar{l}$ and ρ, we find $\lambda=C\sqrt{T}$. The nature of the relation $\lambda=f(T)$ is shown in Fig. 87.

5.153. $D=\frac{\lambda VN_A}{c_vN}=2\times10^{-5}$ m^2/s.

5.154. (1) $\frac{D_1}{D_2}=0.8$, (2) $\frac{\eta_1}{\eta_2}=1.25$, (3) $\frac{\lambda_1}{\lambda_2}=0.96$.

5.155. $p=1.26$ N/m$^2=0.0096$ mm Hg.

5.156. (1) The thermal conductivity coefficient of the air will begin to depend on the pressure when $\bar{l}=d$, where d is the distance between the walls of the bottle. We have $\bar{l}=\frac{kT}{\sqrt{2}\pi\sigma^2p}$, whence $p=\frac{kT}{\sqrt{2}\pi\sigma^2 d}$ when $\bar{l}=d$. Upon inserting the numerical values, we get $p=7.35\times10^{-3}$ mm Hg.

(2) (a) $\lambda=13.1\times10^{-3}$ W/m·deg. (b) If $p=10^{-4}$ mm Hg, the mean free path $\bar{l}$ is greater than the distance between the walls of the bottle. Hence $\lambda=\frac{1}{3}d\bar{v}\rho c_v=$

$$=\frac{1}{3}d\sqrt{\frac{8RT}{\pi\mu}}\,\frac{p\mu}{RT}\,\frac{Ri}{r\mu}=\frac{1}{6}dpi\sqrt{\frac{8R}{\pi\mu T}}=1.78\times10^{-4}\ \text{W/m·deg}.$$

(3) In our case $Q=\lambda\frac{\Delta T}{\Delta x}\Delta A\Delta t$. But $\Delta A=2\pi rh$, where $r=\frac{r_1+r_2}{2}$ Hence $Q=\lambda\frac{\Delta T}{\Delta x}2\pi rh\,\Delta t$. Upon inserting the numerical values, we obtain: (a) $Q=188$ J $=45$ cal, and (b) $Q=2.55$ J $=0.61$ cal The actual losses will be greater due to convection.

5.157. $Q=5.7$ kcal. **5.158.** $Q=78$ J.

5.159. (1) $Q=\frac{M}{\mu}C_p\Delta T=7.92\times10^3$ J $=1,890$ cal,

(2) $\Delta E=\frac{i}{2}p\Delta V=5,660$ J $=1,350$ cal,

(3) $W=p\Delta V=2.26\times10^3$ J $=540$ cal.

Thus, as could be expected, $Q=\Delta E+W$ on the basis of the first law of thermodynamics.

5.160. (1) $W=8.1\times10^3$ J, (2) $\Delta E=20.2\times10^3$ J, (3) $Q=28.3\times10^3$ J ($Q=\Delta E+W$).

5.161. $\Delta E=1,000$ J.

5.162. (1) $\Delta E=2,500$ kJ, (2) $W=830$ kJ, (3) $Q=3,330$ kJ.

5.163. $W=600$ J. **5.164.** $Q=W\left(\frac{i}{2}+1\right)=550$ J.

5.165. $\Delta t=57°$ C. **5.166.** $W=13.2$ J, $\Delta E=39.6$ J.

5.167. (1) $Q=3.32\times10^6$ J, (2) $\Delta E=2.49\times10^6$ J, (3) $W=8.31\times10^5$ J.

5.168. (1) $Q=10.4$ J, (2) $\Delta h=2.8$ cm. 5.169. $Q=360$ J.
5.170. $W=720$ J. 5.171. 2.72 times. 5.172. $\sqrt{\overline{v^2}}=500$ m/s.
5.173. (1) $W=70$ J, (2) $Q=W=70$ J $=16.8$ cal.
5.174. $W=2.2\times10^5$ J. 5.175. $T=207°$ K $=-66°$ C.
5.176. $p_1=9.5\times10^4$ N/m². 5.177. $T=865°$ K $=592°$ C.
5.178. $i=5$. 5.179. $t=123°$ C, $p=52.8\times10^5$ N/m².
5.180. $T=780°$ K. 5.181. $\frac{c_p}{c_v}=1.4$.
5.182. (1) The diagram of the process is shown in Fig. 88. (2) $V_2=0.25$ litre, $p_2=1.32$ at.
5.183. $\frac{c_p}{c_v}=1.4$.
5.184. In the adiabatic process $\Delta E=-W$. (1) $\Delta E=\frac{M}{\mu}\frac{i}{2}R(T_2-T_1)$. The temperature T_2 can be found from Poisson's equation. After calculations we find that $\Delta E=-2.69\times10^6$ J. (2) $W=-\Delta E=2.69\times10^6$ J.

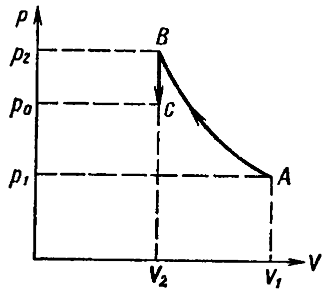


Fig. 88

5.185. The work in adiabatic compression

$$W_{ad}=\frac{M}{\mu}\frac{RT_1}{(\varkappa-1)}\left[1-\left(\frac{V_1}{V_2}\right)^{\varkappa-1}\right]$$

and the work in isothermal compression

$$W_{is}=\frac{M}{\mu}RT\log_e\frac{V_2}{V_1}$$

Whence

$$\frac{W_{ad}}{W_{is}}=\frac{\left[1-\left(\frac{V_1}{V_2}\right)^{\varkappa-1}\right]}{(\varkappa-1)\log_e\frac{V_2}{V_1}} \qquad (1)$$

In our case $V_1=10^{-2}$ m³, $V_2=2\times10^{-3}$ m³, $\varkappa-1=0.4$. Upon introducing these data in Eq. (1), we obtain $\frac{W_{ad}}{W_{is}}=1.4$, i. e., isothermal compression is better.
5.186. By 7°. 5.187. 1.15 times.
5.188. (1) $p_2=5$ atm, $T_2=273°$ K, $W=-1{,}140$ J,
(2) $p_2=9.5$ atm, $T_2=520°$ K, $W=-1{,}590$ J.
5.189. (1) $T_2=T_1=313°$ K $=40°$ C, $p_2=2.0\times10^5$ N/m², $W=-1{,}800$ J,
(2) $T_2=413°$ K $=140°$ C, $p_2=2.6\times10^5$ N/m², $W=-2{,}080$ J.
5.190. (1) 2 times, (2) 1.64 times.
5.191. The monoatomic gas will be heated more, 1.2 times.
5.192. (1) $\frac{V_2}{V_1}=1.33$, (2) $T_2=270°$ $K=-3°$ C, (3) $W=2.3\times10^4$ J.
5.193. (1) $p=\frac{A}{V}$, (2) $p=\frac{B}{V^\varkappa}$, where $\varkappa=\frac{c_p}{c_v}$. The gas pressure p versus the volume V is shown in Fig. 89 by curve *1* for isothermal expansion and by curve *2* for adiabatic expansion.
5.194. (1) $Q=1.55$ kJ, $W=0.92$ kJ, $\Delta E=0.63$ kJ,
(2) $Q=1.88$ kJ, $W=1.25$ kJ, $\Delta E=0.63$ kJ.
5.195. The heat engine operating according to the Carnot cycle performs work equal to $W=Q_1-Q_2=\eta Q_1$, where Q_1 is the amount of heat received by the engine from the hot source, Q_2 the amount of heat rejected to the cold sink and

η the efficiency of the engine. In our case $\eta=\frac{T_1-T_2}{T_1}=0.25$. Hence $W=\eta Q_1=$ $=150$ cal $=630$ J, and $Q_2=Q_1-W=450$ cal $=1{,}880$ J.

5.196. $\eta=18$ per cent.

5.197. (1) $\eta=26.8$ per cent, (2) $Q_1=27.4\times10^4$ J, (3) $Q_2=20.0\times10^4$ J.

5.198. (1) $\eta=20$ per cent, (2) $W=1.26\times10^3$ J.

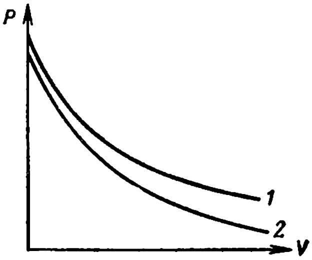


Fig. 89

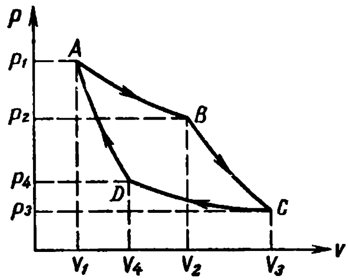


Fig. 90

5.199. The equation of the isothermal line AB (Fig. 90) takes the form:

$$pV=\frac{M}{\mu}RT_1 \tag{1}$$

The coordinates of point A satisfy this equation, i.e.,

$$p_1V_1=\frac{M}{\mu}RT_1$$

whence

$$\frac{M}{\mu}=\frac{p_1V_1}{RT_1}=\frac{7\times1.013\times10^5\times2\times10^{-3}}{8.31\times10^3\times400}\text{ kmole}=0.427\times10^{-3}\text{ kmole}$$

and then equation (1) becomes

$$pV=0.427\times10^{-3}\times8.31\times10^3\times400\text{ J}=1{,}420\text{ J} \tag{2}$$

For point B

$$p_2=\frac{pV}{V_2}=\frac{1{,}420}{5\times10^{-3}}\text{ N/m}^2=2.8\text{ atm}$$

Since the coordinates of points B and C satisfy the adiabatic line BC, then $p_2V_2^{\varkappa}=p_3V_3^{\varkappa}$, whence $p_3=p_2\left(\frac{V_2}{V_3}\right)^{\varkappa}=1.44$ atm. The equation of the isothermal line DC is $pV=\frac{M}{\mu}RT=p_3V_3=1.44\times1.013\times10^5\times8\times10^{-3}$ J $=1{,}170$ J. Hence $T_2=330°$ K. Since the coordinates of points D and A should satisfy the equation of adiabatic line DA, then $\left(\frac{V_4}{V_1}\right)^{\varkappa-1}=\frac{T_1}{T_2}$, whence $V_4=3.22\times10^{-3}$ m³ and

$$p_4=\frac{1{,}170}{3.22\times10^{-3}\times1.013\times10^5}\text{ atm}=3.6\text{ atm}$$

(1) Thus, $V_1=2$ litres, $p_1=7$ atm, $V_2=5$ litres, $p_2=2.8$ atm, $V_3=8$ litres, $p_3=1.44$ atm, $V_4=3.22$ litres, $p_4=3.6$ atm.

(2) The work in the isothermal process AB is

$$W_1=\frac{M}{\mu}RT_1\log_e\frac{V_2}{V_1}=0.427\times10^{-3}\times8.31\times10^3\times400\times0.916\ \text{J}=1{,}300\ \text{J}$$

the work in the adiabatic process BC is

$$W_2=\frac{M}{\mu}\frac{RT_1}{(\varkappa-1)}\left[1-\left(\frac{V_1}{V_2}\right)^{\varkappa-1}\right]=\frac{M}{\mu}\frac{RT_1}{\varkappa-1}\left(1-\frac{T_2}{T_1}\right)=620\ \text{J}$$

the work in the isothermal process CD

$$W_3=\frac{M}{\mu}RT_2\log_e\frac{V_4}{V_3}=-1{,}070\ \text{J}$$

the work in the adiabatic process DA

$$W_4=\frac{M}{\mu}\frac{RT_2}{\varkappa-1}\left(1-\frac{T_1}{T_2}\right)=-620\ \text{J}$$

(3) The work during the entire cycle $W=\sum W_i=230$ J.

(4) The cycle efficiency $\eta=\frac{T_1-T_2}{T_1}=0.175=17.5\%$

(5) The amount of heat received from the hot source per cycle is

$$Q_1=\frac{W}{\eta}=\frac{230}{0.175}=1{,}300\ \text{J}=312\ \text{cal}$$

(6) The amount of heat rejected to the cold sink per cycle is

$$Q_2=Q_1-W=1{,}070\ \text{J}=256\ \text{cal}$$

5.200. 2.1 times.

5.201. During the reverse cycle the external forces perform the work W. The quantity of heat Q_2 rejected from the cold body together with the work W done is equal to the amount of heat Q imparted to the hot body.

(1) $\eta=\frac{T_1-T_2}{T_1}=0.093$,

(2) $Q_2=Q_1-W=\frac{W}{\eta}-W=\frac{1-\eta}{\eta}W$. Here $W=37{,}000$ J $=37$ kJ. Hence $Q_2=\frac{1-\eta}{\eta}W=360$ kJ.

(3) $Q_1=Q_2+W=397$ kJ. Thus, the refrigerator will transmit to the hot body 397 kJ per cycle, of which 37 kJ are due to the conversion of the work into heat and 360 kJ are transferred from the cold body.

5.202. The quantities η_1, η_2 and η_3 are interrelated by the formulas $\eta_1=\frac{1}{1-\eta_3}$, and $\eta_2=\frac{1-\eta_3}{\eta_3}$. In our problem $\eta_1=1.09$, $\eta_2=11.0$, and $\eta_3=0.083$.

5.203. 4.94 kg.

5.204. The heat Q_0 can perform the work $W=\eta_2 Q_0$, where η_2 is the efficiency of the heat engine; $\eta_2=\frac{T_1-T_2}{T_1}$. The refrigerator will impart to the room the heat $Q_1=\frac{W}{\eta_3}$, where η_3 is the refrigerator efficiency, and $\eta_3=\frac{T_1'-T_2'}{T_1'}$. Hence $\frac{Q_1}{Q_0}=\frac{\eta_2 Q_0}{\eta_3 Q_0}=\frac{\eta_2}{\eta_3}=\frac{(T_1-T_2)T_1'}{(T_1'-T_2')T_1}$. Upon inserting the numerical data, we get $\frac{Q_1}{Q_0}=3$, or $\frac{Q_0}{Q_1}=\frac{1}{3}$, i. e., when firewood is burned in the furnace the room receives one-third of the heat received from the refrigerator consuming the same amount of firewood.

5.205. A glance at Fig. 9 shows that

$$W=p_1(V_1-V_0)+\frac{p_1V_1}{\varkappa-1}\left[1-\left(\frac{V_1}{V_2}\right)^{\varkappa-1}\right]-p_0(V_2-V_0)$$

Upon inserting numerical data, we get $W=1.920$ J.

5.206. $\eta_1=20\%$ and $\eta_2=30\%$ **5.207.** 104 cycles.

5.208. $\eta=\frac{W}{Q_1}$, where W is the total work during the entire cycle and Q_1 is the amount of heat evolved during combustion of the fuel. Since $W_{AB}=W_{BA}$ and $W_{CD}=W_{EB}=0$, then

$$W=W_{BC}-W_{DE}=\frac{M}{\mu}\frac{R(T_0-T_3)}{\varkappa-1}\left[1-\left(\frac{V_1}{V_2}\right)^{\varkappa-1}\right]$$

But

$$\frac{R}{\varkappa-1}=C_v \text{ and } \left(\frac{V_1}{V_2}\right)^{\varkappa-1}=\frac{T_1}{T_0}=\frac{T_2}{T_3}$$

therefore

$$W=\frac{M}{\mu}C_v(T_0-T_3)\left(1-\frac{T_2}{T_3}\right)$$

Further $Q=\frac{M}{\mu}C_v(T_2-T_1)$. Hence

$$\eta=\frac{W}{Q_1}=\frac{(T_0-T_3)\left(1-\frac{T_2}{T_3}\right)}{T_2-T_1}=\frac{T_2-T_3}{T_2}=1-\frac{T_3}{T_2}=$$

$$=1-\frac{1}{\left(\frac{V_1}{V_2}\right)^{\varkappa-1}}=0.412=41.2\%$$

5.209. $p=9.3\times10^5$ N/m² and $T=686°$ K $=413°$ C. **5.210.** $n=1.3$.

5.211. (1) Obviously $V_1-V_2=As$, where A is the cross-sectional area of the cylinder and s is the piston stroke. On the other hand $\left(\frac{V_1}{V_2}\right)^{\varkappa}=\frac{p_2}{p_1}$ Upon solving these two equations with respect to V_2 and inserting numerical data, we find that $V_2=1.76\times10^{-4}$ m³,

(2) $\frac{T_1}{T_2}=\left(\frac{p_1}{p_2}\right)^{\frac{\varkappa-1}{\varkappa}}$, whence $T_2=680°$ K $=407°$ C,

(3) $W=\frac{p_1V_1}{\varkappa-1}\frac{T_1-T_2}{T_1}$, but $V_1=As+V_2=1.04\times10^{-3}$ m³ and $W=243$ J

5.212. (1) 36.7%, (2) 44.6%, (3) 49.6%.

5.213. Knowing the consumption and calorific heat of the petrol, let us find the actual efficiency $\eta_{act}=0.216\approx22\%$. The theoretical efficiency is

$$\eta=1-\frac{1}{\left(\frac{V_1}{V_2}\right)^{n-1}}=0.3=30\%$$

Thus the losses due to friction, conductivity, etc., amount to $30\%-22\%=8\%$.

5.214. The work performed during a full cycle is

$$W=Q_1-Q_2 \tag{1}$$

where Q_1 is the amount of heat evolved during fuel combustion (CD in Fig. 11) and Q_2 the amount of heat received by the medium (EB). But since CD is an isobaric line, then

$$Q_1=\frac{M}{\mu}C_p(T_2-T_1) \tag{2}$$

where T_1 is the temperature at the beginning of isobaric expansion and T_2 at its end. Since section EB is an isochoric line, then

$$Q_2=\frac{M}{\mu}C_v(T_3-T_0) \tag{3}$$

where T_3 is the temperature at the beginning of the isochoric process and T_0 at its end. Hence, according to Eq. (1)

$$W=\frac{M}{\mu}C_v\,[\varkappa(T_2-T_1)-(T_3-T_0)] \tag{4}$$

and then the efficiency

$$\eta=\frac{W}{Q_1}=1-\frac{1}{\varkappa}\,\frac{T_3-T_0}{T_2-T_1} \tag{5}$$

Equation (5) may be written in another form. The temperatures T_0, T_1 and T_3 can be expressed through T_2. For the isobaric line CD we have $\frac{T_2}{T_1}=\frac{V_3}{V_1}=\beta$ is the degree of isobaric expansion and, consequently, $T_1=\frac{T_2}{\beta}$ Further, for the adiabatic line DE, $\frac{T_2}{T_3}=\left(\frac{V_2}{V_3}\right)^{\varkappa-1}=\delta^{\varkappa-1}$, where δ is the degree of adiabatic expansion, and therefore $T_3=\frac{T_2}{\delta^{\varkappa-1}}$ For the adiabatic line BC we have $\frac{T_1}{T_0}=\left(\frac{V_2}{V}\right)^{\varkappa-1}=\varepsilon^{\varkappa-1}$, where ε is the degree of adiabatic compression, and, consequently, $T_0=\frac{T_1}{\delta^{\varkappa-1}}=\frac{T_2}{\beta\varepsilon^{\varkappa-1}}$ Upon inserting the obtained values of T_0, T_1, T_3 in Eq. (5) and taking into account that $\beta=\frac{\varepsilon}{\delta}$, we finally obtain

$$\eta=1-\frac{\beta^{\varkappa-1}}{\varkappa\varepsilon^{\varkappa-1}(\beta-1)}$$

5.215. We have

$$\eta=\frac{W}{Q}=\frac{Pt}{mq_0} \tag{1}$$

where m is the mass of the fuel and q_0 its calorific value. On the other hand,

$$\eta=1-\frac{\beta^{\varkappa}-1}{\varkappa\varepsilon^{\varkappa-1}(\beta-1)} \tag{2}$$

In our case $\beta=\frac{\varepsilon}{\delta}=\frac{16}{6.4}=2.5$, $\varkappa=1.3$, $\beta^{\varkappa}=3.29$, $\beta^{\varkappa}-1=2.29$, $\varepsilon^{\varkappa-1}=2.30$ and $\beta-1=1.5$. Upon inserting these data in formula (2), we obtain $\eta=0.49=$ $=49\%$. Hence, $m=5.9$ kg.

5.216. The change in entropy is determined from the formula

$$S_2-S_1=\int_1^2 \frac{dQ}{T} \tag{1}$$

where S_1 and S_2 are the entropies in the first and the second states. In the given case the total change in the entropy is the sum of its changes in the separate processes.

(1) Heating of the mass m of ice from the temperature T_1 to the temperature T_2. Since $dQ=mc_1dT$, where c_1 is the specific heat of the ice, we get from formula (1)

$$\Delta S_1=mc_1 \log_e \frac{T_2}{T_1}$$

(2) Melting of the mass m of ice at the temperature T_2. Since $\int dQ=mH$, where H is the specific heat of fusion, then according to formula (1),

$$\Delta S_2=\frac{mH}{T_2}$$

(3) Heating of the mass m of water from T_2 to T_3

$$\Delta S_3=mc_2 \log_e \frac{T_3}{T_2}$$

where c_2 is the specific heat of the water.

(4) Evaporating of the mass m of water at the temperature T_3

$$\Delta S_4=\frac{mr}{T_3}$$

where r is the specific heat of vaporization.

The total change in the entropy

$$\Delta S=m\left(c_1 \log_e \frac{T_2}{T_1}+\frac{H}{T_2}+c_2 \log_e \frac{T_3}{T_2}+\frac{r}{T_3}\right) \tag{2}$$

In our case, $m=0.01$ kg, $c_1=0.5$ cal/g·deg $=2.1\times10^3$ J/kg·deg, $T_1=253°$ K, $T_2=273°$ K, $T_3=373°$ K, $H=80$ cal/g $=3.35\times10^5$ J/kg, $c_2=1$ cal/g·deg $=4.19\times$ $\times10^3$ J/kg·deg and $r=539$ cal/g $=2.26\times10^6$ J/kg. Upon inserting these data in Eq. (2), we obtain $\Delta S=88$ J/deg $=21$ cal/deg.

5.217. $\Delta S=7.4$ J/deg. **5.218.** $\Delta S=1{,}230$ J/deg.

5.219. $\Delta S=63$ J/deg.

5.220. We have $S_2-S_1=\int_1^2 \frac{dQ}{T}$. But $dQ=\frac{M}{\mu}C_v\,dT+p\,dV$ and, in addition, $pV=\frac{M}{\mu}RT$ Consequently, $S_2-S_1=\int_1^2 \frac{M}{\mu}\frac{C_v\,dT}{T}+\int_1^2 \frac{\frac{M}{\mu}R\,dV}{V}$, or $S_2-S_1=$ $=\frac{M}{\mu}C_v \log_e \frac{T_2}{T_1}+\frac{M}{\mu}R \log_e \frac{V_2}{V_1}=5.4$ J/deg.

5.221. In the previous problem we found the entropy as a function of the parameters T and V. In this problem let us express the entropy through the parameters V and p. We have

$$\Delta S=\frac{M}{\mu}C_v\log_e\frac{T_2}{T_1}+\frac{M}{\mu}R\log_e\frac{V_2}{V_1} \tag{1}$$

But, from the Mendeleyev-Clapeyron equation

$$\frac{T_2}{T_1}=\frac{p_2V_2}{p_1V_1} \tag{2}$$

Upon inserting Eq. (2) into (1), we obtain

$$\Delta S=\frac{M}{\mu}C_v\log_e\frac{p_2}{p_1}+\frac{M}{\mu}C_v\log_e\frac{V_2}{V_1}+$$
$$+\frac{M}{\mu}R\log_e\frac{V_2}{V_1}=\frac{M}{\mu}C_v\log_e\frac{p_2}{p_1}+$$
$$+\frac{M}{\mu}C_p\log_e\frac{V_2}{V_1}$$

Insertion of the numerical data gives $\Delta S=71.0$ J/deg. Express the entropy through the parameters p and T to obtain the following formula

$$\Delta S=\frac{M}{\mu}C_p\log_e\frac{T_2}{T_1}-\frac{M}{\mu}R\log_e\frac{p_2}{p_1}$$

5.222. We have (see the solution to the previous problem) $\Delta S=\frac{M}{\mu}C_v\log_e\frac{p_2}{p_1}+\frac{M}{\mu}C_p\log_e\frac{V_2}{V_1}$. In the isobaric process $p_1=p_2$ and $\Delta S=\frac{M}{\mu}C_p\log_e\frac{V_2}{V_1}$. Upon inserting the numerical data, we get $\Delta S=66.3$ J/deg $=15.8$ cal/deg.

5.223. $\Delta S=38.1$ J/deg.

5.224. We have (see the solution to Problem 5.221) $\Delta S=\frac{M}{\mu}C_p\log_e\frac{T_2}{T_1}-\frac{M}{\mu}R\log_e\frac{p_2}{p_1}$ In the isothermal process $T_1=T_2$, and $\Delta S=-\frac{M}{\mu}R\log_e\frac{p_2}{p_1}=\frac{M}{\mu}R\log_e\frac{p_1}{p_2}$ Upon inserting the numerical data, we get $\Delta S=17.3$ J/deg.

5.225. $\Delta S=2.9$ J/deg.

5.226. (1) $\Delta S=1.76$ J/deg, (2) $\Delta S=2.46$ J/deg.

5.227. (1) $\Delta S=8.5\times10^3$ J/deg, (2) $\Delta S=11.8\times10^3$ J/deg.

5.228. The nitrogen was heated at a constant pressure.

5.229. We propose that the students convince themselves that the change in entropy does not depend on how the transition of the gas from one state to another takes place. In both cases the change in entropy will be equal to 5.45 J/deg.

5.230. $\Delta S\cong500$ J/deg. **5.231.** $Q=4.2\times10^5$ J.

6. Real Gases

6.1. The constant b is expressed in m^3/kmole, and the constant a in $N\cdot m^4/kmole^2$.
6.2.

Gas	$a\times10^{-5}$, $N\cdot m^4/kmole^2$	$b\times10^2$, m^3/kmole
Water vapour	5.56	3.06
Carbon dioxide	3.64	4.26
Oxygen	1.36	3.16
Argon	1.36	3.22
Nitrogen	1.36	3.85
Hydrogen	2.44×10^{-1}	2.63
Helium	3.43×10^{-2}	2.34

6.3. (1) By solving the Mendeleyev-Clapeyron equation with respect to the temperature, we find

$$T=\frac{\mu pV}{MR} \tag{1}$$

In our case $\mu=28$ kg/kmole, $p=2$ atm$=2\times1.013\times10^5$ N/m^2, $V=8.2\times10^{-4}$ m^3, $M=2\times10^{-3}$ kg. Upon inserting these data in Eq. (1), we get $T=280°$ K.

(2) By solving the Van der Waals equation with respect to the temperature, we find

$$T=\frac{\mu}{MR}\left(p+\frac{aM^2}{\mu^2V^2}\right)\left(V-\frac{M}{\mu}b\right) \tag{2}$$

Upon inserting the numerical data into Eq. (2), we shall obtain $T=280°$K with an accuracy of three significant digits. Thus, the gas behaves as an ideal gas at low pressures. At great pressures the gas parameters do not obey the Mendeleyev Clapeyron equation (see the condition of and answer to the next problem).

6.4. (1) $T=281°$ K, (2) $T=289°$ K.

6.5. (1) $T=482°$ K, (2) $T=204°$ K.

6.6. (1) For the real gas: (a) $p=2.87\times10^6$ N/m^2 and (b) $p=2.73\times10^8$ N/m^2, (2) for the ideal gas: (a) $p=3.09\times10^6$ N/m^2 and (b) $p=6.18\times10^7$ N/m^2.

A comparison of the results obtained shows that when the pressures are not too high, real gases are more compressible than ideal ones (the effect of the forces of attraction between the molecules); at high pressures, real gases are less compressible than ideal ones (the effect of the own volume of the molecules).

6.7. $\frac{T_2}{T_1}=\frac{2p+p_i}{p+p_i}=1.85$. Here $p_i=\frac{aN^2}{V^2}$, where N is the number of kilomoles. If the gas obeys the Mendeleyev-Clapeyron equation, $\frac{T_2}{T_1}=2$.

6.8. A third-power equation has to be solved to find the volume from the Van der Waals equation. One of the three roots of this equation which corresponds to the gaseous state of a substance can be found by the method of approximations. It follows from the Van der Waals equation that

$$V=\frac{RT}{p+\frac{a}{V^2}}+b=\frac{RT}{p+p_i}+b \tag{1}$$

As a first approximation let us take $V=V_1$, which is the volume obtained from the Mendeleyev-Clapeyron equation

$$V_1=\frac{MRT}{\mu p} \qquad (2)$$

In our case $\frac{M}{\mu}=1$ kmole, $R=8.31\times10^3$ J/kmole·deg, $T=300°K$ and $p=10^7$ N/m². Upon inserting these data into Eq. (2), we obtain $V_1=0.24$ m³. Hence, $p_i=\frac{a}{V_1^2}=\frac{1.36\times10^5}{0.24^2}$ N/m² $=0.24\times10^7$ N/m². By inserting p_i into Eq. (1), we get the second approximation

$$V_2=\frac{8.31\times10^3\times300}{1.24\times10^7}\text{ m}^3+3.16\times10^{-2}\text{ m}^3=0.232\text{ m}^3.$$

Hence

$$p_i=\frac{a}{V_2^2}=\frac{1.36\times10^5}{0.232^2}=0.253\times10^7\text{ N/m}^2$$

and

$$V_3=\left(\frac{8.31\times10^3\times300}{1.253\times10^7}+3.16\times10^{-2}\right)\text{ m}^3=0.231\text{ m}^3$$

In the same way we get the fourth, etc., approximations. It is easy to see that already the fourth approximation practically coincides with the third one. Thus, the volume sought is $V=0.231$ m³ $=231$ litres.

6.9. $V=0.49$ m³ (see the solution to the previous problem)

6.10. The constant b in the Van der Waals equation is approximately equal to four times the own volume of the molecules. On the other hand $b=\frac{T_{cr}R}{8p_{cr}}$. Hence the volume of one molecule $V'=\frac{RT_{cr}}{32\,N_Ap_{cr}}=\frac{4}{3}\pi r^3=\frac{1}{6}\pi\sigma^3$, where σ is the effective diameter of the molecules. Since $\frac{R}{N_A}=k$ is Boltzmann's constant, we finally obtain $\sigma=\sqrt[3]{\frac{3kT_{cr}}{16\pi p_{cr}}}$. Substitution of the numerical data gives $\sigma=2.94\times10^{-10}$ m $=2.94$ Å. This value of σ closely coincides with the one obtained by other methods (see the solution to Problem 5.141).

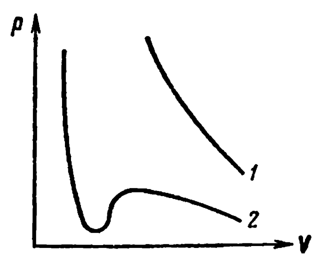


Fig. 91

6.11. (1) $\sigma=2.97\times10^{-10}$ m $\cong 3.0$ Å, (2) $\sigma=3.13\times10^{-10}$ m $\cong 3.1$ Å. Thus the results obtained by the two different methods are in good agreement.

6.12. $\bar{l}=7.9\times10^{-8}$ m.

6.13. $D=3.5\times10^{-5}$ m²/s.

6.14. Fig. 91 shows the relation $p=f(V)$ plotted for 1 kmole of carbon dioxide at 0° C. Curve *1* corresponds to the equation of an ideal gas, and curve *2* to that of a real gas

6.15. $p_i=\frac{27T_{cr}^2p^2}{64p_{cr}T^2}=1.31\times10^3$ N/m²

6.16. (1) $p\left(V-\frac{M}{\mu}b\right)=\frac{M}{\mu}RT$, (2) $x=\frac{N-N'}{N'}=\frac{pb}{RT}=0.33=33\%$. Here, N and N' are the numbers of kilomoles without and with account taken of the own weight of the molecules.

6.17. (1) $\frac{p_i}{p}=4.95\%$, (2) $\frac{V_i}{V}=0.86\%$.

6.18. The work performed against the forces of interaction of the molecules is

$$W=\int_{V_1}^{V_2} p_i\,dV, \quad \text{where } p_i=\frac{M^2a}{\mu^2V^2}$$

Thus,

$$W=\frac{M^2a}{\mu^2}\int_{V_1}^{V_2}\frac{dV}{V^2}=\frac{M^2a}{\mu^2}\left(\frac{1}{V_1}-\frac{1}{V_2}\right)=\frac{M^2a\,(V_2-V_1)}{\mu^2V_1V_2} \qquad (1)$$

whence

$$a=\frac{W\mu^2V_1V_2}{M^2\,(V_2-V_1)}=\frac{WV_1V_2}{N^2\,(V_2-V_1)} \qquad (2)$$

where N is the number of kilomoles. Upon inserting the numerical data into Eq. (2), we obtain $a=1.36\times10^5$ N·m^4/kmole^2.

6.19. $\Delta T=\frac{aN\,(V_2-V_1)2}{V_1V_2iR}$, where i is the number of degrees of freedom of the gas molecules and N is the number of kilomoles of the gas. Substitution of the numerical data gives $\Delta T=2.33°$.

6.20 $a=3.64\times10^5$ N·m^4/kmole^2.

6.21. (1) (a) Since the temperature $t=31°$ C is the critical temperature of the carbon dioxide gas, the required pressure $p=p_{cr}=73$ atm. (b) Since the temperature $t=50°$ C is higher than the critical temperature, CO_2 cannot be liquefied at 50° C under any pressure.

(2) $V_x=\frac{3b}{\mu}=2.9\times10^{-3}$ m^3

(3) $p=p_{cr}=73$ atm.

6.22. $\rho_{cr}=\frac{\mu}{3b}=196$ kg/m^3. **6.23.** $\rho_{cr}=\frac{8\mu p_{cr}}{3T_{cr}R}=57$ kg/m^3.

6.24. From the Van der Waals equation in the transformed form we have

$$\tau=\frac{\left(\pi+\frac{3}{\omega^2}\right)(3\omega-1)}{8} \qquad (1)$$

In our case $\pi=\frac{p}{p_{cr}}=\frac{920}{50}=18.4$. For oxygen $V_{0ox}=3b=\frac{3T_{cr}R}{8p_{cr}}=9.5\times$ $\times10^{-2}$ m^3/kmole, $\omega=\frac{V_0}{V_{0ox}}=\frac{0.056}{0.095}=0.59$. By inserting these values in Eq. (1), we obtain $\tau=2.6$ and, therefore, $T=\tau T_{cr}=2.6\times154°$ K $=400°$ K, or $t=127°$ C.

6.25. $p=2.7\times10^6$ N/m^2. **6.26.** $\pi=\frac{p}{p_{cr}}=2.45$.

7. Saturated Vapours and Liquids

7.1. The amount of water vapours M can be found from the Mendeleyev-Clapeyron formula

$$M=\frac{pV\mu}{RT} \quad (1)$$

where p is the elasticity of the water vapours which saturate the space at the temperature T. When $T=50°\,C=323°\,K$, the elasticity $p=92.5$ mm Hg $=92.5\times\times133.3$ N/m². Since $\mu=18$ kg/kmole and $V=1$ m³, then by inserting these data in Eq. (1), we obtain $M=8.2\times10^{-2}$ kg $=82$ g.

7.2. $\rho=8.2\times10^{-2}$ kg/m³. **7.3.** 74,000 times. **7.4.** 12 times.

7.5. The relative humidity is determined from the formula $w=\frac{p}{p_s}$, where p is the pressure of the water vapours in the air, and p_s the pressure of the water vapours saturating the space at the given temperature. The mass of the water vapours in the volume V of the air at the temperature T is

$$M=\frac{pV\mu}{RT}=\frac{wp_sV\mu}{RT} \quad (1)$$

In our case $w=0.75$, $\mu=18$ kg/kmole, $V=1$ m³, $T=30°\,C=303°\,K$. At the temperature $t=30°\,C$ the pressure of the saturated vapours is $p_s=31.8$ mm Hg $=31.8\times133.3$ N/m². Upon inserting these data into formula (1), we find $M=22.5\times10^{-3}$ kg. Thus, the weight of the water vapours in the conditions of the problem is 22.5×10^{-3} kgf.

7.6. $M=6.9\times10^{-3}$ kg. **7.7.** $t=7°\,C$. **7.8.** $n=10^{18}$ cm⁻³.

7.9. (1) $w=60.4$ per cent, (2) $M=86\times10^{-6}$ kg.

7.10. (1) Before expansion the saturated water vapours are at a temperature of 20° C and therefore (see the tables) the pressure of these vapours is $p_1=17.5$ mm Hg $=17.5\times133.3$ N/m².

(2) The amount of water vapours in the chamber before expansion is

$$M_1=\frac{p_1\mu V_1}{RT_1} \quad (1)$$

Substitution of the numerical data in formula (1) gives $M_1=17.2\times10^{-6}$ kg $=17.2$ mg.

(3) $\rho_1=\frac{p_1\mu}{RT_1}\doteq1.72\times10^{-2}$ kg/m³

(4) $T_2=\frac{T_1}{\left(\frac{V_2}{V_1}\right)^{\varkappa-1}}=268°$ K

(5) At a temperature of —5° C the pressure of the saturated water vapours is $p_2=3$ mm Hg. The amount of vapours in the chamber which corresponds to this pressure is $M_2=\frac{p_2\mu V_2}{RT_2}$, where $V_2=1.25V_1$. Upon inserting the numerical data, we find $M_2=4$ mg. Therefore, the amount of condensed vapours $\Delta M=M_1-M_2=(17.2-4.0)$ mg $=13.2$ mg.

(6) $\rho_2=\frac{p_2\mu}{RT_2}=3.2\times10^{-3}$ kg/m³.

(7) Since the density of the water vapours after expansion (but before condensation) is

$$\rho_3=\frac{M_1}{V_2}=\frac{17.2\times10^{-6}}{1.25\times10^{-3}}\ \text{kg/m}^3=13.7\times10^{-3}\ \text{kg/m}^3$$

the degree of supersaturation is $s=\frac{\rho_3}{\rho_2}=\frac{13.7\times10^{-3}}{3.2\times10^{-3}}=4.3$.

7.11. $V_l=10^{-3}$ m³/kg $=1$ cm³/g, $V_v=1.25$ m³/kg $=1.25\times10^3$ cm³/g.

7.12. During vaporization heat is spent not only to overcome the forces of interaction of the molecules, but also for the work of expansion against the external pressure. Thus, according to the first law of thermodynamics, we have

$$r_0=\Delta E+W \qquad (1)$$

where r_0 is the molecular heat of vaporization, ΔE the change in the internal energy of the forces of interaction during vaporization, and W the work performed against the external pressure. We have

$$W=p_s(V_2-V_1) \qquad (2)$$

where p_s is the pressure of the saturated vapour at the temperature of vaporization, V_1 the volume of one kilomole of the liquid and V_2 the volume of one kilomole of the vapour. Obviously, $V_1=\frac{\mu}{\rho}$, where μ is the mass of 1 kmole of the water and ρ its density. In our case

$$V_1=\frac{18\ \text{kg/kmole}}{1{,}000\ \text{kg/m}^3}=18\times10^{-3}\ \text{m}^3\text{/kmole}.$$

Since according to the condition $\frac{M}{\mu}=1$ kmole, then from the Mendeleyev-Clapeyron equation $V_2=\frac{RT}{p_s}$. When $T=200°\,\text{C}=473°\,\text{K}$ we have (see Table VI): $p_s=15.3$ atm $=15.3\times1.013\times10^5$ N/m² and $V_2=\frac{RT}{p_s}=2.5$ m³/kmole.

Assuming that the change in the internal energy of the forces of interaction in vaporization corresponds to the Van der Waals equation (see Problem 6.18), we have

$$\Delta E=\frac{a(V_2-V_1)}{V_1V_2} \qquad (3)$$

where $a=\frac{27T_{cr}^2R^2}{64p_{cr}}=5.56\times10^5$ N·m⁴/kmole². Since $V_1\ll V_2$, we obtain from Eqs. (1), (2) and (3)

$$r_0=\frac{a}{V_1}+p_sV_2=\frac{a\rho}{\mu}+RT=(3.1+0.4)\times10^7\ \text{J/kmole}=3.5\times10^7\ \text{J/kmole}$$

Hence, the specific heat of vaporization $r=\frac{r_0}{\mu}=1.94\times10^6$ J/kg $=465$ cal/g.

From Table VII we have $r=464$ cal/g for a temperature of $t=200°$ C. Thus, despite the fact that the Van der Waals equation and, therefore, formula (3) are approximate, the results are in good agreement.

7.13. $x=\frac{\Delta E}{r_0}=\frac{r_0-W}{r_0}=1-\frac{RT}{r_0}=92.4\%$.

7.14. $\Delta E=7.22\times10^3$ J.

7.15. From the Clausius-Clapeyron equation

$$\frac{dp}{dT}=\frac{r_0}{T(V_v-V_l)} \qquad (1)$$

Assuming that saturated vapours obey the Mendeleyev-Clapeyron equation, we have (for one kmole) $V_v=\frac{RT}{p}$ Since (see Table VI) $p=6.54$ mm Hg at a temperature of 5° C, it is easy to find that $V_v=2.65\times10^3$ m³/kmole. Besides, $V_l=\frac{\mu}{\rho}=18\times10^{-3}$ m³/kmole. Thus, we see that $V_l \ll V_v$, and then equation (1) may be written as

$$\frac{dp}{dT}=\frac{r_0 p}{RT^2}$$

or

$$\frac{dp}{p}=\frac{r_0}{R}\frac{dT}{T^2} \qquad (2)$$

For a small temperature range T_2-T_1, the heat of vaporization may be considered constant and then, upon integrating equation (2), we get

$$\log_e\frac{p_2}{p_1}=\frac{r_0(T_2-T_1)}{RT_1T_2} \qquad (3)$$

whence

$$r_0=\frac{RT_1T_2\log_e\frac{p_2}{p_1}}{T_2-T_1} \qquad (4)$$

In formula (4), p_1 and p_2 are the pressures of the saturated vapours at the temperatures T_1 and T_2, respectively. Our task is to find the value of r_0 at the temperature $t=5°$ C. For this reason the values $t_1=4°$ C and $t_2=6°$ C may be taken for T_1 and T_2. Then, from the data in Table VI we have $p_1=6.10$ mm Hg, $p_2=7.01$ mm Hg and $\frac{p_2}{p_1}=1.15$. Upon inserting the numerical data in Eq. (4), we obtain $r_0=\frac{8.31\times10^3\times277\times279\log_e 1.15}{2}$ J/kmole $=45\times10^6$ J/kmole $=$ $=10.7\times10^3$ kcal/kmole. Hence, the specific heat of vaporization $r=\frac{r_0}{\mu}=595$ cal/g. After plotting the diagram $r=f(t)$ from the data in Table VII, it can easily be seen that when $t=5°$ C, the value of r will be equal to 592 cal/g, which is in good agreement with the determined value.

7.16. $r=72.2$ cal/g. 7.17. $p=650$ mm Hg.

7.18 $\Delta S=2.86$ J/deg $=0.683$ cal/deg. 7.19. By 4.5 mm Hg.

7.20. To the pressure $p=7\times10^{-4}$ mm Hg, i. e., to a pressure equal to the pressure of the saturated mercury vapours at 15° C.

7.21 In our case $\rho_0=\frac{M}{V_0}$ and $\rho=\frac{M}{V}$ But since $V=V_0(1+\gamma t)$, then finally $\rho=\frac{\rho_0}{1+\gamma t}$. Upon inserting the numerica data, we obtain $\rho=1.29\times10^4$ kg/m³ $=$ $=12.9$ g/cm³.

7.22. $t=222°$ C. 7.23. $\rho=1{,}055$ kg/m³.

7.24 $\Delta p=\frac{\gamma\Delta t}{k}=1.4\times10^{6}$ N/m² $=13.8$ atm.

7.25. $k=3.9\times10^{-6}$ atm^{-1}. 7.26. $\Delta h=16.4$ mm.

7.27 $\Delta t=\frac{h(1+\gamma t)}{(H-h)\gamma}=56°$ 7.28. $M=0.884$ kg.

7.29. Let us denote the coefficients of volume expansion of the mercury and the glass by γ_1 and γ_2, respectively. Heating increased the volume of the vessel to $V=V_0(1+\gamma_2 t)$, and the density of the mercury became equal to

$$\rho=\frac{M}{V}=\frac{M}{V_0(1+\gamma_2 t)} \qquad (1)$$

On the other hand (see the solution of Problem 7.21)

$$\rho=\frac{\rho_0}{1+\gamma_1 t}=\frac{M_0}{V_0(1+\gamma_1 t)} \qquad (2)$$

By comparing Eqs. (1) and (2), we find that

$$M=\frac{M_0(1+\gamma_2 t)}{1+\gamma_1 t}=0.887 \text{ kg}$$

7.30. $\gamma_x=7\times10^{-4}$ deg^{-1}.

7.31. $x=\frac{\gamma-\gamma'}{\gamma}=5\%$, where γ and γ' are the coefficients of volume expansion of the oil found with and without account of the expansion of the glass, respectively.

7.32. 765 mm Hg.

7.33. (1) The force requirea to tear the ring away from the water surface consists of the weight of the ring and the force of surface tension, i. e., $F=F_1+F_2$. The weight of the ring $F_1=\rho h\frac{\pi}{4}(d_2^2-d_1^2)g=40.0\times10^{-3}$ N. When the ring is torn away, the surface film breaks along the external and internal circumferences of the ring, and therefore the force of surface tension $F_2=\pi\alpha(d_1+d_2)=$ $=23.5\times10^{-3}$ N. Thus, $F=63.5\times10^{-3}$ N.

(2) $x=\frac{F_2}{F}=37\%$.

7.34. $\alpha=32.4\times10^{-3}$ N/m. 7.35. (1) $d=1.2$ mm, (2) $l=5$ cm.

7.36. The weight of a drop at the moment of fall should break the surface film over the length $l=2\pi r$, where r is the radius of the drop neck. Hence, the weight of a drop $G=2\pi r\alpha=\pi d\alpha$. The number of drops of alcohol contained in M grammes is $N=\frac{Mg}{G}=\frac{Mg}{\pi d\alpha}$ Substitution of the numerical data gives $N=780$ drops. According to the condition, the interval between drops is 1 second, and therefore the alcohol will flow out in $t=7.8\times10^{3}$ s $=13$ min.

7.37. $\alpha=59\times10^{-3}$ N/m 7.38 By 34 cm.

7.39. $R=\sqrt[3]{\frac{3r\alpha}{2\rho g}}=2.2\times10^{-3}$ m $=2.2$ mm.

7.40. Upon the merging of two drops of mercury the energy evolved is $\Delta E=\alpha\Delta A$, where ΔA is the change in the area of the surface ($\Delta A=4\pi r^2\times2-4\pi R^2$, where r is the radius of the small drops, and R the radius of the large drop). The radius R can be found by equating the volume of the large drop to the sum of the volumes of the merged drops: $2\frac{4}{3}\pi r^3=\frac{4}{3}\pi R^3$, whence $R=r\sqrt[3]{2}$. Hence $\Delta A=4\pi r^2(2-\sqrt[3]{4})$ and

$$\Delta E=\alpha\Delta A=\alpha 4\pi r^2\left(2-\sqrt[3]{4}\right) \tag{1}$$

The energy evolved is used to heat the mercury drop, and therefore

$$\Delta E=cm\Delta t=c\rho\frac{4}{3}\pi R^3\Delta t=c\rho\frac{8}{3}\pi r^3\Delta t \tag{2}$$

By comparing Eqs. (1) and (2), we finally find

$$\Delta t=\frac{3\alpha\left(2-\sqrt[3]{4}\right)}{c\rho 2r} \tag{3}$$

or after inserting the numerical data, $\Delta t=1.65\times10^{-4}$ deg.

7.41. $W=1.47\times10^{-5}$ J. **7.42.** $W=6.4\times10^{-5}$ J. **7.43.** $W=4.32\times10^{-4}$ J.

7.44. The air pressure in the bubble will consist of the atmospheric pressure p_1, the hydrostatic pressure of the water $p_2=\rho gh$ and the additional pressure $p_3=\frac{2\alpha}{r}=\frac{4\alpha}{d}$ caused by the curvature of the surface. Thus, $p=p_1+\rho gh+\frac{2\alpha}{r}$. In our case $p_1=765$ mm Hg, $p_2=1{,}970$ N/m^2=14.7 mm Hg and $p_3=2.92\times10^4$ N/m^2= =219 mm Hg. Thus, the air pressure in the bubble $p=$ =999 mm Hg.

7.45. $D=\frac{8\alpha}{\Delta p}=2.6\times10^{-3}$ m=2.6 mm. **7.46.** $h=4.9$ m.

7.47. 4.4 times.

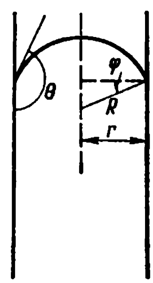


Fig. 92

7.48. The radius of the meniscus R is related to the tube radius r as follows (Fig. 92): $r=R\cos\varphi=R\cos(180°-\theta)=-R\cos\theta$, where θ is the contact angle. The additional pressure caused by the curvature of the meniscus $\Delta p=-\frac{2\alpha\cos\theta}{r}$ Since for mercury $\theta>\frac{\pi}{2}$, i. e., $\cos\theta<0$, this additional pressure is positive, and the mercury level in the capillary tube will be lower than in the vessel. The difference in the levels $\Delta h=-\frac{4\alpha\cos\theta}{\rho gd}$, whence $-\cos\theta=\frac{\Delta h\rho gd}{4\alpha}$. Substitution of the numerical values gives $-\cos\theta=0.740$. Hence the radius of curvature of the mercury meniscus $R=-\frac{r}{\cos\theta}=2\times10^{-3}$ m=2 mm.

7.49. (1) $R=0.53$ mm, (2) $\Delta h=2.98$ cm. **7.50.** $h=13.9$ mm.

7.51. (1) $d=1.5$ mm, (2) $d=8.8$ mm. **7.52.** $\Delta h=7.5$ mm.

7.53. $d=0.15$ mm. **7.54.** $\alpha=0.07$ N/m.

7.55. $p=p_0+\frac{2\alpha}{r}=102.2\times10^3$ N/m^2=767 mm Hg.

7.56. Let us denote the air pressure in the capillary tube before it is submerged into the water by p_0 and the pressure after submergence by p_1. Correspondingly, V_0 and V_1 are the volumes of the air in the tube before and after submergence. According to Boyle-Mariotte's law,

$$p_0V_0=p_1V_1 \tag{1}$$

In equation (1) $p_1 = p_0 + \frac{2\alpha}{r}$ and $V_0 = Ah_0$, where A is the cross-sectional area of the capillary tube and h_0 its length; $V_1 = Ah_1$, where h_1 is the length of the tube projecting above the liquid after submergence. Upon inserting these values into Eq. (1), we obtain $p_0h_0 = \left(p_0 + \frac{2\alpha}{r}\right)h_1$, whence

$$r = \frac{2\alpha h_1}{p_0(h_0 - h_1)} \tag{2}$$

According to the initial condition, $\frac{h_0 - h_1}{h_0} = 0.015$, or $\frac{h_1}{h_0 - h_1} = 67.5$. Upon inserting the numerical data into Eq. (2), we get $r = 10^{-4}$ m $= 0.1$ mm.

7.57. (a) $h = 755$ mm, and (b) $h = 757$ mm. Thus, if the tube is narrow, the atmospheric pressure cannot be determined directly from the height of the mercury column h, since the pressure of the convex meniscus of the mercury in the tube is added to the pressure of the column.

7.58. The height of the mercury column should be increased by 2 mm.

7.59. (1) $x = \frac{H-h}{h} = 0.4\%$, (2) $x = \frac{H-h}{h} = 0.2\%$.

7.60. For the needle to remain on the water surface, the pressure exerted by the weight of the needle on its supporting surface should not exceed the pressure caused by the curvature of the liquid surface in the recess under the needle and directed upwards (the loss of weight according to Archimedes' law is neglected). The pressure of the needle on the water $p_1 = \frac{mg}{ld} = \frac{\rho Vg}{ld} = \frac{\rho\pi dg}{4}$, where d is the needle diameter, l its length and V its volume. The pressure due to the curvature of the liquid surface can be found from the Laplace formula $p_2 = \alpha\left(\frac{1}{R_1} + \frac{1}{R_2}\right)$. In our case the surface of the liquid is cylindrical, i. e., $R_1 = \infty$ and $R_2 = r$, which is the radius of the needle. Hence, $p_2 = \frac{\alpha}{r} = \frac{2\alpha}{d}$.

Since it is necessary that $p_1 \leqslant p_2$, then $\frac{\rho\pi gd}{4} \leqslant \frac{2\alpha}{d}$, whence $d \leqslant \sqrt{\frac{8\alpha}{\rho\pi g}}$. Insertion of the numerical data gives $d \leqslant 1.6$ mm.

7.61. It will not. **7.62.** $d = 0.5$ mm. **7.63.** $M = 1.22$ kg. **7.64.** 27.5×10^{-6} kgf.

7.65. The surface of the wetting liquid between the plates is cylindrical in shape with a radius of curvature $R = \frac{d}{2}$, where d is the distance between the plates (Fig. 93). Hence, the additional negative pressure under the cylindrical concave surface $p = \frac{\alpha}{R} = \frac{2\alpha}{d}$. The quantity p is the excess external pressure acting on the area A. Hence, the force which should be applied to detach the plates is

$$F = pA = \frac{2\alpha}{d}A = \frac{2 \times 0.073 \times 1.08 \times 10^{-2}}{5 \times 10^{-5}} \text{ N} = 31.5 \text{ N} = 3.2 \text{ kgf}$$

7.66. $\rho = 790$ kg/m³. **7.67.** $\alpha = 0.5$ N/m.

7.68. When the capillary tube is in its vertical position, the upper meniscus is concave and the pressure caused by the curvature of this meniscus is always directed upwards and is equal to $p_1 = \frac{2\alpha}{R_1}$, where R_1 is the radius of curvature of the upper meniscus. With complete wetting $p_1=\frac{2\alpha}{r}$, where r is the radius of the capillary tube. The hydrostatic pressure of the liquid column is always directed downward and is equal to $p_2=\rho gh$. If $p_1>p_2$, the resulting pressure directed upward makes the lower meniscus concave. Hence the pressure p_3 caused by the curvature of the lower meniscus is directed downward and is equal to $p_3=\frac{2\alpha}{R_2}$, where R_2 is the radius of curvature of the lower meniscus. In equilibrium

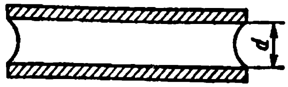


Fig. 93

$$p_1=p_2+p_3 \qquad (1)$$

If $p_1<p_2$, the resulting pressure is directed downwards and the lower meniscus will be convex. Now the pressure $p_3=\frac{2\alpha}{R_2}$ will be directed upward, and

$$p_1+p_3=p_2 \qquad (2)$$

If, finally,

$$p_1=p_2 \qquad (3)$$

the lower meniscus is flat, and $p_3=0$. Upon inserting the numerical data, it is easy to find that:

(1) $R_1=0.5$ mm and $R_2=-1.52$ mm,
(2) $R_1=0.5$ mm and $R_2=1.46$ mm,
(3) $R_1=0.5$ mm and $R_2=\infty$.

7.69. $M=2.2\times10^{-4}$ kg.

7.70. (1) $h=11.5$ mm, (2) $h=12.9$ mm, (3) $h=17.2$ mm. (See the solution of Problem 7.68.)

7.71. (1) $\Delta h=6.8$ mm, (2) $\Delta h=8.5$ mm, (3) $\Delta h=17$ mm, (4) $\Delta h=23.8$ mm. When $\Delta h>23.8$ mm the liquid will begin to flow out from tube a.

7.72. If the capillary tube were long enough it would be easy to see that the water in it rises to the height $h=2.98$ cm. But the height of the capillary tube is $h_1<h$. Now, the meniscus is acted upon by the pressure p_1 caused by the curvature of the meniscus, directed upward and equal to $p_1=\frac{2\alpha}{R}$, and the hydrostatic pressure $p_2=\rho gh_1$. For any height h_1

$$\rho gh_1=\frac{2\alpha}{R}$$

Upon inserting the numerical data we obtain $R=0.75\times10^{-3}$ m.

7.73. The aerometer floating in water is acted upon by its own weight G directed downwards, the force of surface tension

$$f_1=2\pi r\alpha=\pi d\alpha \qquad (1)$$

directed downward in case of complete wetting (with the complete absence of wetting it is directed upwards), and the Archimedean force f_2 directed upward and equal to

$$f_2=\rho g(V+Ah) \qquad (2)$$

where ρ is the density of the liquid, V the volume of the non-cylindrical part of the aerometer, A the cross-sectional area of the aerometer tube and h the length of the cylindrical tube in the liquid. In equilibrium

$$G+f_1=f_2 \qquad (3)$$

Since several drops of alcohol will not change the density of the water, we can write from Eqs. (1), (2) and (3) for water

$$G+d\pi\alpha_1=\rho g\,(V+Ah_1) \qquad (4)$$

and for alcohol

$$G+d\pi\alpha_2=\rho g\,(V+Ah_2) \qquad (5)$$

From Eqs. (4) and (5) it is easy to obtain that

$$\Delta h=\frac{4\Delta\alpha}{\rho g d}=\frac{4\times(73-20)\times10^{-3}}{1{,}000\times9.81\times9\times10^{-3}}\text{ m}=2.4\times10^{-3}\text{ m}=2.4\text{ mm}.$$

7.74. The aerometer will rise by $\Delta h=3.5$ mm.
7.75. $T=313°\text{K}=40°\text{C}$. **7.76.** 1,000 molecules. **7.77.** $p=2.9\times10^6$ N/m².
7.78. $M=2\times10^{-3}$ kg. **7.79** (1) $\alpha=55\%$, (2) 4×10^{19} cm^{-3}.
7.80. $p=92.1$ mm Hg. **7.81.** $p=147.6$ mm Hg. **7.82.** 50 molecules.
7.83. Raoult's law can be used to determine the mass of 1 kmole of a substance. Indeed, this law can be written as

$$\frac{p_0}{p_0-p}=\frac{z}{z'}+1$$

or

$$\frac{p_0}{p_0-p}-1=\frac{p}{p_0-p}=\frac{z}{z'} \qquad (1)$$

Since $z=\frac{M}{\mu}$ and $z'=\frac{M'}{\mu'}$, it is easy to obtain from Eq. (1)

$$\mu'=\mu\frac{M'}{M}\frac{p}{p_0-p} \qquad (2)$$

Here M is the mass of solvent, μ the mass of one kilomole of the solvent, M' the mass of the dissolved substance and μ' is the mass of one kilomole of the dissolved substance. Upon inserting the numerical data, we get $\mu'=92$ kg/kmole.
7.84. $p=9.25\times10^5$ N/m².

8. Solids

8.1. We find from the Clausius-Clapeyron equation that

$$\Delta T=\frac{\Delta pT\,(V_l-V_s)}{q_0} \qquad (1)$$

On the other hand, the change in entropy is

$$\Delta S=\frac{mH_0}{T}=\frac{Nq_0}{T} \qquad (2)$$

where H_0 is the specific heat of melting and q_0 the molecular heat of melting; m is the mass and N the number of kilomoles. From Eqs. (1) and (2) we have

$$\Delta T=\frac{\Delta p\,(V_l-V_s)\,N}{\Delta S} \qquad (3)$$

In our case $V_l=\frac{\mu}{\rho_1}=\frac{18}{1{,}000}\text{ m}^3=18\times10^{-3}\text{ m}^3$, $V_s=\frac{\mu}{\rho_2}=\frac{18}{900}\text{ m}^3=2\times10^{-2}\text{ m}^3$, $N=1$ kmole, $\Delta S=22.2\times10^3$ J/deg and $\Delta p=10^6$ N/m². Insertion of the numerical data into Eq. (3) gives $\Delta T=0.009°$.
8.2. $\Delta S=15.8\times10^3$ J/deg. **8.3.** By 1.03×10^{-3} m³.

8.4. (1) 390 J/kg·deg, (2) 450 J/kg·deg, (3) 930 J/kg·deg.

8.5. The mass of one kg-atom of the ball material is equal to 107 kg/kg-atom. Therefore the ball is made of silver.

8.6. 7.2 times. **8.7.** By 66°.

8.8. The amount of heat which passed through the copper and iron plates put together can be determined from the formula

$$Q=\lambda_1\frac{t_1-t_x}{d_1}At=\lambda_2\frac{t_x-t_2}{d_2}At$$

whence

$$t_x=\frac{\lambda_1 t_1 d_2+\lambda_2 t_2 d_1}{\lambda_1 d_2+\lambda_2 d_1}$$

Upon inserting the numerical data, we obtain $t_x=34.5°$ C.

8.9. $\lambda=1.28$ W/m·deg $=1.1$ kcal/m·h·deg.

8.10. $Q=1.9\times10^5$ J. **8.11.** (1) 2 cal/s, (2) 60 g. **8.12.** $Q=11.7$ J.

8.13. 106° C. **8.14.** In 28.4 hours.

8.15. When heated from 0° C to t° C, the rod will expand by

$$\Delta l=l-l_0=l_0\alpha t \tag{1}$$

To prevent expansion of the rod, the force $F=\frac{\Delta lEA}{l_0}$ should be applied to it, whence

$$\Delta l=\frac{l_0F}{EA} \tag{2}$$

where E is Young's modulus of the rod material and A is the cross-sectional area of the rod. From formulas (1) and (2) we have $F=EA\alpha t$. Upon inserting the numerical data, we obtain $F=7.1\times10^4$ N.

8.16. $G=149$ N. **8.17.** At 20° C. **8.18.** $\alpha=1.8\times10^{-5}$ deg^{-1}.

8.19. For the steel rod

$$l_1=l_{01}(1+\alpha_1 t)=l_{01}+l_{01}\alpha_1 t \tag{1}$$

and for the copper rod

$$l_2=l_{02}(1+\alpha_2 t)=l_{02}+l_{02}\alpha_2 t \tag{2}$$

According to the condition,

$$l_1-l_2=L \tag{3}$$

and

$$l_{01}-l_{02}=L \tag{4}$$

where $L=5$ cm. Upon subtracting Eq. (2) from Eq. (1) and bearing in mind the conditions of Eqs. (3) and (4), we obtain

$$\alpha_1 l_{01}=\alpha_2 l_{02} \tag{5}$$

The length of the rods at 0° C can be easily found from equations (4) and (5)

$$l_{02}=\frac{L\alpha_1}{\alpha_2-\alpha_1}=11\text{ cm},\quad l_{01}=l_{02}+L=16\text{ cm}$$

8.20. 1.02 times. **8.21.** 2.94×10^7 N/m². **8.22.** $d=4.0\times10^{-3}$ m.

8.23. $l=2{,}900$ m **8.24.** $l=180$ m. **8.25.** $l=11.9$ km.

8.26. (1) 250 kgf, (2) by 4 cm, (3) there will not, since the specific load is less than the elastic limit.

8.27. $\gamma=75°30'$ **8.28.** 3.4 rev/s.

8.29. The centrifugal force acting on the rod is

$$F=\int_0^l r\omega^2\,dm$$

where l is the rod length, ω the angular velocity of rotation, and r the distance from the element of mass dm to the axis of rotation. For a homogeneous rod $dm=\rho A\,dr$, where ρ is the density of the rod material and A its cross section. After integration we obtain

$$F=\frac{\rho A\omega^2 l^2}{2}$$

whence the maximum number of revolutions per second is

$$\nu=\frac{1}{\pi l}\sqrt{\frac{F}{2\rho A}}=38\text{ rev/s}$$

8.30. $p=5.7\times10^8$ N/m²

8.31. According to Hooke's law, $\frac{\Delta l}{l}=\frac{1}{E}p_s=\frac{1}{E}\frac{F}{A}$, whence

$$F=\frac{AE}{l}\Delta l \tag{1}$$

But for elastic forces

$$F=k\Delta l \tag{2}$$

By comparing Eqs. (1) and (2), we see that

$$k=\frac{AE}{l} \tag{3}$$

Then

$$W=\frac{k\Delta l^2}{2}=\frac{AE\Delta l^2}{2l} \tag{4}$$

Upon calculating the value of Δl from formula (1) and inserting the remaining numerical data in equation (4), we finally obtain $W=0.706$ J.

8.32. $E=2\ 94\times10^6$ N/m².

8.33. The hose can be stretched by Δl by the force

$$F=EA\frac{\Delta l}{l} \tag{1}$$

In this case the internal diameter of the hose will be reduced by $\Delta d=\beta d_0\frac{F}{A}$. But from Eq. (1) $\frac{F}{A}=\frac{E\Delta l}{l}$ Hence, $\Delta d=\beta d_0\frac{E\Delta l}{l}=\frac{\mu d_0\Delta l}{l}$, where $\mu=\beta E$ is Poisson's ratio. Upon inserting the numerical data, we get $\Delta d=1$ mm, and therefore $d_2=d_0-\Delta d=9$ mm.

8.34. $x=0.3$ m. **8.35.** $M=2.26\times10^{-7}$ N·m.

8.36. The twisting moment of the wire $M=\frac{\pi G d^4 \varphi}{2L16}$, and $\tan 2\varphi=\frac{l}{D}$. When φ is small, we may assume that $\tan\varphi=\varphi$ and then $\varphi=\frac{l}{2D}=\frac{32LM}{\pi G d^4}$. Hence $M=\frac{l\pi G d^4}{64DL}=1.96\times10^{-13}$ N·m.

8.37. To twist the wire through the angle $d\varphi$, it is necessary to do the work

$$dW=M\,d\varphi$$

where M is the twisting moment. Since $M=\frac{\pi G r^4 \varphi}{2L}$, then

$$W=\int_0^{\varphi}\frac{\pi G r^4\varphi}{2L}\,d\varphi=\frac{\pi G r^4 \varphi^2}{4L}$$

Upon inserting the numerical data, we find that $W=1.25\times10^{-12}$ J. This work will be converted into the potential energy of the twisted wire.

8.38. Over 1.74×10^{-2} m.

8.39. Poisson's ratio $\mu=\beta E=\frac{\Delta r}{r}\frac{l}{\Delta l}$, where r is the radius of the wire and l is its length. Before expansion the volume of the wire is $V_1=\pi r^2 l$, and it is V_2 after expansion

$$V_2=\pi(r-\Delta r)^2(l+\Delta l)$$

If during expansion the volume does not change, then $\pi r^2 l=\pi(r-\Delta r)^2(l+\Delta l)$. Upon opening the parentheses and neglecting the squares of the quantities Δr and Δl, we find $\pi r^2\Delta l=2\pi r\Delta r l$, whence $\mu=0.5$.

8.40. When the rod is not compressed, its density is $\rho_1=\frac{m}{V_1}$, where $V_1=\pi r^2 l$. The density of the compressed rod $\rho_2=\frac{m}{V_2}$, where $V_2=\pi(r+\Delta r)^2(l-\Delta l)$. Therefore the change in density is

$$\Delta\rho=\rho_2-\rho_1=m\left(\frac{1}{V_2}-\frac{1}{V_1}\right)=\frac{m\Delta V}{V_2V_1}$$

Since compression is not considerable, it may be approximately assumed that $V_2V_1=V_1^2$, i.e., $\Delta\rho=\frac{m\Delta V}{V_1^2}$. Hence the relative change in density $\frac{\Delta\rho}{\rho_1}=\frac{\Delta V}{V_1}$. Let us find the change in the volume $\Delta V=\pi r^2 l-\pi(r+\Delta r)^2(l-\Delta l)$. Upon opening the parentheses and neglecting the squares of the quantities Δr and Δl, we obtain $\Delta V=V_1\frac{\Delta l}{l}(1-2\mu)$ where μ is Poisson's ratio. Hence $\frac{\Delta\rho}{\rho_1}=\frac{\Delta V}{V_1}=\frac{\Delta l}{l}(1-2\mu)$. According to Hooke's law, $\frac{\Delta l}{l}=\frac{p_s}{E}$. Then, finally $\frac{\Delta\rho}{\rho_1}=\frac{p_s}{E}(1-2\mu)$. In our case $p_s=10^3$ kgf/cm^2 $=9.81\times10^7$ N/m^2, $E=1.18\times10^{11}$ N/m^2 and $\mu=0.34$. Upon inserting these data, we obtain $\frac{\Delta\rho}{\rho_1}=0.027\%$

8.41. By 1 mm^3.

Chapter 3

ELECTRICITY AND MAGNETISM

9. Electrostatics

9.1. $F = 9.23 \times 10^{-8}$ N. **9.2** $r = 8.94 \times 10^{-2}$ m.
9.4. 1.25×10^{36} times. **9.5.** $F = 0.7$ N.

9.6. The electrostatic energy of the balls $W_e = \frac{Q^2}{4\pi\varepsilon_0\varepsilon_r r}$ and their mutual gravitational energy $W_{gr} = \gamma\frac{m_1 m_2}{r}$. According to the condition of the problem, $\frac{Q^2}{4\pi\varepsilon_0\varepsilon_r r} = n\frac{\gamma m_1 m_2}{r}$, where $n = 10^6$. Hence $Q = \sqrt{n\varepsilon_0\varepsilon_r 4\pi\gamma m_1 m_2}$ Substitution of numerical data gives $Q = 1.7 \times 10^{-8}$ C.

9.7. (1) $\frac{W_e}{W_{gr}} = 4.17 \times 10^{42}$, (2) $\frac{W_{e.pr}}{W_{gr}} = 1.24 \times 10^{36}$.

9.8. How the energy W_e of two point charges depends on the distance r between them is shown in Fig. 94.

9.9. $E = 5.04 \times 10^4$ V/m.

9.10. $Q = -2.23 \times 10^{-9}$ C.

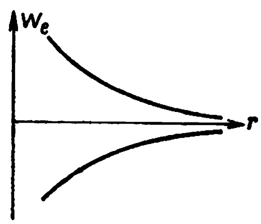


Fig. 94

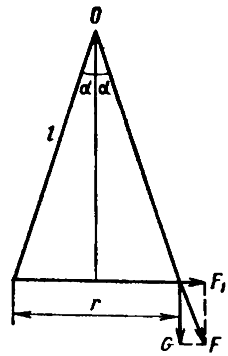


Fig. 95

9.11. Depending on the arrangement of the charges: (1) $E = 0$, (2) $E = 6 \times 10^4$ V/m, (3) $E = 3 \times 10^4$ V/m.

9.12. $E = 0$. **9.13.** $E = 1.12 \times 10^6$ V/m.

9.14. Let us designate the angle between the threads by 2α (Fig. 95). Each ball is acted upon by two forces: the weight of the ball G and the force of Coulomb repulsion F_1. The resultant of these forces is F. But $F_1 = G \tan\alpha = \frac{Q^2}{4\pi\varepsilon_0\varepsilon_r r^2}$ and $\frac{r}{2} = l\sin\alpha$. Hence finally

$$G = \frac{F_1}{\tan\alpha} = \frac{Q^2}{4\pi\varepsilon_0\varepsilon_r 4l^2\sin^2\alpha\tan\alpha}$$

Each ball carries the charge $Q = \frac{Q_0}{2}$. Upon inserting the numerical data, we find $G = 0.0157\ \text{N} = 1.6\times10^{-3}$ kgf.

9.15. $Q = 1.1\times10^{-6}$ C.

9.16. For a ball in air the following equation is true (see the solution to Problem 9.14)

$$G = \frac{Q^2}{4\pi\varepsilon_0\varepsilon_{r1}4l^2\sin^2\alpha_1\tan\alpha_1} \tag{1}$$

When the balls are immersed in kerosene, each ball is acted upon by the Archimedean force G_1. For a ball in kerosene

$$G - G_1 = \frac{Q^2}{4\pi\varepsilon_0\varepsilon_{r2}4l^2\sin^2\alpha_2\tan\alpha_2} \tag{2}$$

In equation (2)

$$G - G_1 = (\rho_1 - \rho_2)Vg \tag{3}$$

where ρ_1 is the density of the ball material, ρ_2 is the density of kerosene, V is the volume of the ball, g is the gravitational acceleration. From Eqs. (1), (2) and (3) we have

$$\frac{G - G_1}{G} = \frac{\sin^2\alpha_1\tan\alpha_1\varepsilon_{r1}}{\sin^2\alpha_2\tan\alpha_2\varepsilon_{r2}} = \frac{\rho_1 - \rho_2}{\rho_1}$$

whence

$$\rho_1 = \rho_2\frac{\sin^2\alpha_2\tan\alpha_2\varepsilon_{r2}}{\sin^2\alpha_2\tan\alpha_2\varepsilon_{r2} - \sin^2\alpha_1\tan\alpha_1\varepsilon_{r1}}$$

Upon inserting the numerical data, we obtain $\rho_1 = 2{,}550$ kg/m^3.

9.17. $\rho = \frac{\varepsilon_r\rho_1}{\varepsilon_r - 1}$. **9.18.** $\alpha = 13°$.

9.19. $\sigma = \frac{2\varepsilon_0\varepsilon_r\sqrt{F^2 - G^2}}{Q} = 7.8\times10^{-6}$ C/m^2.

9.20. (1) 2×10^{-5} N, (2) 12.6×10^{-5} N, (3) 6.28×10^{-5} N.

9.22. $E = 3.6\times10^{10}$ V/m. **9.23.** $F = 3.4$ N.

9.24. (1) $\frac{F}{l} = 8.1$ N/m, (2) $\frac{W}{l} = 0.112$ J/m.

9.25. $E = 3.12\times10^6$ V/m. The field is directed perpendicular to the plane passing through both filaments.

9.26. $\frac{F}{A} = 5.1\times10^3$ N/m^2.

9.27. The ball is acted upon by three forces: the force of the electric field F_1 directed upwards, the force of gravity G directed downwards, and the Archimedean force F_2 directed upwards. In equilibrium

$$G = F_1 + F_2 \tag{1}$$

we know that

$$G = mg = \rho_1 \frac{4}{3} \pi r^3 g \qquad (2)$$

where ρ_1 is the density of copper,

$$F_1 = EQ \qquad (3)$$

and

$$F_2 = \rho_2 \frac{4}{3} \pi r^3 g \qquad (4)$$

where ρ_2 is the density of oil. From Eqs. (1), (2), (3) and (4) we have

$$Q = \frac{4\pi r^3 g (\rho_1 - \rho_2)}{3E} = 1.1 \times 10^{-8}\,\mathrm{C}$$

9.28. $r = 4.4 \times 10^{-7}$ m.

9.29. We have

$$E = \frac{\tau \sin\theta}{2\pi\varepsilon_0\varepsilon_r a} \qquad (1)$$

It can easily be seen from a drawing that

$$\sin\theta = \frac{\frac{l}{2}}{\sqrt{a^2 + \left(\frac{l}{2}\right)^2}} \qquad (2)$$

where l is the length of the filament and a is the distance from the point under consideration to the filament. Upon inserting Eq. (2) into (1), we obtain

$$E = \frac{\tau l}{4\pi\varepsilon_0\varepsilon_r a \sqrt{a^2 + \left(\frac{l}{2}\right)^2}} \qquad (3)$$

(1) If $a \ll l$, then $\sqrt{a^2 + \left(\frac{l}{2}\right)^2} \cong \frac{l}{2}$. In this case formula (3) gives $E = \frac{\tau}{2\pi\varepsilon_0\varepsilon_r a}$, which is the intensity of an infinitely long filament.

(2) If $a \gg l$, then $\sqrt{a^2 + \left(\frac{l}{2}\right)^2} \cong a$. Besides, since $\tau l = Q$, formula (3) gives us $E = \frac{Q}{4\pi\varepsilon_0\varepsilon_r a^2}$, which is the intensity of the field of a point charge.

9.30. $\frac{a}{l} = \frac{\sqrt{1-(1-\delta)^2}}{2(1-\delta)} \cong \frac{1}{1-\delta}\sqrt{\frac{\delta}{2}}$. When $\delta = 0.05$ and $l = 0.25$ m, the maximum distance $a = 4.18 \times 10^{-2}$ m.

9.31. (1) $l = 0.49$ m, (2) $E = 1{,}350$ V/cm, (3) $\tau = 4.1 \times 10^{-9}$ C/m.

9.32. This problem is similar to Problem 2.159. (1) Let us take the ring element dl (see Fig. 81). This element carries the charge dQ. The intensity of the electric field produced by this element at point A $dE = \frac{dQ}{4\pi\varepsilon_0\varepsilon_r x^2}$. The intensity is directed along line x connecting the ring element dl to point A. Obviously, the intensity of the entire ring can be found by geometrical summation of $d\mathbf{E}$ produced by all the elements. The vector $d\mathbf{E}$ can be resolved into two components dE_t and dE_n. The components dE_n produced by each two diametrally opposite elements are mutually cancelled, and then

$$E=\int dE_t;\ \text{but}\ dE_t=dE\cos\alpha=dE\,\frac{l}{x}=\frac{l\,dQ}{4\pi\varepsilon_0\varepsilon_r x^2}$$

Hence $E=\frac{l}{4\pi\varepsilon_0\varepsilon_r x^3}\int dQ=\frac{lQ}{4\pi\varepsilon_0\varepsilon_r x^3}$. But $x=\sqrt{R^2+l^2}$ and finally

$$E=\frac{lQ}{4\pi\varepsilon_0\varepsilon_r (R^2+l^2)^{3/2}} \qquad (1)$$

is the intensity of the electric field on the ring axis.

If $l \gg R$, then $E=\frac{Q}{4\pi\varepsilon_0\varepsilon_r l^2}$, i. e., the charged ring may be regarded as a point charge at great distances. Upon inserting the numerical data into Eq. (1), we obtain $E=0$; 1,600; 1,710; 1,600; and 1,150 V/m, respectively.

(2) Let us express the quantities x and l through the angle α. We have $R=x\sin\alpha$ and $l=x\cos\alpha$. Now formula (1) becomes

$$E=\frac{Q}{4\pi\varepsilon_0\varepsilon_r R^2}\cos\alpha\sin^2\alpha$$

To determine the maximum of E, let us take the derivative $\frac{dE}{d\alpha}$ and equate it to zero:

$$\frac{dE}{d\alpha}=\frac{Q}{4\pi\varepsilon_0\varepsilon_r R^2}(\cos^2\alpha\, 2\sin\alpha-\sin^3\alpha)=0$$

or $\tan^2\alpha=2$. Then the distance l from the centre of the ring to point A where the intensity of the electric field is maximum is equal to $l=\frac{R}{\tan\alpha}=\frac{R}{\sqrt{2}}$. In our case $R=0.1$ m and, hence, $l=7.1\times10^{-2}$ m.

9.33. 1.3 times. Compare this problem with Problem 2.161.

9.34. (1) When $a \ll R$, the quantity $\frac{R}{a}$ is very high, and

$$\left[1-\frac{1}{\sqrt{1+\left(\frac{R}{a}\right)^2}}\right]\cong 1$$

Hence $E=\frac{\sigma}{2\varepsilon_0\varepsilon_r}$, i. e., for points close to the disk the latter may be considered as an infinitely long plane.

(2) When $a \gg R$, the quantity $\frac{R}{a}$ is small, and

$$\sqrt{1+\left(\frac{R}{a}\right)^2}\simeq 1-\frac{R^2}{2a^2}$$

Hence $E=\frac{\sigma}{2\varepsilon_0\varepsilon_r}\frac{R^2}{2a^2}$. But since $\sigma=\frac{Q}{\pi R^2}$, then $E=\frac{Q}{4\pi\varepsilon_0\varepsilon_r a^2}$, i. e., for points remote from the disk the latter may be considered as a point charge.

9.35. $\frac{a}{R}=\frac{\delta}{\sqrt{1-\delta^2}}\cong\delta$. When $\delta=0.05$ and $R=0.25$ m, we have $a=1.2\times\times10^{-2}$ m.

9.36. (1) $R=2.5$ m, (2) $E=11.3\times10^4$ V/m, (3) 1.1 times.

9.37. (1) $R=0.2$ m, (2) $\delta=10\%$.

9.38. $\frac{mv^2}{2}=\frac{Q_1Q_2}{4\pi\varepsilon_0\varepsilon_r r}$, where m is the mass of the ball moving with the velocity v, Q_1 is the charge of this ball, Q_2 is the point charge forming the field, r is the distance between the charges. Upon inserting the numerical data, we find $r=6\times10^{-2}$ m.

9.39. $r=5.1\times10^{-10}$ m. **9.40.** $r=6.1\times10^{-14}$ m.

9.41. (1) $r\cong6\times10^{-15}$ m, (2) $v=1.6\times10^{7}$ m/s. **9.42.** $W=1.2\times10^{-6}$ J.

9.44. (1) $U=11.3$ V, (2) $U=30$ V. **9.45.** $W=1.13\times10^{-4}$ J.

9.46. $v_1=16.7\times10^{-2}$ m/s.

9.47. It is convenient to give the answer in the form of the following table:

U, V	1	5	10	100	1,000
v, m/s	5.93×10^{5}	1.33×10^{6}	1.87×10^{6}	5.93×10^{6}	1.87×10^{7}

9.48. $E_k=8.5\times10^{-13}$ J $=5.32$ MeV, $U=2.66\times10^{6}$ V.

9.49. We have $dW=Q\,dU$, but $dU=-E\,dr=\frac{\tau dr}{2\pi\varepsilon_0\varepsilon_r r}$, and

$$W=-\int_{r_1}^{r_2}\frac{Q\tau dr}{2\pi\varepsilon_0\varepsilon_r r}=\frac{Q\tau}{2\pi\varepsilon_0\varepsilon_r}\log_e\frac{r_1}{r_2}$$

whence

$$\tau=\frac{2\pi\varepsilon_0\varepsilon_r W}{Q\log_e\frac{r_1}{r_2}} \qquad (1)$$

In our case $\varepsilon_0=8.85\times10^{-12}$ F/m, $\varepsilon_r=1$, $W=50\times10^{-7}$ J, $Q=\frac{2}{3\times10^{9}}$ C and $\frac{r_1}{r_2}=2$. Upon inserting these data in Eq. (1), we obtain $\tau=6\times10^{-7}$ C/m.

9.50. $\tau=3.7\times10^{-6}$ C/m. **9.51.** $v=2.97\times10^{7}$ m/s.

9.52. $\sigma=\frac{2W\varepsilon_0\varepsilon_r}{Q\Delta r}=6.6\times10^{-6}$ C/m^2. **9.53.** $d=4.8\times10^{-3}$ m.

9.54. $m=5.1\times10^{-16}$ kg.

9.55. In the absence of a field

$$mg=6\pi\eta r v_1 \qquad (1)$$

In the presence of a field

$$mg-EQ=6\pi\eta r v_2 \qquad (2)$$

We find from Eqs. (1) and (2) that $mg-EQ=\frac{v_2}{v_1}mg$, or

$$Q=\frac{mg}{E}\left(1-\frac{v_2}{v_1}\right)=\frac{mgd}{U}\left(1-\frac{v_2}{v_1}\right)=4.1\times10^{-18}\text{ C}$$

9.56. In the absence of an electric field

$$mg=6\pi\eta r v_1 \qquad (1)$$

In the presence of a field, the speck of dust is acted upon by the horizontal force $F=QE$. This force accelerates the speck of dust, but, owing to friction, the motion in the horizontal direction will also occur at a certain constant velocity v_2, and

$$QE=6\pi\eta r v_2 \quad (2)$$

The resultant of the velocities v_1 and v_2 is directed at the angle α, and $\tan\alpha=\frac{v_2}{v_1}=\frac{QE}{mg}$. Obviously, $\frac{v_2}{v_1}=0.5\frac{d}{l}$, whence the sought distance l can be found from the formula

$$l=\frac{0.5v_1 d}{v_2}=\frac{0.5mgd}{QE}=2\times10^{-2}\ \text{m}$$

Further, $v_2=\frac{v_1 d}{2l}=10^{-2}$ m/s. The sought time can be found from the formula $t=\frac{d}{2v_2}$, or $t=\frac{l}{v_1}$ By inserting the numerical data into any of these formulas, we obtain $t=1$ s.

9.57. $l=2\times10^{-2}$ m, $t=6.4\times10^{-2}$ s. **9.58.** $r=10^{-6}$ m, $Q=7.3\times10^{-18}$ C.

9.59. $Q=1.73\times10^{-9}$ C. **9.60.** 22 kV.

9.61. 2.2×10^{-5} m $=0.022$ mm. **9.62.** 5×10^{-3} m $=0.5$ cm.

9.63. (1) $U=2.8$ V, (2) $E=530$ V/m, (3) $\sigma=4.7\times10^{-9}$ C/m^2.

9.64. $v=\sqrt{\frac{2QU(r_1-r_2)}{md}}=2.53\times10^6$ m/s.

9.65. (1) $E=5.7$ V/m, (2) $v=10^6$ m/s, (3) $W=4.5\times10^{-19}$ J, $U=2.8$ V.

9.66. (1) $F=9.6\times10^{-14}$ N, (2) $a=1.05\times10^{17}$ m/s^2, (3) $v=3.24\times10^7$ m/s, (4) $\sigma=5.3\times10^{-6}$ C/m^2.

9.67 The electron will move in the plane capacitor along a parabola similar to a body thrown horizontally in a gravitational field. Indeed, the electron in the capacitor is acted upon by the constant force $F=eE$ and accelerated to $a=\frac{eE}{m}$ The electron covers the length l of the capacitor during the time $t=\frac{l}{v}$ and is deflected over the distance

$$y=\frac{at^2}{2}=\frac{eEl^2}{2mv^2} \quad (1)$$

For the electron not to fly out of the capacitor, the distance y should be greater than or equal to $\frac{d}{2}$, where d is the distance between the capacitor plates. Hence $v_0 \leqslant l\sqrt{\frac{eE}{md}}$. Upon inserting the numerical data, we obtain for the electron $v_0=3.64\times10^7$ m/s and for the α-particle $v_0=6\times10^5$ m/s.

9.68. (1) In 4.8×10^{-7} s, (2) $s_x=0.22$ m $=22$ cm.

9.69. $a_t=15.7\times10^{14}$ m/s^2, $a_n=8\times10^{14}$ m/s^2, $a_{tot}=17.6\times10^{14}$ m/s^2.

9.70. Two times.

9.71. The deflection of the proton and the α-particle is the same.

9.72. $v=\sqrt{v_x^2+v_y^2}=1.33\times10^7$ m/s, $\alpha=41°20'$.

9.73. $U_1=\frac{2Uyd}{l\left(l_1+\frac{l}{2}\right)}=28$ V. **9.74.** Over 0.01 m. **9.75.** 2.24 times.

9.76. $E_1=\frac{\varepsilon_{r2}U}{d_1\varepsilon_{r2}+d_2\varepsilon_{r1}}=60$ kV/m, $E_2=\frac{\varepsilon_{r1}E_1}{\varepsilon_{r2}}=10$ kV/m.

9.77. $C=7.1\times10^{-4}$ F, $\Delta U=1{,}400$ V. **9.78.** 2.5×10^{-20} kg.

9.79. The charge of n drops $Q_0=nQ$. This charge will be on the big drop. The radius of the big drop can be found from the condition $n\frac{4}{3}\pi r^3\rho=$ $=\frac{4}{3}\pi R^3\rho$, whence $R=r\sqrt[3]{n}$. Hence the potential of this drop will be $U=\frac{Q_0}{C}=$ $=\frac{nQ}{4\pi\varepsilon_0\varepsilon_r R}=\frac{nQ}{4\pi\varepsilon_0\varepsilon_r r\sqrt[3]{n}}$. In our case $n=8$, $Q=10^{-10}$ C, $\varepsilon_0=8.85\times10^{-12}$ F/m, $\varepsilon_r=1$, $r=10^{-3}$ m. Upon inserting these data, we obtain $U=3{,}600$ V.

9.80. $U=19{,}500$ V$=19.5$ kV. **9.81.** $r=2.1\times10^{-2}$ m$=2.1$ cm.

9.82. (1) $U=E_0R$ is a linear relationship, (2) $U=1.5\times10^6$ V.

9.83. $W_e=26.6\times10^{-7}$ J. **9.84.** $C=5.9\times10^{-9}$ F.

9.85. $\sigma=1.77\times10^{-6}$ C/m^2. **9.86.** $D=0.03$ m$=3$ cm.

9.87. In this case $Q_1=Q_2$, where Q_1 and Q_2 are the charges on the capacitor plates before and after they are filled, respectively. Thus, $Q=$const. Therefore the charge surface density on the plates is $\sigma=\frac{Q}{A}=$const.

(1) Since $E=\frac{\sigma}{\varepsilon_0\varepsilon_r}=\frac{U}{d}$, then $\sigma d=U_1\varepsilon_0\varepsilon_{r1}$ before filling, and $\sigma d=U_2\varepsilon_0\varepsilon_{r2}$ after filling. Since $\sigma=$const and $d=$const, then $U_1\varepsilon_{r1}=U_2\varepsilon_{r2}$, and $U_2=\frac{U_1\varepsilon_{r1}}{\varepsilon_{r2}}=115$ V.

(2) $C_1=\frac{\varepsilon_0\varepsilon_{r1}A}{d}=1.77\times10^{-11}$ F, $C_2=\frac{\varepsilon_0\varepsilon_{r2}A}{d}=4.6\times10^{-11}$ F.

(3) $\sigma_1=\sigma_2=\frac{Q}{A}=\frac{CU}{A}=5.31\times10^{-7}$ C/m^2.

9.88. In our case $U_1=U_2=U$. (1) $U_1=U_2=300$ V, (2) $C_1=1.77\times10^{-11}$ F, $C_2=4.6\times10^{-11}$ F, (3) $\sigma_1=5.31\times10^{-7}$ C/m^2, $\sigma_2=1.38\times10^{-6}$ C/m^2.

9.89. (1) Let us designate the intensity of the electric field in each layer by E_1 and E_2, and the potential drop in each layer by U_1 and U_2 Hence

$$\varepsilon_{r1}E_1=\varepsilon_{r2}E_2 \quad (1)$$

$$U_1+U_2=U \quad (2)$$

Equation (2) can be written as

$$E_1d_1+E_2d_2=U \quad (3)$$

From Eqs. (1) and (3) we have

$$E_1=\frac{U\varepsilon_{r2}}{\varepsilon_{r1}d_2+\varepsilon_{r2}d_1}=1.5\times10^4 \text{ V/m and } E_2=\frac{\varepsilon_{r1}E_1}{\varepsilon_{r2}}=4.5\times10^4 \text{ V/m}$$

(2) $U_1=75$ V, $U_2=225$ V

(3)
$$\frac{1}{C}=\frac{1}{C_1}+\frac{1}{C_2} \quad (4)$$

where

$$C_1=\frac{\varepsilon_0\varepsilon_{r1}A}{d_1} \text{ and } C_2=\frac{\varepsilon_0\varepsilon_{r2}A}{d_2} \quad (5)$$

By solving Eqs. (4) and (5) simultaneously, we get $C=\frac{\varepsilon_0\varepsilon_{r1}\varepsilon_{r2}A}{\varepsilon_{r1}d_2+\varepsilon_{r2}d_1}=$ $=2.66\times10^{-11}$ F.

(4) The charge on one of the plates $Q=\sigma A=C_1U_1=C_2U_2=CU$, whence $\sigma=\frac{CU}{A}=8\times10^{-7}$ C/m².

9.90. $U=1{,}800$ V. **9.91.** 2.14×10^{-4} μF/m.

9.92. We have $E=\frac{\tau}{2\pi\varepsilon_0\varepsilon_r x}$, where τ is the charge per unit length of the cable, and x is the distance from the cable axis. The quantity τ can be found from the following relation

$$C=\frac{2\pi\varepsilon_0\varepsilon_r l}{\log_e\frac{R}{r}}=\frac{Q}{U_0}=\frac{\tau l}{U_0},\quad \text{whence}\quad \tau=\frac{2\pi\varepsilon_0\varepsilon_r U_0}{\log_e\frac{R}{r}}$$

where U_0 is the potential difference between the core and the sheath. Hence the intensity of the field $E=\frac{U_0}{x\log_e\frac{R}{r}}$. Insertion of the data gives $E=136$ kV/m.

9.93. The work of the forces of the electric field is converted into the kinetic energy of the electron, i. e., $W=\frac{mv^2}{2}$. We have $dW=QdU=-QEdx$. Since

$$E=\frac{U_0}{x\log_e\frac{R}{r}},\quad \text{then}\quad W=-\int_{l_1}^{l_2}\frac{QU_0dx}{x\log_e\frac{R}{r}}=\frac{QU_0\log_e\frac{l_1}{l_2}}{\log_e\frac{R}{r}}=\frac{mv^2}{2},\quad \text{whence}$$

$$v=\sqrt{\frac{2QU_0\log_e\frac{l_1}{l_2}}{m\log_e\frac{R}{r}}}.$$ Upon inserting the numerical data, we obtain $V=1.46\times10^7$ m/s.

9.94. Inside the cylindrical capacitor the field intensity $E=\frac{U_0}{x\log_e\frac{R}{r}}$. Hence the potential drop in the first layer will be

$$U_1=-\int_{r+d_1}^{r}Edx=-\int_{r+d_1}^{r}\frac{U_0}{x\log_e\frac{R}{r}}dx=\frac{U_0\log_e\frac{r+d_1}{r}}{\log_e\frac{R}{r}}$$

The potential drop in the second layer is

$$U_2=\frac{U_0\log_e\frac{R}{r+d_1}}{\log_e\frac{R}{r}}$$

Whence

$$\frac{U_1}{U_2}=\frac{\log_e\frac{r+d_1}{r}}{\log_e\frac{R}{r+d_1}}=1.35$$

9.95. $C=9.6\times10^{-7}\ \mu F$. **9.96.** (1) $U=300$ V, (2) $U=75$ V.
9.97. $C=1.17\times10^{-9}$ F, $R=2.1$ m.
9.98. $E=\frac{UR_1R_2}{(R_2-R_1)x^2}=44.5$ kV/m.
Note. This problem is similar to Problem 9.92, but the field of a concentric spherical capacitor is taken.
9.99. $v=\sqrt{\frac{2QR_1R_2U(r_1-r_2)}{m(R_2-R_1)r_1r_2}}=1.54\times10^7$ m/s.
Note. This problem is similar to Problem 9.93, but the field of a concentric spherical capacitor is taken.
9.100. $C=0.33\ \mu F$ **9.101.** $C_1:C_2=3$.
9.102. $Q_1=Q_2=8\times10^{-6}$ C, $U_1=4$ V, $U_2=2$ V.
9.103. From 1×10^{-8} C to 1.7×10^{-7} C when connected in parallel and from 2.23×10^{-9} C to 3.27×10^{-9} C when connected in series.
9.104. From 20 pF to 900 pF when connected in parallel and from 5 pF to 225 pF when connected in series
9.105. $W_e=0.1$ J. **9.106** $W_e=0.05$ J.
9.107. (1) $R=7\times10^{-3}$ m, (2) $Q=7.0\times10^{-9}$ C, (3) $C=1.55\times10^{-6}\ \mu F$, (4) $W_e=1.58\times10^{-5}$ J.
9.108. (1) 5×10^{-5} J,
(2) the energy of each sphere is 1.25×10^{-5} J and the work of discharge during connection is 2.5×10^{-5} J,
(3) the energy of each sphere is 31.25×10^{-7} J and the work of discharge is 62.5×10^{-7} J.
9.109. (1) $U_1'=3$ kV, (2) $Q_2'=2\times10^{-8}$ C, (3) $W_{e1}'=1.5\times10^{-5}$ J and $W_{e2}'=9\times10^{-5}$ J, (4) $Q_1''=1.8\times10^{-8}$ C and $U_1''=5.4$ kV, (5) $Q_2''=1.2\times10^{-8}$ C, $U_2''=5.4$ kV, (6) $W_e=8.1\times10^{-5}$ J, (7) $W=2.4\times10^{-5}$ J.
9.110. $Q=2.7\times10^{-6}$ C.
9.111. (1) $Q=1.77\times10^{-7}$ C, (2) $E=3{,}330$ V/cm, (3) $W_e=2.94$ J/m^3.
9.112. $p=26.5$ N/m^2. **9.113.** $U=15$ kV.
9.114. (1) $E=560$ V/cm, (2) $d=5\times10^{-3}$ m $=5$ mm, (3) $v=10^7$ m/s, (4) $W_e=6.95\times10^{-7}$ J, (5) $C=1.77\times10^{-11}$ F, (6) 13.9×10^{-5} N.
9.115. $U=21.7$ kV
9.116. $E=6\times10^4$ V/m, $W_{e1}=2\times10^{-5}$ J, $W_{e2}=0.8\times10^{-5}$ J.
9.117 $E_2=E_1=150$ kV/m, $W_{e1}=2\times10^{-5}$ J, $W_{e2}=5\times10^{-5}$ J.
9.118. (1) $W_{e1}=4.43\times10^{-7}$ J, $W_{e2}=1.78\times10^{-6}$ J, (2) $W_{e1}=4.43\times10^{-7}$ J, $W_{e2}=1.11\times10^{-6}$ J.
9.119 $\varepsilon_r=4.5$.
9.120. (1) (a) The capacitance decreased by 1.1 pF, (b) the intensity flux decreased by 750 V, (c) the volume density of the energy decreased by 4.8×10^{-2} J/m^3.
(2) (a) As in the first case, the capacitance decreased by 1.1 pF, (b) the intensity flux did not change ($\Delta N_E=0$), (c) the volume density of the energy also remained the same ($\Delta W_0=0$).
9.121. (1) $W_0=\frac{\sigma^2R^4}{2\varepsilon_0\varepsilon_r(R+x)^4}$, where R is the radius of the ball and x is the distance to the point under consideration from the surface of the ball Upon inserting the numerical data, we obtain $W_0=9.7\times10^{-2}$ J/m^3, (2) $W_0=\frac{\sigma^2}{8\varepsilon_0\varepsilon_r}=1.97$ J/m^3, (3) $W_0=\frac{\tau^2}{8\pi^2\varepsilon_0\varepsilon_r x^2}=0.05$ J/m^3.
9.122. Let us denote the surface density of the charge on the capacitor plates without the dielectric by σ_1, with the dielectric by σ_d, and the surface density

of the bound (polarizing) charges by σ_b. The joint action of the charges σ_d and σ_b is such as if on the boundary between the conductor and the dielectric there is a charge distributed with the density

$$\sigma' = \sigma_d - \sigma_b \tag{1}$$

Thus, σ' is the surface density of the "effective" charges, i.e., the charges which determine the total resulting field in the dielectric. Obviously, the quantities σ are related to the corresponding fields as follows: the field without the dielectric

$$E_0 = \frac{\sigma_0}{\varepsilon_0} = \frac{U_1}{d} \tag{2}$$

and the resulting field in the dielectric

$$E = \frac{\sigma_d}{\varepsilon_0 \varepsilon_r} = \frac{\sigma'}{\varepsilon_0} = \frac{U_2}{d} \tag{3}$$

From Eq. (1), $\sigma_b = \sigma_d - \sigma'$, or from Eq. (3)

$$\sigma_b = \varepsilon_0 \varepsilon_r E - \varepsilon_0 E = \varepsilon_0 (\varepsilon_r - 1) E = \varepsilon_0 (\varepsilon_r - 1) \frac{U_2}{d}$$

(1) In this case $U_1 = U_2 = U$, and thus

(a) $\sigma_b = \varepsilon_0 (\varepsilon_r - 1) \dfrac{U}{d} = \dfrac{8.85 \times 10^{-12} \times 6 \times 10^3}{3 \times 10^{-3}}$ C/m^2 $= 1.77 \times 10^{-5}$ C/m^2,

(b) $\sigma_d - \sigma_0 = \varepsilon_0 \varepsilon_r E - \varepsilon_0 E_0$, and since with the power source connected $E = E_0 = \dfrac{U}{d}$, then

$$\sigma_d - \sigma_0 = \varepsilon_0 (\varepsilon_r - 1) \frac{U}{d} = \sigma_b = 1.77 \times 10^{-5} \text{ C/m}^2$$

Thus, the power source causes additional charges to appear on the capacitor plates, which compensate for the drop in the charge due to polarization of the dielectric.

(2) In this case $Q = \text{const}$ and $U_2 = \dfrac{\varepsilon_{r1} U_1}{\varepsilon_{r2}}$ (see the solution to Problem 9.87) and hence

(a) $\sigma_b = \varepsilon_0 (\varepsilon_r - 1) \dfrac{U_2}{d} = \varepsilon_0 (\varepsilon_r - 1) \dfrac{\varepsilon_{r1} U_1}{\varepsilon_{r2} d} = \dfrac{8.85 \times 10^{-12} \times 6 \times 1 \times 10^3}{7 \times 3 \times 10^{-3}}$ C/m^2 =

$= 2.53 \times 10^{-6}$ C/m^2,

(b) since $Q = \text{const}$, then $\sigma_b = \sigma_0$, i.e., the surface density of the charge on the capacitor plates does not change.

9.123. The polarization vector P numerically equal to the surface density of the bound charges σ_b is proportional to the intensity of the field in the dielectric, i.e., $P = \sigma_b = \varkappa' E$. In the MKSA system the coefficient $\varkappa'$ is not a dimensionless quantity, but is measured in F/m. It can be shown that $\varkappa' = 4\pi\varepsilon_0 \varkappa$, where $\varkappa$ is a dimensionless quantity (the tabulated value of the electric susceptibility). Hence

$$\sigma_b = 4\pi\varepsilon_0 \varkappa E = 4\pi\varepsilon_0 \varkappa \frac{U}{d} = \frac{4 \times 3.14 \times 8.85 \times 10^{-12} \times 0.08 \times 4 \times 10^3}{5 \times 10^{-3}} \text{ C/m}^2 = 7.1 \times 10^{-6} \text{ C/m}^2$$

Let us find the relative permittivity (dielectric constant). Since $\sigma_b = \varepsilon_0 (\varepsilon_r - 1) E$ (see the solution to the previous problem), then $\sigma_b = 4\pi\varepsilon_0 \varkappa E = \varepsilon_0 (\varepsilon_r - 1) E$, whence $\varepsilon_r - 1 = 4\pi\varkappa$, or $\varepsilon_r = 1 + 4\pi\varkappa = 1 + \dfrac{\varkappa'}{\varepsilon_0}$, whence $\varepsilon_r = 1 + 4\pi \times 0.08 = 2$.

Hence $E=\frac{U}{d}=\frac{\sigma_d}{\varepsilon_0\varepsilon_r}$ and the surface density of the charge on the capacitor plates is

$$\sigma_d=\frac{U\varepsilon_0\varepsilon_r}{d}=\frac{4\times10^3\times8.85\times10^{-12}\times2}{5\times10^{-3}}\text{ C/m}^2=1.4\times10^{-5}\text{ C/m}^2$$

9.124. (1) $E=3$ kV/cm, (2) $\sigma_d=1.59\times10^{-5}$ C/m², (3) $\sigma_b=1.33\times10^{-5}$ C/m², (4) $\varkappa'=\frac{\sigma_b}{E}=4.44\times10^{-11}$ F/m, $\varkappa=\frac{\varkappa'}{4\pi\varepsilon_0}=0.4$.

9.125. $U=1{,}750$ V. **9.126.** $\sigma_b=6\times10^{-6}$ C/m².

9.127. (1) $E=7.52\times10^5$ V/m, $D=\varepsilon_0\varepsilon_r E=1.33\times10^{-5}$ C/m², (2) $\sigma_b=6.7\times10^{-6}$ C/m², (3) $\sigma_d=1.33\times10^{-5}$ C/m², (4) $W_0=5$ J/m³, (5) $\varkappa'=8.9\times10^{-12}$ F/m, $\varkappa=0.08$.

9.128. (1) $\sigma_b=5.3\times10^{-6}$ C/m², (2) $\varkappa'=1.77\times10^{-11}$ F/m, $\varkappa=0.159$.

9.129. (1) $W=1.97\times10^{-5}$ J, (2) $W=9.8\times10^{-5}$ J.

10. Electric Current

10.1. (1) $Q=\int_{t_1}^{t_2} I\,dt=\int_{t_1}^{t_2}(4+2t)\,dt=48$ C, (2) $I=12$ A.

10.2. (1) $R=70\ \Omega$, (2) (a) 87.5 Ω, (b) 116.7 Ω, (c) 175 Ω, (d) 350 Ω.

10.3. $N=200$ turns. **10.4.** $l=500$ m, $d=10^{-3}$ m $=1$ mm.

10.5. $R=0.0018\ \Omega$. **10.6.** 2.22 times.

10.7. $R_1=R_0(1+\alpha t_1)$, where R_0 is the resistance at 0°C (and not at the initial temperature). Hence $R_0=\frac{R_1}{1+\alpha t_1}=32.8\ \Omega$. Also $R_2=\frac{U}{I}=364\ \Omega$, and since $R_2=R_0(1+\alpha t_2)$, then $t_2=\frac{R_2-R_0}{R_0\alpha}=2200$°C.

10.8. 17.5 mA. **10.9.** To a temperature of $t=70$°C.

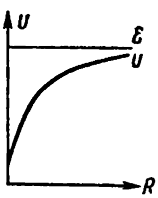


Fig. 96

10.10 $U=5.4$ V.

10.11. $U_1=12$ V, $U_2=U_3=4$ V, $I_2=2$ A, $I_3=1$ A.

10.12. (1) $I=0.11$ A, (2) $U_1=0.99$ V, (3) $U_2=0.11$ V, (4) $\eta=0.9$.

10.13 $U=\frac{\mathscr{E}}{R+r}R=\frac{1.1}{1+R}R$. The curve in Fig. 96 shows how the potential drop U in the external circuit depends on the external resistance R. The curve asymptotically approaches the straight line $U=\mathscr{E}=1.1$ V.

10.14 $U=0.125$ V, $R=7.5\ \Omega$. **10.15.** $\eta=25\%$

10.16. $U=2.7$ V, $r=0.9\ \Omega$.

10.17. $x=\frac{U}{\mathscr{E}}=\frac{n}{1+n}$; (1) $x=9.1\%$, (2) $x=50\%$, (3) $x=91\%$.

10.18. $\eta=80\%$.

10.19. In series connection of the elements $I'=\frac{2\mathscr{E}}{2r+R}$, and in parallel connection $I''=\frac{\mathscr{E}}{0.5r+R}$

(1) $I'=\frac{2\times2}{0.6+0.2}$ A $=5$ A, $I''=\frac{2}{0.15+0.2}$ A $=5.7$ A,

(2) $I'=\frac{4}{0.6+16}$ A $=0.24$ A, $I''=\frac{2}{0.15+16}$ A $=0.124$ A.

Thus, when the external resistance is small, it will be good to connect the elements in parallel, and when it is great — in series.

10.20. (1) $\frac{\Delta R}{R}=1\%$, (2) $\frac{\Delta R}{R}=10\%$, (3) $\frac{\Delta R}{R}=100\%$.

10.21. (1) $\frac{\Delta R}{R}=20\%$, (2) $\frac{\Delta R}{R}=2\%$, (3) $\frac{\Delta R}{R}=0.2\%$.

10.22. $I_1=0.6$ A, $I_2=0.4$ A, $I=I_1+I_2=1$ A.

10.23. The intensity of the current in the circuit $I=\frac{2\mathcal{E}}{R+r_1+r_2}=\frac{4}{3}$ A. The potential difference across the terminals of the first element $U_1=\mathcal{E}-Ir_1=$ $=\frac{2}{3}$ V. The potential difference across the terminals of the second element $U_2=\mathcal{E}-Ir_2=0$. Determine in the general form the relationship between R, r_1 and r_2 with which the potential difference across the terminals of one of the elements will be equal to zero.

10.24. $R_1=1.5\ \Omega$, $R_2=2.5\ \Omega$, $U_1=7.5$ V and $U_2=12.5$ V.

10.25. $\mathcal{E}=2$ V, $r=0.5\ \Omega$.

10.26. $I=0.2$ A.

10.27. $R_1=60\ \Omega$.

10.28. (1) $I=0.4$ A, (2) $U=32$ V.

10.29. $R_2=60\ \Omega$.

10.30. (1) $I=2$ A, (2) $U=2$ V.

10.31. 80 V.

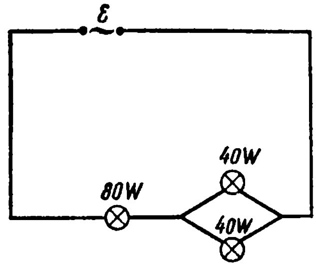


Fig. 97

10.32. $\mathcal{E}=170$ V.

10.33. (1) 0.22 A and 110 V, (2) 0.142 A and 53.2 V; (3) 0.57 A and 110 V, (4) 0.089 A and 35.6 V.

10.34. $I=40$ A.

10.35. (1) The resistance $R=0.02\ \Omega$ should be connected in parallel with the ammeter, (2) the graduation of the ammeter will change and become 1 A/div instead of 0.1 A/div.

10.36. (1) The resistance $R=3{,}000\ \Omega$ should be connected in series with the voltmeter, (2) the graduation of the voltmeter will change and become 0.5 V/div instead of 0.2 V/div.

10.37. (1) The resistance $R=0.555\ \Omega$ is connected in parallel with the instrument, (2) the resistance $R=9{,}950\ \Omega$ is connected in series with the instrument.

10.38. $R=300\ \Omega$, $l=21.2$ m.

10.39. See the diagram in Fig. 97; $I_1=I_2=0.365$ A and $I_3=0.73$ A.

10.40. By 6.8 V. **10.41.** 208 W. **10.42.** $A=7.2\times10^{-4}$ m^2.

10.43. (1) $\frac{Q_c}{Q_s}=0.17$, (2) $\frac{U_c}{U_s}=0.17$. **10.44.** (1) $\frac{Q_c}{Q_s}=5.9$, (2) $\frac{U_c}{U_s}=1$.

10.45. $Q=1.08$ kJ.

10.46. (1) 2.4 kW, (2) 2.3 kW, (3) 96%.

10.47. $r=1\ \Omega$, $\eta_1=83.3\%$, $\eta_2=16.7\%$.

10.48. Compile a table using the crosses on the curve in Fig. 34:

I, A	0	1	2	3	4	5	6	7	8	9	10
P, W	0	1.8	3.2	4.2	4.8	5	4.8	4.2	3.2	1.8	0

The power produced in the external circuit (net power) reaches its maximum when the external resistance is equal to the internal resistance of the element. The potential drop in the external circuit is $U=\frac{\mathscr{E}}{2}$, where $\mathscr{E}$ is the e.m.f. of the element. Thus the efficiency of the element is $\eta=0.5$. In our case $P_{max}=IU=5$ W. Hence $U=\frac{P_{max}}{I}=\frac{5}{5}$ V$=1$ V, and therefore the sought e.m.f. of the element $\mathscr{E}=2U=2$ V. Since $I=\frac{\mathscr{E}}{2r}$, the sought internal resistance of the element $r=\frac{\mathscr{E}}{2I}=0.2\ \Omega$. The potential drop in the external circuit $U=\frac{P}{I}$, and the efficiency of the element $\eta=\frac{U}{\mathscr{E}}=\frac{P}{\mathscr{E}I}$.

10.49. By using the crosses on the curve in Fig. 34, we find (see the solution to the previous problem) $\mathscr{E}=2$ V and $r=0.2\ \Omega$. The values of $\mathscr{E}$ and r being known, it is easy to determine the required quantities η, P_1 and P_2.

10.50. $\mathscr{E}=4$ V, $r=1\ \Omega$.

10.51. For the dependence of U, P_1 and P_2 on R see the solutions to Problems 10.48 and 10.49.

10.52. $\mathscr{E}=6$ V, $r=1\ \Omega$. **10.53.** 60 W.

10.54. 1 A. **10.55.** 16 W. **10.56.** $\mathscr{E}=100$ V.

10.57. The potential difference across the lamp terminals changes from 30 to 54.5 V. The power consumed by the lamp changes from 30 to 9.9 W.

10.58. (1) $Q_1=6.37$ J, $Q_2=3.82$ J, (2) $Q_1=16.2$ J, $Q_2=27.2$ J.

10.59. More power (1.5 times) is consumed by the lamp with the smaller resistance.

10.60. By 36°. **10.61.** 2.9 litres. **10.62.** (1) 1.2 kW, (2) 12 Ω.

10.63. $Q=2.5\times10^5$ J$=60$ kcal.

10.64. (1) 25 min, (2) 50 min, (3) 12.5 min.

10.65. (1) 45 min, (2) 10 min. **10.66.** In 22 min.

10.67. (1) 5.4 Ω, (2) 2,100 J/kg·deg, (3) 49.6 Ω. **10.68.** $\eta=80\%$.

10.69. (1) 14.4 Ω, (2) 11.3 m, (3) 1 kW. **10.70.** By 3°.

10.71. 133 kopeks. **10.72.** In 49 min. **10.73.** $R=33\ \Omega$.

10.74. The amount of heat liberated in the copper wire is

$$Q_1=m_1c_1\Delta t=\delta_1 l_1 A_1 c_1 \Delta t \qquad (1)$$

where δ is the density of copper, l_1 the length of the wire, A_1 its cross-sectional area, c_1 the specific heat of copper, and Δt is the increase in the temperature of the wire.

The amount of heat liberated in the lead fuse link is

$$Q_2=\delta_2 l_2 A_2 (c_2\Delta t_1+r) \qquad (2)$$

where r is the specific heat of fusion of lead, δ_2 the density of lead, l_2 the length of the fuse link, A_2 its cross-sectional area, c_2 the specific heat of lead, and $\Delta t_1=t_f-t_0$.

Since both wires are connected to the circuit in series, we have

$$I_1=I_2 \text{ and } \frac{Q_1}{Q_2}=\frac{R_1}{R_2}=\frac{l_1A_2\rho_1}{l_2A_1\rho_2}. \qquad (3)$$

where ρ_1 and ρ_2 are the resistivities of copper and lead, respectively. From Eqs. (1), (2) and (3) we have

$$\frac{\delta_1 l_1 A_1 c_1 \Delta t}{\delta_2 l_2 A_2 (c_2\Delta t_1+r)}=\frac{\rho_1 l_1 A_2}{\rho_2 l_2 A_1}$$

whence the sought temperature difference is

$$\Delta t = \frac{\rho_1 \delta_2 A_2^2 (c_2 \Delta t_1 + r)}{\rho_2 \delta_1 A_1^2 c_1}$$

In our case (see the relevant tables in the Appendix) $\rho_1 = 1.7 \times 10^{-8}\ \Omega$ m, $\rho_2 = 2.2 \times 10^{-7}\ \Omega \cdot$m, $\delta_1 = 8{,}600$ kg/m³, $\delta_2 = 11{,}300$ kg/m³, $c_1 = 395$ J/kg deg, $c_2 = 126.0$ J/kg·deg, $t_f = 327°$C, $r = 2.26 \times 10^4$ J/kg, $t_f - t_0 = 327° - 17° = 310°$ Upon inserting these data, we obtain $\Delta t = 1.8°$.

10.75. 1.55×10^3 J/m³·s. **10.76.** $I_1 = I_2 = 26.7$ mA, $I_3 = I_4 = 4$ mA.

10.77. Let us use Kirchhoff's law for the given branched circuit. First of all, let us mark the direction of the currents with arrows in Fig. 98.

Assume that the currents flow in the direction of these arrows. According to Kirchhoff's first law, for the junction C

$$I_3 = I_1 + I_2 \tag{1}$$

(An identical equation is obtained for junction A.) According to Kirchhoff's second law, for circuit ABC

$$I_3 R_3 + I_1 R_1 = \mathscr{E}_1 \tag{2}$$

and for circuit ACD

$$I_1 R_1 - I_2 R_2 = \mathscr{E}_2 \tag{3}$$

(Circuit $ABCD$ could be taken instead of circuit ACD or ABC.)

Now, we have three equations for determining three unknown quantities I_1, I_2 and I_3. When problems are solved on the basis of Kirchhoff's laws, it is more convenient to present equations (1), (2) and (3) in their numerical form. In our case these equations will take the form:

$$I_3 = I_1 + I_2 \tag{1a}$$

$$10 I_3 + 45 I_1 = 2.1 \tag{2a}$$

$$45 I_1 - 10 I_2 = 1.9 \tag{3a}$$

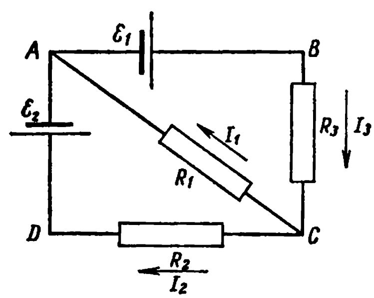


Fig. 98

Upon solving these equations, we get $I_1 = 0.04$ A, $I_2 = -0.01$ A and $I_3 = 0.03$ A. The negative sign of I_2 shows that the direction of the current we have taken is wrong, the current I_2 will actually flow from D to C, and not vice versa, as was assumed when compiling the equation.

10.78. $U = 1.28$ V. **10.79.** $R = \frac{2}{3}\ \Omega$, $I_2 = 0.5$ A, $I_R = 1.5$ A.

10.80. $R = 0.75\ \Omega$, $I_2 = 2$ A, $I_R = 4$ A. **10.81.** $I = 0.4$ A. **10.82.** 2 A.

10.83. $R_1 = 20\ \Omega$. **10.84.** $I = 0.45$ mA.

10.85. $I = 0.001$ A $= 1$ mA.

10.86. $I_1 = 0.385$ A, $I_2 = 0.077$ A, $I_3 = 0.308$ A.

10.87. $I_1 = 0.3$ A, $I_2 = 0.5$ A, $I_3 = 0.8$ A, $R_3 = 7.5\ \Omega$.

10.88. $\mathscr{E}_2 = 35$ V, $\mathscr{E}_3 = 55$ V. **10.89.** $I = 9$ A.

10.90. $\mathscr{E}_1 = 24$ V, $\mathscr{E}_2 = 12$ V, $I_2 = 1.2$ A, $I_3 = 0.3$ A.

10.91. (1) 2.22 A, (2) 0.44 A, (3) 1.78 A.

10.92. $\mathscr{E}_1 = \mathscr{E}_2 = 4$ V, $r_1 = r_2 = 1\ \Omega$. **10.93.** 100 V.

10.94. $\mathscr{E}_1 = \mathscr{E}_2 = 200$ V. **10.95.** 75 mA.

10.96. (1) $U_1 = 120$ V, $U_2 = 80$ V, (2) $U_1 = U_2 = 100$ V.

10.97. In 2 hours. **10.98.** (1) 10 min, (2) 4.6×10^{-6} m.
10.99. $J=56$ A/m^2. **10.100.** $K=1.04\times10^{-8}$ kg/C.
10.101. The ammeter shows 0.04 A less. **10.102.** 53 mg.
10.103. (1) 149 h, (2) 1.49×10^4 kW·h. **10.104.** $W_e=1{,}800$ J.
10.105. The energy required to liberate a mass M of a substance in electrolysis is

$$W_e=IUt=\frac{MUZF}{M_A} \tag{1}$$

where F is Faraday's number, M_A the mass of a kilogram-atom, Z the valency and U the applied potential difference. To decompose 2 kmoles of water, i. e., to produce 4 kg of hydrogen, 5.75×10^8 J of energy are required. Thus, in our case, $M=4$ kg, $W_e=5.75\times10^8$ J. Upon inserting the numerical data into Eq. (1), we obtain $U=1.5$ V.

10.106. In weak solutions $\alpha\approx1$, i. e., all the molecules are dissociated. Therefore, the equivalent conductivity $\Lambda_\infty=F(u_++u_-)$. In our case $F=96.5\times\times10^6$ C/kg-eq, $u_+=3.26\times10^{-7}$ m^2/V·s and $u_-=6.4\times10^{-8}$ m^2/V·s. Upon inserting these data, we get $\Lambda_\infty=37.6$ m^2/Ω·kg-eq.
10.107. $Q_+=100$ C, $Q_-=20$ C.
10.108. (1) $\alpha=94\%$, (2) $\eta=10^{-2}$ kg-eq/m^3 $=10^{-2}$ g-eq/l $=0.01$ N, (3) $u_++u_-=1.35\times10^{-7}$ m^2/V·s.
10.109. $R=1.8\times10^6$ Ω. **10.110.** $R=5.2\times10^6$ Ω.
10.111. 3.9 m^2/Ω·kg-eq. **10.112.** 92%.
10.113. $n_+=n_-=5.5\times10^{26}$ m^{-3}. **10.114.** 10^{-6}.
10.115. (1) $J=2.4\times10^{-7}$ A/m^2, (2) $\frac{I_+}{I}=0.01\%$. **10.116.** $I_s=10^{-7}$ A.

10.117. The maximum possible number of ionic pairs in 1 cm^3 of the chamber is obtained if the number of ions diminishes only due to their recombination. In this case $N=\alpha n^2$ and $n=\sqrt{\frac{N}{\alpha}}=3.2\times10^7$

10.118. $R=3.4\times10^{14}$ Ω. **10.119.** $I=3.3\times10^{-9}$ A, $\frac{I}{I_s}=3.3\%$.

10.120. The ionization potential of an atom is the potential difference which must be passed by an electron to ionize it when it collides with the atom. For this reason the velocity of the electron can be found from the equation $\frac{mv^2}{2}=eU$, or $v=\sqrt{\frac{2eU}{m}}$. Upon inserting the numerical data, we obtain $v=2.2\times10^6$ m/s.
10.121. At 80,000° K. **10.122.** 39.2×10^{-19} J.
10.123. (1) 8.3×10^5 m/s, (2) 1.4×10^6 m/s.
10.124. At a temperature of T_1 the emissivity of tungsten is

$$j_1=BT_1^2e^{-\frac{W}{kT_1}} \tag{1}$$

and at a temperature of T_2

$$j_2=BT_2^2e^{-\frac{W}{kT_2}} \tag{2}$$

Upon dividing Eq. (2) by Eq. (1), we obtain

$$\frac{j_2}{j_1}=\left(\frac{T_2}{T_1}\right)^2 e^{-\frac{W}{k}\left(\frac{1}{T_2}-\frac{1}{T_1}\right)} \tag{3}$$

In our case $T_1=2{,}400°$ K, $T_2=2{,}500°$ K, $W=4.54$ eV $=4.54\times1.6\times10^{-19}$ J, and $k=1.38\times10^{-23}$ J/deg. Insertion of these data into Eq. (3) gives $\frac{j_2}{j_1}=2.6$.

10.125. 11,000 times.

10.126. The emissivity of pure tungsten at a temperature of $T_1=2{,}500°$ K is equal to $j_1=B_1T_1^2e^{-\frac{W_1}{kT_1}}=2.84\times10^3$ A/m². The emissivity of thoriated tungsten at a temperature of T_x is equal to $j_2=B_2T_x^2e^{-\frac{W_2}{kT_x}}$. According to the given condition, $j_1=j_2$, i. e.,

$$B_2T_x^2e^{-\frac{W_2}{kT_x}}=2.84\times10^3 \text{ A/m}^2 \qquad (1)$$

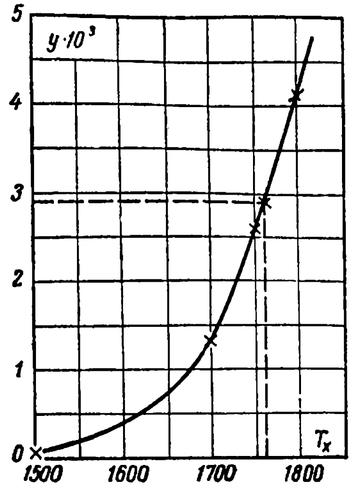


Fig. 99

Equation (1) can be solved in one of two ways: (1) graphically, and (2) by the method of successive approximations. Let us consider each of them.

1. *Graphical method.* Let us plot the values of T_x on the axis of abscissas and those of $y\times10^{-3}=B_2T_x^2e^{-\frac{W_2}{kT_x}}$ on the axis of ordinates (Fig. 99). The abscissa of the point of intersection of this curve with the horizontal straight line $y=2.84\times10^3$ will give us the sought temperature. It is convenient to tabulate the results of calculations as shown below.

T_x °K	$z=\frac{W_2}{kT_x}$	e^{-z}	$y\times10^{-3}$
1500	20.3	1.6×10^{-9}	0.11
1700	17.7	1.6×10^{-8}	1.38
1750	17.1	3.7×10^{-8}	2.54
1800	16.7	5.6×10^{-8}	4.25

The diagram in Fig. 99 shows that the solution to equation (1) is $T_x\simeq1760$ °K.

(2) *Method of successive approximations.* Since the relation between the emissivity and the temperature is mainly determined by the exponential factor $e^{-\frac{W}{kT}}$, and not by the factor T^2, it may be assumed to a first approximation that

$$B_2T_1^2e^{-\frac{W_2}{kT_x}}=B_2\,(2500)^2\,e^{-\frac{W_2}{kT_x}}=2.84\times10^3 \text{ A/m}^2$$

whence $e^{-\frac{W_2}{kT_x}}=\frac{2.84\times10^3}{B_2T_1^2}=1.86\times10^{-8}$ and $T_x=1690$ °K is the first approximation.

To a second approximation

$$B_2 (1690)^2 e^{-\frac{W_2}{kT_x}} = 2.84\times 10^3 \text{ A/m}^2$$

whence $T_x = 1770$°K is the second approximation.

Further

$$B_2 (1770)^2 e^{-\frac{W_2}{kT_x}} = 2.84\times 10^3 \text{ A/m}^2$$

whence $T_x = 1750$°K is the third approximation.

Similarly,

$$B_2 (1750)^2 e^{-\frac{W_2}{kT_x}} = 2.84\times 10^3 \text{ A/m}^2$$

whence $T_x = 1760$°K is the fourth approximation.

It is easy to see that the fifth approximation coincides with the fourth approximation with an accuracy to the third digit. Hence, the solution sought is $T_x = 1760$°K.

11. Electromagnetism

11.1. $H = 39.8$ A/m. **11.2.** $H = 50$ A/m.

11.3. $H_1 = 120$ A/m, $H_2 = 159$ A/m, $H_3 = 135$ A/m.

11.4. $H_1 = 199$ A/m, $H_2 = 0$, $H_3 = 183$ A/m.

11.5. The point at which the intensity of the magnetic field is zero lies between points I_1 and I_2 at a distance of 3.3 cm from A.

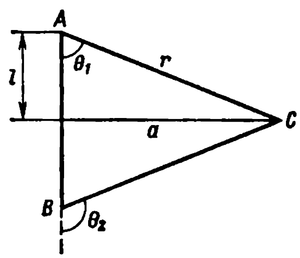


Fig. 100

11.6. The points at which the intensity of the magnetic field is zero lie to the right of point A at distances of 1.8 cm and 6.96 cm from it.

11.7. $H_1 = 8$ A/m, $H_2 = 55.8$ A/m.

11.8. $H_1 = 35.6$ A/m, $H_2 = 57.4$ A/m.

11.9. $H = 8$ A/m. The intensity of the magnetic field is directed perpendicular to the plane passing through both conductors.

11.10. The resulting field will be directed vertically upward if the field of the current compensates for the horizontal component of the terrestrial magnetic field. Since

$$H = H_h = \frac{I}{2\pi r}, \text{ then } r = \frac{I}{2\pi H_h} = 0.08 \text{ m.}$$

11.11. The intensity of the magnetic field at point C will be (see Fig. 100) $H = \int_{\theta_2}^{\theta_1} \frac{I \sin\theta dl}{4\pi r^2}$. But $l = a\cot\theta$ and $dl = -\frac{a\,d\theta}{\sin^2\theta}$. Further $r = \frac{a}{\sin\theta}$ Hence

$H = -\frac{I}{4\pi a}\int_{\theta_2}^{\theta_1} \sin\theta\, d\theta = \frac{I}{4\pi a}(\cos\theta_1 - \cos\theta_2)$. According to the initial condition, $I = 20$ A, $a = 5\times 10^{-2}$ m, $\theta_1 = 60°$, and $\theta_2 = 180° - 60° = 120°$. Upon inserting these data, we obtain $H = 31.8$ A/m.

11.12. $H = 56.5$ A/m. **11.13.** $a \leq 5$ cm.

11.14. (1) $l \geq 0.245$ m, (2) $H = 358$ A/m. **11.15.** $H = 77.3$ A/m.

11.16. $U = \frac{\pi\rho I^2}{AH} = 0.12$ V. **11.17.** $H = 12.7$ A/m.

11.18. $H = 25.7$ A/m. **11.19.** (1) $H = 12.2$ A/m, (2) $H = 0$.
11.20. (1) $H = 62.2$ A/m, (2) $H = 38.2$ A/m. **11.22.** $H = 177$ A/m.
11.23. $H = 35.8$ A/m. **11.24.** $U_2 = 4U_1$. **11.25.** $L = 0.2$ m.
11.26. $r = 8\times10^{-2}$ m. **11.27.** $H = 6{,}670$ A/m. **11.28.** $H = 1{,}250$ A/m.
11.29. 4 layers. **11.30.** (1) $NI = 200$ At, (2) 2.7 V.
11.31. $\frac{L}{D} = \frac{1-\delta}{\sqrt{1-(1-\delta)^2}} \cong \frac{1-\delta}{\sqrt{2\delta}}$; when $\delta \leqslant 0.05$ we get $\frac{L}{D} \geqslant 3$.
11.32. $\delta = 3\%$.
11.33. The relation $H = f(x)$ is shown in Fig. 101.
11.34. $H_h = 16$ A/m.
Note. See the solution of a similar problem on p. 110.
11.35. $n = 100$ s^{-1}. **11.36.** $\Phi = 1.13\times10^{-4}$ Wb. **11.37.** $\Phi = 0.157$ Wb.
11.38. (1) $\Phi = 1.6\times10^{-4}\cos(4\pi t + \theta)$ Wb, where θ is the angle between a normal to the frame and the direction of the magnetic field at the initial moment of time, (2) $\Phi_{max} = 1.6\times\times10^{-4}$ Wb.

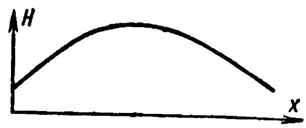


Fig. 101

11.39. We have

$$\mu_r = \frac{B}{H\mu_0} \qquad (1)$$

According to the condition, $H = 10$ Oe $= 796$ A/m $\cong 800$ A/m. From the graph $B = f(H)$ given in the Appendix we find that $B = 1.4$ T corresponds to $H = 0.8\times10^3$ A/m. Upon inserting the values of μ_0, H and B in Eq. (1), we obtain $\mu_r = 1{,}400$.
11.40. 500 At. **11.41.** 955 At. **11.42.** $\mu_r = 440$.
11.43. $IN = 5{,}000$ At. **11.44.** $B = 1.8$ T, $\mu_r = 200$.
11.45. The magnetic induction is the same in the core and in the air-gap, i. e.,

$$B_2 = B_1 = \frac{\Phi}{A} = \frac{IN\mu_0}{\frac{l_1}{\mu_{r1}} + \frac{l_2}{\mu_{r2}}} \qquad (1)$$

Since

$$B_2 = \mu_0\mu_{r2}H_2 \qquad (2)$$

then from Fq. (1)

$$B_1\frac{l_1}{\mu_{r1}} + \mu_0 H_2 l_2 = IN\mu_0 \qquad (3)$$

Equation (3) is an equation of a straight line in coordinate axes (H, B). But the quantities H and B are also related by a curve $B = f(H)$. The ordinate of the point of intersection of straight line (3) and the curve $B = f(H)$ gives us the magnetic induction $B_2 = B_1$. To plot the straight line according to equation (3), we find: at $H = 0$

$$B = \frac{IN\mu_0\mu_{r1}}{l_1} = 0.84 \text{ T}$$

and at $B = 0$

$$H = \frac{IN}{l_2} = 2{,}000 \text{ A/m}$$

The sought point of intersection gives us $B_2 = B_1 = 0.78$ T. Hence for the air-gap we have $H_1 = \frac{B_1}{\mu_0\mu_{r1}} = 6.2\times10^5$ A/m.
11.46. 1.9 times (see the solution to the previous problem).

11.48. $p=1\ m^2\cdot A$. **11.49.** $\Phi=1.8\times10^{-5}$ Wb.

11.50. $H=\frac{I}{2\pi x}$. Let us take element dA of the ring cross-sectional area equal to $dA=hdx$. The magnetic flux through this element will be $d\Phi=BdA=$ $=\mu_0\mu_r\frac{I}{2\pi x}hdx$. The flux through the entire cross section of the ring is

$$\Phi=\frac{\mu_0\mu_r Ih}{2\pi}\int_{l_1}^{l_2}\frac{dx}{x}=\frac{\mu_0\mu_r Ih}{2\pi}\log_e\frac{l_2}{l_1}$$

After finding μ_r and inserting the other data, we obtain $\Phi=1.8\times10^{-5}$ Wb.

11.51. $I=620$ A. **11.52.** $I=60$ A. **11.53.** (1) $I=11.3$ A, (2) $\mu_r=457$

11.54. (1) $B=\frac{IN\mu_0}{l_1/\mu_{r1}+l_2/\mu_{r2}}$, whence the required number of ampere-turns $IN=\frac{B}{\mu_0}\left(\frac{l_1}{\mu_{r1}}+\frac{l_2}{\mu_{r2}}\right)=\frac{Bl_1}{\mu_0\mu_{r1}}+Hl_2$. The curve $B=f(H)$ shows that $H=800$ A/m corresponds to $B=14{,}000$ Gs $=1.4$ T Hence, $IN=1.14\times10^4$ At Further, $I=\frac{\mathcal{E}}{R}=\frac{\mathcal{E}A}{\rho\pi DN}$, whence $\mathcal{E}=\frac{IN\rho\pi D}{A}=31$ V.

(2) Since the wire diameter $d=\sqrt{\frac{4A}{\pi}}=1.13\times10^{-3}$ m, the solenoid will accommodate $N=\frac{40\times10^{-2}}{1.13\times10^{-3}}=354$ turns. Since $I=jA=3$ A and $N=3{,}830$ turns, the required number of layers will be $\frac{3{,}830}{354}\cong11$ and, seeing that the wire diameter is 1.13×10^{-3} m, the 11 layers will occupy a thickness of 1.2×10^{-2} m $=1.2$ cm.

11.55. $F=4.9$ N.

11.56. $W=\int_{x_1}^{x_2}Fdx=\int_{x_1}^{x_2}\frac{\mu_0\mu_r I_1I_2l}{2\pi x}dx=\frac{\mu_0\mu_r I_1I_2l}{2\pi}\log_e\frac{x_2}{x_1}$ and the work per unit length of the conductors $\frac{W}{l}=\frac{\mu_0\mu_r I_1I_2}{2\pi}\log_e\frac{x_2}{x_1}=8.3\times10^{-6}$ J/m.

11.57. $I_1=I_2=20$ A. **11.58.** (1) 3.53×10^{-4} N·m, (2) 4.5×10^{-4} N·m.

11.59. (1) 0.125%. (2) By 3.2×10^{-5} N.

11.60. (1) $M=2.4\times10^{-9}$ N·m, (2) $M=1.2\times10^{-9}$ N·m.

11.61. The rotating moment acting on the magnetic needle is $M=pB\sin\alpha$, where p is the magnetic moment of the needle and $B=\mu_0\mu_r H=\frac{I\mu_0\mu_r}{2\pi a}$ is the induction of the magnetic field of the current. The rotating moment M turns the thread through the angle $\varphi=\frac{2lM}{\pi Gr^4}$, where l is the length of the thread, r its radius, and G the shear modulus of the thread material. Since $\sin\alpha=1$, then $M=pB=p\frac{I\mu_0\mu_r}{2\pi a}$. Thus $\varphi=\frac{\mu_0\mu_r Ilp}{a\pi^2Gr^4}$. In our case $I=30$ A, $l=0.1$ m, $p=10^{-2}$ A·m², $a=0.2$ m, $G=600$ kgf/mm² $=5.9\times10^9$ N/m² and $r=0.05$ mm $=$ $=5\times10^{-5}$ m. Upon inserting these data, we obtain $\varphi=0.52$ rad, or $\varphi=30°$.

11.62. $I=10^{-7}$ A. **11.63.** 5×10^{10} N/m². **11.64.** $A=5\times10^{-4}$ J.

11.65. (1) $A=0.2$ J, (2) $P=2\times10^{-2}$ W.

11.66. (1) The force acting on the radius ab (see Fig. 57) is equal to $F=BIr$. The work performed during one revolution of the disk $W=BIA$, where A is the area described by the radius during one revolution, i. e., the area of the disk. The power of such a motor is $P=\frac{W}{t}=\nu B I \pi r^2=2.36\times10^{-2}$ W. (2) The disk rotates counterclockwise. (3) The force acting on an element with the radius dx is determined by the formula $dF=BIdx$. The rotational moment acting on this element $dM=x\,dF=BIx\,dx$, where x is the distance to the element dx from the axis of rotation. The rotational moment acting on the entire disk is $M=\int_0^r BIx\,dx=\frac{BIr^2}{2}$. In our case $B=0.2$ T, $I=5$ A, and $r=5\times10^{-2}$ m. Upon inserting these data, we obtain $M=12.5\times10^{-4}$ N·m.

11.67. $I=15.3$ A. **11.68.** $\Phi=1$ Wb.

11.69. (1) $R=9\times10^{-2}$ m. (2) We have $T=\frac{2\pi R}{v}$, where $R=\frac{mv}{eB}$. Therefore $T=\frac{2\pi m}{eB}$, i. e., the period does not depend on the velocity of the electron. Upon inserting the numerical data, we find that $T=3\times10^{-8}$ s. (3) 1.5×10^{-24} kg·m²/s.

11.70. $F=4\times10^{-16}$ N.

11.71. $F=4.7\times10^{-12}$ N.

11.72. $a_t=0$ during the entire time of motion, $a_n=\text{const}=7\times10^{15}$ m/s².

11.73. $W_e=17.3$ MeV.

11.74. $\frac{R_1}{R_2}=\frac{m_1}{m_2}=1{,}840$.

11.75. $\frac{R_1}{R_2}=\sqrt{\frac{m_1}{m_2}}=\sqrt{1{,}840}=42.9$.

11.76. $W_e=88$ keV.

11.77. $Q=3.2\times10^{-19}$ C.

11.78. Two times.

11.79. (1) $F=5\times10^{-15}$ N, (2) $R=3.2\times10^{-2}$ m $=3.2$ cm, (3) $T=1.3\times10^{-6}$ s.

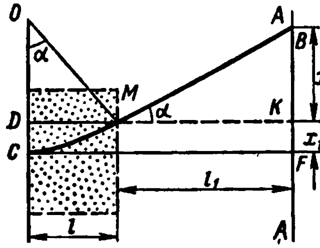


Fig. 102

11.80. $W_e=500$ eV. **11.81.** $R_1=0.195$ m, $R_2=0.200$ m.

11.82. $Q/m=4.8\times10^7$ C/kg. For the electron $Q/m=1.76\times10^{11}$ C/kg, for the proton $Q/m=9.6\times10^7$ C/kg, and for the α-particle $Q/m=4.8\times10^7$ C/kg.

11.83. The total displacement of the electron is $x=x_1+x_2$, where x_1 is the displacement of the electron in the magnetic field (Fig. 102) In the magnetic field the electron moves along a circle whose radius of curvature is $R=\frac{mv}{eB}$. The displacement x_1 can be found from the relation $x_1=DC=OC-OD$. But $OC=R$ and $OD=\sqrt{OM^2-DM^2}=\sqrt{R^2-l^2}$ Thus, $x_1=R-\sqrt{R^2-l^2}$. The displacement x_2 can be found from the proportion $\frac{x_2}{l_1}=\frac{DM}{DO}$, whence $x_2=l_1\frac{l}{\sqrt{R^2-l^2}}$. Hence the total displacement will be $x=R-\sqrt{R^2-l^2}+l_1\frac{l}{\sqrt{R^2-l^2}}$. In our case $R=\frac{mv}{eB}=\frac{1}{B}\sqrt{\frac{2Um}{e}}$, where U is the applied potential difference. Substitution of the data gives $R=4\times10^{-2}$ m $=4$ cm, $x=4.9\times10^{-2}$ m $=4.9$ cm.

11.84. (1) $a_n=0$, $a=a_t=\frac{eE}{m}=1.76\times10^{14}$ m/s², (2) $a_t=0$, $a=a_n=$
$=\sqrt{\left(\frac{evB}{m}\right)^2+\left(\frac{eE}{m}\right)^2}=2.5\times10^{14}$ m/s².

11.85. (1) $v=2\times10^6$ m/s, (2) $R=2.3\times10^{-2}$ m.

11.86. The velocity of the electron flying into the magnetic field is $v=\sqrt{\frac{2eU}{m}}$. Let us resolve the velocity v into two components: v_t directed along the force lines of the field, and v_n directed normal to them. The projection of the electron path onto a plane perpendicular to B is a circle whose radius is equal to the sought radius of the helix and can be determined from the formula

$$R=\frac{mv_n}{eB}=\frac{mv\sin\alpha}{eB} \qquad (1)$$

where α is the angle between the direction of the electron velocity v and the direction of the field. Since the period of electron rotation is $T=\frac{2\pi R}{v\sin\alpha}=\frac{2\pi m}{eB}$, the pitch of the helical trajectory of the electron will be

$$l=v_tT=\frac{2\pi mv\cos\alpha}{eB} \qquad (2)$$

Upon inserting the numerical data into Eqs. (1) and (2), we get: (1) $R=10^{-2}$ m $=1$ cm, (2) $l=11\times10^{-2}$ m $=11$ cm.

11.87. $W_e=433$ eV. **11.88.** (1) $R=5$ mm, (2) $l=3.6$ cm.

11.89. $l=3.94\times10^{-2}$ m $=3.94$ cm.

11.90. (1) $n=\frac{IB}{Uea}=8.1\times10^{28}$ m^{-3}, (2) $\bar{v}=\frac{j}{ne}=\frac{I}{Ane}=3.1\times10^{-4}$ m/s.

11.91. $U=2.7\times10^{-6}$ V. **11.92.** $u=0.65$ m^2/V·s.

11.93. $\mathcal{E}=-\frac{d\Phi}{dt}=-\frac{1}{dt}(Bldx)=-Blv=-0.15$ V.

11.94. $\mathcal{E}_m=78.5$ V. **11.95.** $\mathcal{E}=165$ mV.

11.96. Upon each revolution of the rod, the magnetic flux which it intersects is equal to $\Phi=BA=B\pi l^2$, where l is the length of the rod. If the rod makes ν rev/s, then $\mathcal{E}=B\pi l^2\nu=B\pi l^2\frac{\omega}{2\pi}=Bl^2\frac{\omega}{2}$, where ω is the angular velocity of rotation. Upon inserting the numerical data, we obtain $\mathcal{E}=0.5$ V.

11.97. $v=0.5$ m/s. **11.98.** $\mathcal{E}_m=1$ V.

11.99. $\mathcal{E}_{max}=\Phi_0\omega=BAN2\pi\nu$, where N is the number of coil turns and ν is the number of revolutions per second. Upon inserting the numerical data, we get $\mathcal{E}_{max}=3.14$ V.

11.100. $\mathcal{E}_{max}=0.09$ V. **11.101.** $\mathcal{E}=4.7$ mV. **11.102.** At 6.4 rev/s.

11.103. $\mathcal{E}_m=0.018$ V. **11.104.** $\mathcal{E}_m=5.1$ V. **11.105.** $\mathcal{E}_m=1.57$ V.

11.106. $\mathcal{E}_{max}=250$ mV. **11.107.** (1) $L=0.9$ mH, (2) $L=0.36$ H.

11.108. $L=5.5\times10^{-5}$ H.

11.109. (1) $L=7.1\times10^{-4}$ H, (2) $\Phi=3.55\times10^{-6}$ Wb.

11.110. $N=380$ turns. **11.111.** $\mu_r=1{,}400$. **11.112.** At $I=1$ A.

11.113. $N=500$. **11.114.** (1) $\mu_r=1{,}400$, (2) $I=1.6$ A.

11.115. (1) $\mu_r=640$, (2) $L=6.4\times10^{-2}$ H.

11.116. (1) $L=9.0$ H, (2) $L=5.8$ H, (3) $L=0.83$ H.

11.117. We have

$$L_1=\mu_0\mu_r n_1^2 lA \qquad (1)$$

and

$$L_2=\mu_0\mu_r n_2^2 lA \qquad (2)$$

The mutual inductance of coils with a common core is

$$L_{1,2}=\mu_0\mu_r n_1 n_2 l A \tag{3}$$

Upon multiplying Eq. (1) by Eq. (2), we obtain $L_1L_2=(\mu_0\mu_r lA)^2 n_1^2 n_2^2$, whence

$$n_1n_2=\frac{\sqrt{L_1L_2}}{\mu_0\mu_r lA} \tag{4}$$

Upon inserting Eq. (4) into Eq. (3), we find that $L_{1,2}=\sqrt{L_1L_2}$ Since $\mathscr{E}_2=-L_{1,2}\frac{dI_1}{dt}$, the mean current intensity in the second coil $I_2=\frac{L_{1,2}}{R}\frac{\Delta I}{\Delta t}=\frac{\sqrt{L_1L_2}}{R}\frac{\Delta I}{\Delta t}$. Upon inserting the numerical data, we obtain $I_2=0.2$ A.

11.118. The quantity of electricity induced in the frame is

$$Q=-\frac{1}{R}\int\limits_{\Phi_1}^{\Phi_2}d\Phi=-\frac{1}{R}(\Phi_2-\Phi_1) \tag{1}$$

where Φ_1 is the magnetic flux through the frame in its first position, and Φ_2 in its second position. In our case $\Phi_2=0$ and, besides,

$$R=\frac{\rho l}{A_w}=\frac{\rho 4a}{A_w}=\frac{\rho 4\sqrt{A_f}}{A_w} \tag{2}$$

In equation (2) a is a side of the frame, A_f the area of the frame and A_w the cross-sectional area of the wire. Since $\Phi_1=BA_f$, then finally $Q=\frac{BA_w\sqrt{A_f}}{4\rho}=0.074$ C.

11.119. $Q=1.5\times10^{-4}$ C. **11.120.** $Q=2.5\times10^{-4}$ C.

11.121. $C_b=10^{-8}$ C/div. **11.122.** $B=0.2$ T.

11.123. The intensity of the magnetic field in the toroid is

$$H=\frac{IN_1}{l} \tag{1}$$

If the direction of the current in the primary coil is reversed, the quantity of electricity passing through the galvanometer will be $Q=\frac{2\Phi N_2}{R}$, where Φ is the magnetic flux piercing the cross-sectional area of the toroid, and R is the resistance of the secondary circuit. But $\Phi=BA=\mu_0\mu_r HA=\mu_0\mu_r A\frac{IN_1}{l}$, hence $Q=\frac{2N_2\mu_0\mu_r AIN_1}{Rl}$, and $\mu_r=\frac{QRl}{2\mu_0 N_1 N_2 AI}$ But $Q=C_b\alpha$ and we finally obtain

$$\mu_r=\frac{C_b\alpha Rl}{2\mu_0 N_1 N_2 AI} \tag{2}$$

Upon inserting the various values of I and the corresponding values of α from the table given in the problem into equations (1) and (2), we get the following table:

I, A	0.1	0.2	0.3	0.4	0.5
H, A/m	133	266	400	533	667
μ_r	1,440	2,190	2,050	1,790	1,520

11.124. $\mu_r = 1{,}200$. **11.125.** In 0.126 s. **11.126** In 2.5×10^{-4} s.
11.127. 1.5 times. **11.128.** In 0.01 s.
11.130. (1) $\Phi = B_0 A \sin\omega t = 2.5\times10^{-5} \sin 100\,\pi t$ Wb, $\Phi_{max} = 2.5\times10^{-5}$ Wb,
(2) $\mathcal{E} = -7.85\times10^{-3}\cos 100\pi t$ V, $\mathcal{E}_{max} = 7.85\times10^{-3}$ V,
(3) $I = -2.3\cos 100\pi t$ A, $I_{max} = 2.3$ A.
11.131. (1) $\mathcal{E} = -33\cos 100\pi t$ V,
(2) $W_e = \frac{LI^2}{2} = 0.263\sin^2 100\pi t$ J.
11.132. (1) $\mathcal{E}_2 = -L_{1,2}\frac{dI}{dt} = -L_{1,2} I_0 \omega \cos\omega t = -15.7\cos 100\pi t$ V,
(2) $\mathcal{E}_{max} = 15.7$ V.

Chapter 4

OSCILLATIONS AND WAVES

12. Harmonic Oscillatory Motion and Waves

12.1. $x=5\sin\left(5\pi t+\frac{\pi}{4}\right)$ cm. **12.2.** $x=0.1\sin 0.5\pi t$ m.

12.3. (1) $x=50\sin\left(\frac{\pi t}{2}+\frac{\pi}{4}\right)$ mm, (2) $x_1=35.2$ mm, $x_2=0$.

12.4. (1) $x=5\sin\frac{\pi t}{4}$ cm, (2) $x=5\sin\left(\frac{\pi t}{4}+\frac{\pi}{2}\right)$ cm, (3) $x=5\sin\left(\frac{\pi t}{4}+\pi\right)$cm, (4) $x=5\sin\left(\frac{\pi t}{4}+\frac{3\pi}{2}\right)$ cm, (5) $x=5\sin\frac{\pi t}{4}$ cm.

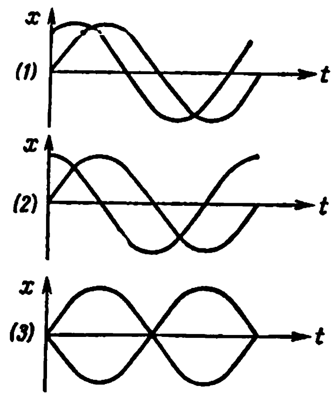
12.5. See Fig. 103.

12.6. We have $x=A\sin\left(\frac{2\pi t}{T}+\varphi\right)$. According to the initial condition, $x=\frac{A}{2}$; in addition, $T=24$ s and $\varphi=0$. Hence $0.5=\sin\left(\frac{\pi}{12}t\right)$, i.e., $\left(\frac{\pi}{12}t\right)=30°=\frac{\pi}{6}$, whence $t=2$ s.

12.7. $t=\frac{1}{6}T$. **12.8.** In 1 second.

12.9 $v_{max}=7.85\times10^{-2}$ m/s, $a_{max}=12.3\times\times10^{-2}$ m/s^2.

12.10. (1) 4 s, (2) 3.14×10^{-2} m/s, (3) $4.93\times\times10^{-2}$ m/s^2.



(4) Both sinusoids coincide

Fig. 103

12.11 According to the given condition $x==\sin\frac{\pi}{6}t$. Hence the velocity $v=\frac{dx}{dt}==\frac{\pi}{6}\cos\left(\frac{\pi}{6}t\right)$. The velocity will be maximum if $\cos\left(\frac{\pi}{6}t\right)=1$, i.e., when $\frac{\pi}{6}t=n\pi$, where $n=0$, 1, 2, etc. Thus, the maximum velocity is attained in $t=0$, 6, 12 s, and so on. The acceleration will be maximum if $\sin\left(\frac{\pi}{6}t\right)=1$, i.e., when $\frac{\pi}{6}t=(2n+1)\frac{\pi}{2}$. Thus, the maximum acceleration is attained in $t=3$, 9, 15 s, and so on.

12.12. $v=0.136$ m/s. **12.13.** $x=5\times10^{-2}\times\sin\left(\pi t+\frac{\pi}{6}\right)$ m.

12.14. $A=3.1\times10^{-2}$ m, $T=4.1$ s. **12.15** $F_{max}=24.6\times10^{-5}$ N.

12.16. $F_{max}=19.7\times10^{-5}$ N, $E_{tot}=4.93\times10^{-6}$ J.

12.17. The kinetic, potential and total energies of a point oscillating in conformity with the equations given in the problem versus the time is illustrated in Fig. 104. The diagram is given within the limits of one period and shows that the period of oscillations of the energy is half that of the oscillatory motion itself.

12.18. (1) $\frac{E_k}{E_p}=3$, (2) $\frac{E_k}{E_p}=1$, (2) $\frac{E_k}{E_p}=\frac{1}{3}$.

12.19. (1) $\frac{E_k}{E_p}=15$, (2) $\frac{E_k}{E_p}=3$, (3) $\frac{E_k}{E_p}=0$.

12.20. $x=0.04\sin\left(\pi t+\frac{\pi}{3}\right)$ m. **12.21.** $x=\frac{FA^2}{2E}=1.5\times10^{-2}$ m.

12.22. The period of oscillations of the ball $T=2\pi\sqrt{\frac{l}{g}}=2.8$ s. With small deflections from the position of equilibrium, the amplitude of the oscillations can be found as follows: $A=l\sin\alpha=2\times0.0698$ m $=0.14$ m. The equation of motion of the ball can then be written as $x=A\sin\left(\frac{2\pi}{T}t\right)=0.14\sin\frac{2\pi t}{2.8}$ m if the time is counted from the position of equilibrium. When the ball passes through the equilibrium position, its velocity will be maximum. Since $v=\frac{0.14\times2\pi}{2.8}\cos\frac{2\pi t}{2.8}$ m/s, then $v_{max}=\frac{0.14\times2\pi}{2.8}$ m/s $=0.31$ m/s. This velocity can also be found from the relation $mgh=\frac{mv^2}{2}$, where h is the height which the ball is raised to. Hence $v=\sqrt{2gh}$. It can easily be seen that $h=l(1-\cos\alpha)$, where l is the length of the thread. Thus, $v=\sqrt{2gl(1-\cos\alpha)}=0.31$ m/s. When the pendulum is greatly deflected from the equilibrium position, its oscillations will no longer be harmonic.

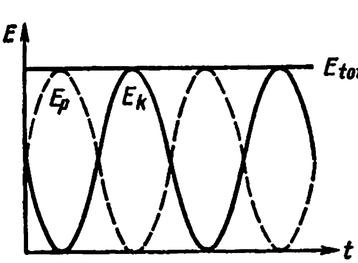


Fig. 104

12.23. 0.78 s. **12.24.** $k=805$ N/m.

12.25. The period will be halved.

12.26. The period will be reduced to five-ninths.

12.27. We have $T_1=2\pi\sqrt{\frac{m}{k}}$

or

$$T_1^2 = 4\pi^2\frac{m}{k} \qquad (1)$$

After more weights Δm are added, we have

$$T_2=2\pi\sqrt{\frac{m+\Delta m}{k}}, \text{ or } T_2^2=4\pi^2\frac{m+\Delta m}{k} \qquad (2)$$

By subtracting Eq. (1) from Eq. (2), we get $T_2^2-T_1^2=4\pi^2\frac{\Delta m}{k}$. But $k=\frac{F}{\Delta l}=\frac{\Delta mg}{\Delta l}$, where F is the force causing the elongation Δl. Thus, $T_2^2-T_1^2=4\pi^2\frac{\Delta l}{g}$, or

$\Delta l = \frac{g}{4\pi^2}(T_2^2 - T_1^2)$. Upon inserting the numerical data we obtain $\Delta l = 2.7 \times$ $\times 10^{-2}$ m $= 2.7$ cm.

12.28. $T = 0.93$ s.

12.29. The floating aerometer is acted upon by the force of gravity (downward) and the Archimedean force (upward). Therefore, in equilibrium $G = \rho g (V + Sh)$, where $(V + Sh)$ is the portion of the aerometer in the liquid. If the aerometer is submerged to the depth x, the resulting force of expulsion will be $F = \rho g [V + S(h + x)] - G = \rho g [V + S(h + x)] - \rho g (V + Sh) = \rho g S x = kx$, where $k = \rho g S$. Thus, since $T = 2\pi \sqrt{\frac{m}{k}}$, then $T = \frac{4}{d}\sqrt{\frac{m\pi}{\rho g}}$, whence $\rho = \frac{16\pi m}{T^2 d^2 g} = 890$ kg/m^3.

12.30. $x = 3.7 \times 10^{-2} \sin\left(\frac{\pi t}{4} + \frac{\pi}{8}\right)$ m.

12.31. $A = 4.6 \times 10^{-2}$ m, $\varphi = 62°46'$. **12.32.** $\Delta\varphi = \frac{2\pi}{3}$.

12.33. (1) $A = 5$ cm, $\varphi = 36°52' \cong 0.2\pi$, (2) $x = 5 \sin(\pi t + 0.2\pi)$ cm.

12.34. (1) From the spectrum of the complex oscillation (Fig. 61) it follows that the first oscillation has an amplitude of $A_1 = 0.03$ m and a frequency of $\nu_1 = 0.2$ s^{-1}, the second $A_2 = 0.02$ m and $\nu_2 = 0.5$ s^{-1}, and the third $A_3 = 0.01$ m and $\nu_3 = 1$ s^{-1}. Thus, the equations of these oscillations will be as follows:

$$x = 0.03 \sin 0.4\pi t \text{ m}$$
$$x = 0.02 \sin \pi t \text{ m}$$
$$x = 0.01 \sin 2\pi t \text{ m}$$

(2) Qualitative diagrams of these oscillations are illustrated in Fig. 105.

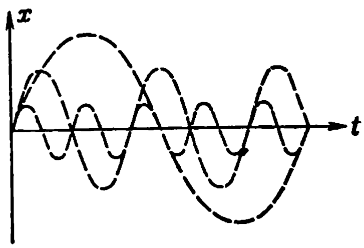


Fig. 105

(3) Compile tables showing $x = f(t)$ for all these oscillations and draw a diagram of the complex oscillation by summation of the ordinates of the sine curves for a number of points on the axis of abscissas.

12.35. The spectrum of the complex oscillation is shown in Fig. 106.

12.36. We have

$$x = A \sin 2\pi\nu_1 t \qquad (1)$$

and

$$A = A_0 (1 + \cos 2\pi\nu_2 t) \qquad (2)$$

Upon inserting Eq. (2) into (1), we obtain

$$x = A_0(1+\cos 2\pi\nu_2 t)\sin 2\pi\nu_1 t = A_0 \sin 2\pi\nu_1 t + A_0\cos 2\pi\nu_2 t \sin 2\pi\nu_1 t =$$

$$= A_0 \sin 2\pi\nu_1 t + \frac{A_0}{2}\sin[2\pi(\nu_1-\nu_2)t] + \frac{A_0}{2}\sin[2\pi(\nu_1+\nu_2)t]$$

Thus, the oscillation being considered can be resolved into three harmonic oscillatory motions with frequencies of ν_1, $(\nu_1-\nu_2)$ and $(\nu_1+\nu_2)$, and amplitudes,

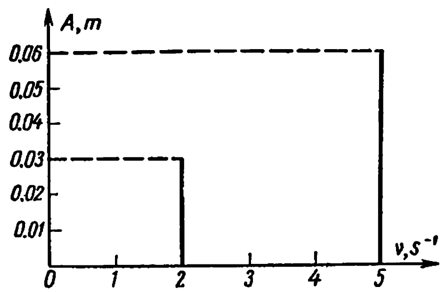


Fig. 106

respectively, of A_0, $\frac{A_0}{2}$ and $\frac{A_0}{2}$ The amplitude of the complex oscillation will vary with time. This kind of oscillation is no longer harmonic oscillatory motion and is known as a modulated oscillation.

12.37. When two mutually perpendicular oscillations having the same period are summated, the equation of the trajectory of the resulting oscillation is

$$\frac{x^2}{A_1^2}+\frac{y^2}{A_2^2}-\frac{2xy}{A_1A_2}\cos(\varphi_2-\varphi_1)=\sin^2(\varphi_2-\varphi_1) \qquad (1)$$

Since in our case $(\varphi_2-\varphi_1)=0$, equation (1) may be written as

$$\frac{x^2}{A_1^2}+\frac{y^2}{A_2^2}-\frac{2xy}{A_1A_2}=0, \text{ or } \left(\frac{x}{A_1}-\frac{y}{A_2}\right)^2=0$$

whence $y=\frac{A_2}{A_1}x$ is the equation of a straight line. In this way, the resulting oscillation will take place along a straight line. The angle of inclination of this line can be found from the equation $\tan\alpha=\frac{A_2}{A_1}=0.5$, whence $\alpha=26°34'$ The period of the resulting oscillation is equal to the period of the constituent oscillations, while the amplitude of the resulting oscillation $A=\sqrt{A_1^2+A_2^2}=$ $=0.112$ m. Hence, the equation of the resulting oscillation takes the form $s=0.112\sin\left(10\pi t+\frac{\pi}{3}\right)$ m.

12.38. (1) 7 cm, (2) 5 cm.

12.39. $\frac{x^2}{4}+\frac{y^2}{4}=1$. This is the equation of a circle with a radius of 2 m.

12.40. We have

$$x=\cos\pi t \qquad (1)$$

and

$$y=\cos\frac{\pi t}{2}=\sqrt{\frac{1+\cos\pi t}{2}}$$

or

$$2y^2-1=\cos\pi t \qquad (2)$$

Upon dividing Eq. (2) by (1), we obtain $\frac{2y^2-1}{x}=1$, or $2y^2-x=1$, which is the equation of a parabola.

12.41. $\frac{x^2}{1}+\frac{y^2}{4}=1$. This is the equation of an ellipse.

12.42. $y=-0.75x$, which is the equation of a straight line.

12.43. (1) The equation of damped oscillations is

$$x=Ae^{-\delta t}\sin(\omega t+\varphi) \qquad (1)$$

In our case $\omega=\frac{2\pi}{T}=\frac{\pi}{2}$, $\varphi=0$ and $\delta=\frac{\delta T}{T}=\frac{1.6}{4}=0.4$. The amplitude A can be found from the condition $x=4.5$ cm at $t=\frac{T}{4}=1$ s. It can be easily found from Eq. (1) that $A=6.7$ cm. Thus, equation (1) becomes

$$x=6.7e^{-0.4t}\sin\frac{\pi}{2}t \qquad (2)$$

(2) To plot the diagram let us find the moments of time $t_1, t_2, t_3, \ldots$ which correspond to the maximum displacements x. The maximum of x can be determined from the condition $v=\frac{dx}{dt}=0$. From equation (1), when $\varphi=0$, we have $v=A\omega e^{-\delta t}\cos\omega t-A\delta e^{-\delta t}\sin\omega t=0$. Hence

$$\tan\omega t=\frac{\omega}{\delta}=\frac{2\pi}{\delta T} \qquad (3)$$

It can be seen from equation (3) that only in the case of undamped oscillations, when $\delta T=0$, the value of $\tan\omega t=\infty$, or $\omega t=\frac{\pi}{2}$, i. e., $\frac{2\pi}{T}t=\frac{\pi}{2}$, or $t=\frac{T}{4}$. In our case $\tan\omega t=\frac{2\pi}{\delta T}=3.925$, i. e., $\omega t=75°42'\cong 0.421\pi$, whence $t=\frac{0.421\pi}{\omega}=$ $=0.842$ s. Thus, $x=x_{max}$ at $t_1=0.842$ s, $t_2=t_1+\frac{T}{2}=2.842$ s, $t_3=t_1+T=$ $=4.842$ s and $t_4=t_1+\frac{3T}{2}=6.842$ s, etc. Upon inserting the found values of t in equation (2), we can easily find the corresponding values of $x_1, x_2, x_3, \ldots$

12.44. See the solution to the previous problem.

12.45. $v_1=7.85$ m/s, $v_2=2.88$ m/s, $v_3=1.06$ m/s, $v_4=0.39$ m/s and $v_5=0.14$ m/s.

12.46. From the formulas for damped oscillations we have $A_1=A_0e^{-\delta t}$ and $A_2=A_0e^{-\delta(t+T)}$, whence $\frac{A_1}{A_2}=e^{\delta T}$. According to the condition $\delta T=0.2$, whence $\frac{A_1}{A_2}=1.22$.

12.47. $\delta T = 0.023$. **12.48.** (1) 120 s, (2) 1.22 s.
12.49. 1.22 times. **12.50.** Eight times. **12.51.** $t = 6.4$ s.

12.52. (1) $\delta = 0.46\ \text{s}^{-1}$, (2) $\delta = 10\ \text{s}^{-1}$, (3) $\delta = \frac{\delta T}{T} = \frac{\delta T \omega_0}{\sqrt{4\pi^2 + (\delta T)^2}} = 6.9\ \text{s}^{-1}$

12.53. The equation of natural oscillations has the form

$$x = A_0 e^{-\delta t} \sin \omega_0 t \qquad (1)$$

Let us find ω_0. According to the condition, the phase shift between the natural and forced oscillations is -0.75π, and therefore

$$\tan \varphi = \frac{2\delta\omega}{\omega_0^2 - \omega^2} = \tan(-0.75\pi) = 1$$

Hence

$$\omega_0 = \sqrt{\omega^2 + 2\delta\omega} \qquad (2)$$

In our case $\omega = 10\pi$ and $\delta = 1.6\ \text{s}^{-1}$. Upon inserting these values in Eq. (2), we obtain $\omega_0 = 33 = 10.5\pi$ and thus the equation of natural oscillations takes the form

$$x = 7e^{-1.6t} \sin 10.5\pi t$$

(2) The equation of an external periodic force is

$$F = F_0 \sin \omega t$$

Let us find the maximum external periodic force F_0. We have

$$F_0 = Am\sqrt{(\omega_0^2 - \omega^2)^2 + 4\delta^2\omega^2}$$

Upon inserting the numerical data into this equation, we obtain $F_0 \cong 7.2 \times 10^{-2}$ N, and then the equation of the external periodic force becomes $F = 7.2 \times 10^{-2} \sin 10\pi t$ N.

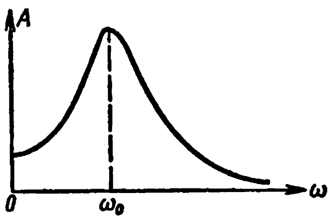


Fig. 107

12.54. The nature of the relation between the amplitude A of the forced oscillations and the frequency ω of the external periodic force is illustrated in Fig. 107.

12.55. The pram will begin to swing heavily if the time between two consecutive jolts on the depressions is equal to the period of natural oscillations of the pram, which can be found from the formula $T = 2\pi\sqrt{\frac{m}{k}}$. In our case $m = \frac{10\ \text{kg}}{2} = 5$ kg, which is the mass per leaf spring; $k = \frac{F_0}{x_0} = \frac{1\ \text{kgf}}{2\ \text{cm}} = 490$ N/m, and therefore $T = 0.63$ s. The time between two consecutive jolts $t = \frac{l}{v}$, where v is the speed of the pram and l the distance between the depressions. In our case $t = \frac{l}{v} = T$, whence $v = \frac{l}{T} = \frac{0.3}{0.63}$ m/s $= 1.7$ km/h.

12.56. $\lambda = 3 \times 10^{-6}$ m. **12.57.** (1) 350 m/s, (2) 0.785 m/s.

12.58. (1) In our case the equation of the wave takes the form

$$x = 10 \sin\left(0.5\pi t - \frac{\pi l}{6\times 10^4}\right) \text{ cm} \qquad (1)$$

Thus, $x = f(t, l)$, i. e., the displacement of the points lying on the ray depends on the time t and the distance l from the point to the source of oscillations.

(2) For a point at a distance of 600 m from the source of oscillations, equation (1) can be written as $x = 10 \sin(0.5\pi t - \pi)$ cm, i. e., at $l = \text{const}$ we obtain $x = f_1(t)$, hence, the displacement of a fixed point lying on the ray changes with time.

(3) When $t = 4$ s, equation (1) becomes $x = 10 \sin\left(2\pi - \frac{\pi}{6\times 10^4}\right)$ cm. In this case $t = \text{const}$ and $x = f_2(l)$, i. e., different points lying on the ray have different displacements at a given moment of time.

12.59. $x = 0.04$ m. 12.60. $x = 0$, $v = 7.85\times 10^{-2}$ m/s, $a = 0$.

12.61. $\Delta\varphi = \pi$—the points oscillate in opposite phases.

12.62. $\Delta\varphi = 4\pi$—the points oscillate in the same phases.

12.63. $x = 0.025$ m. 12.64. $\lambda = 0.48$ m.

12.65. (1) The position of the nodes is determined by the coordinates $x = 3, 9, 15, \ldots$ cm and that of the antinodes by the coordinates $x = 0, 6, 12, 18, \ldots$ cm. (2) The position of the nodes $x = 0, 6, 12, 18, \ldots$ cm and that of the antinodes $x = 3, 9, 15, \ldots$ cm.

12.66. $\lambda = 0.1$ m.

13. Acoustics

13.1. $\lambda = 0.78$ m. 13.2. From $\lambda_1 = 17$ mm to $\lambda_2 = 17$ m.

13.3. $c = 5{,}300$ m/s. 13.4. $c = 3{,}700$ m/s.

13.5. Since Young's modulus E is related to the compression coefficient β by the formula $\beta = \frac{1}{E}$, then $\beta = \frac{1}{\rho c^2}$. Upon inserting the numerical data, we obtain $\beta = 7.1\times 10^{-10}$ m^2/N.

13.6. 1,810 m. 13.7. (1) 318 m/s, (2) 330 m/s, (3) 343 m/s.

13.8. 1.12 times. 13.9. $c = 315$ m/s. 13.10. $c = 330$ m/s.

13.11. $c = 336$ m/s. 13.12. $t = -54°$ C.

13.13. $n = \frac{c_1}{c_2} = 0.067$. 13.14. 3°51′.

13.15. $\frac{I_2}{I_1} = 1.26$ (see Example 2 in the introduction to this chapter).

13.16. $\frac{\Delta p_2}{\Delta p_1} = 1.12$. 13.17. $\frac{I_1}{I_2} = 1{,}000$.

13.18. (1) $\Delta L = 30$ db, (2) $\frac{\Delta p_2}{\Delta p_1} = 31.6$.

13.19. (1) $L = 100$ phons, (2) $\Delta p = 2$ N/m^2.

13.20. (1) By 34.8 phons, (2) by 44.8 phons.

13.21. The groove pitch on a phonograph record can be found from the formula $l = \frac{\omega r}{v}$, where ω is the angular velocity of the record in rotation. Upon inserting the numerical data we obtain: (1) $l = 2.25\times 10^{-3}$ m $= 2.25$ mm, (2) $l = 7.5\times 10^{-4}$ m $= 0.75$ mm.

13.22. (1) $l = 8.15$ mm, (2) $l = 0.41$ mm.

13.23. When oscillations are generated in the steel bar, it will develop a standing wave with nodes at the points of clamping and antinodes at the free ends. The distance between adjacent antinodes in the standing wave of the air column is equal to half the length of the generated sound wave. Denoting all the values pertaining to the steel bar by the subscript 1 and those pertaining to the air column by 2, we obtain

$$\frac{\lambda_1}{\lambda_2}=\frac{c_1}{c_2} \qquad (1)$$

On this basis, the sought length l_2 of the air column can be found from the condition

$$n\frac{\lambda_2}{2}=l_2 \qquad (2)$$

where n is the number of antinodes. From Eqs. (1) and (2) we have $l_2 = \frac{n\lambda_1 c_2}{2c_1}$. Hence (1) $\lambda_1=2l_1$ and $l_2=0.392$ m, (2) $\lambda_1=4l_1$ and $l_2=0.784$ m.

13.24. $l=0.715$ m.

13.25. Approximately up to 43,000 Hz—ultrasonic frequency.

13.26. (1) $\nu'=666$ Hz, (2) $\nu'=542$ Hz. **13.27.** 10%.

13.28. (1) 28.3 km/h, (2) 14.7 km/h. **13.29.** 4 times.

13.30. $v=71$ km/h. **13.31.** $\nu_1=4.50\times10^4$ Hz and $\nu_2=4.66\times10^4$ Hz.

13.32. $l=0.63$ m. **13.33.** $F=7.3$ N. **13.34.** $\nu=158$ Hz.

13.35. We have

$$\frac{\nu_1}{\nu_2}=\sqrt{\frac{F_1}{F_2}}=\sqrt{\frac{15}{16}} \qquad (1)$$

and

$$\nu_2-\nu_1=8\ \text{s}^{-1} \qquad (2)$$

Upon solving Eqs. (1) and (2) simultaneously, we get

$$\nu_2=252 \text{ Hz}$$

13.36. $\nu=250$ Hz or $\nu=254$ Hz. **13.37.** $\nu=250$ Hz.

13.38. (1) A standing wave with antinodes at both ends is formed in an open tube. Obviously, in this case n halfwaves can be accommodated over the tube length l, where $n=1, 2, 3, \ldots$, i.e., $l=n\frac{\lambda}{2}$. The frequency of the sound wave will thus be $\nu=\frac{c}{\lambda}=\frac{nc}{2l}$ When $n=1$, the frequency of the fundamental tone is $\nu=\frac{c}{2l}$.

(2) In a closed tube a standing wave has a node at one end and an antinode at the other. Obviously, in this case $l=n\frac{\lambda}{4}$ and $\nu=\frac{c}{\lambda}=\frac{nc}{4l}$ When $n=1$, the frequency of the fundamental tone is $\nu=\frac{c}{4l}$.

13.39. $\nu=261$ Hz, $l=0.65$ m.

14. Electromagnetic Oscillations and Waves

14.1. $\lambda = 2{,}500$ m. **14.2.** From $\lambda_1 = 700$ m to $\lambda_2 = 1{,}950$ m.
14.3. $L = 12.7$ MH. **14.4.** $\varepsilon_r = 6$.
14.5. (1) $U = 100 \cos(2\pi \times 10^3 t)$ V, $I = -15.7 \sin(2\pi \times 10^3 t)$ mA, (2) $U_1 = 70.7$ V and $I_1 = -11.1$ mA, $U_2 = 0$ and $I_2 = -15.7$ mA, $U_3 = -100$ V and $I_3 = 0$.
14.6. (1) $E_{el} = 12.5 \times 10^{-5} \times \cos^2(2\pi \times 10^3 t)$ J, $E_m = 12.5 \times 10^{-5} \times \sin^2(2\pi \times 10^3 t)$ J, $E_{tot} = \text{const} = 12.5 \times 10^{-5}$ J.
(2) $E'_{el} = 6.25 \times 10^{-5}$ J, $E'_m = 6.25 \times 10^{-5}$ J and $E'_{tot} = 12.5 \times 10^{-5}$ J; $E''_{el} = 0$, $E''_m = 12.5 \times 10^{-5}$ J, $E''_{tot} = 12.5 \times 10^{-5}$ J; $E'''_{el} = 12.5 \times 10^{-5}$ J, $E'''_m = 0$ and $E'''_{tot} = 12.5 \times 10^{-5}$ J.
14.7. (1) $T = 2 \times 10^{-4}$ s, (2) $L = 10.15$ MH, (3) $I = -157 \sin 10^4 \pi t$ mA, (4) $\lambda = 6 \times 10^4$ m.
14.8. (1) $T = 5 \times 10^{-3}$ s, (2) $C = 6.3 \times 10^{-7}$ F, (3) $U_{max} = 25.2$ V, (4) $E_m = 2 \times 10^{-4}$ J, (5) $E_{el} = 2 \times 10^{-4}$ J.
14.9. We have $U = U_0 \cos \omega t$ and $I = C \frac{dU}{dt} = -CU_0 \omega \sin \omega t$. Consequently,

$$E_m = \frac{LI^2}{2} = \frac{1}{2} LC^2 U_0^2 \omega^2 \sin^2 \omega t, \qquad E_{el} = \frac{CU^2}{2} = \frac{1}{2} CU_0^2 \cos^2 \omega t$$

Hence

$$\frac{E_m}{E_{el}} = \frac{LC\omega^2 \sin^2 \omega t}{\cos^2 \omega t} = LC\omega^2 \tan^2 \omega t$$

When $t = \frac{T}{8}$, the value of $\sin \omega t = \frac{\sqrt{2}}{2}$ and of $\cos \omega t = \frac{\sqrt{2}}{2}$. Besides, since $LC = \frac{T^2}{4\pi^2} = \frac{1}{\omega^2}$, then finally $\frac{E_m}{E_{el}} = \frac{\sin^2 \omega t}{\cos^2 \omega t} = 1$.
14.10. (1) $T = 8 \times 10^{-3}$ s, (2) $\delta T = 0.7$, (3) $U = 80 e^{-87t} \cos 250\pi t$ V, (4) $U_1 = -56.5$ V, $U_2 = 40$ V, $U_3 = -28$ V, $U_4 = 20$ V.
14.11. (1) Assuming the resistance to be sufficiently small, let us find the period of oscillations from the formula $T = 2\pi \sqrt{LC} = 2 \times 10^{-4}$ s. Further we have $U_1 = U_0 e^{-\delta t}$, whence $\delta t = \delta T \frac{t}{T} = \log_e \frac{U_0}{U_1}$ According to the initial condition, $U_1 = \frac{U_0}{3}$ or $\frac{U_0}{U_1} = 3$ when $t = 10^{-3}$ s. Therefore,

$$\delta T = \frac{T \log_e \frac{U_0}{U_1}}{t} = \frac{\log_e 3 \times 2 \times 10^{-4}}{10^{-3}} = 0.22$$

(2) $R = 11.1\ \Omega$. It is easy to see that this value of R agrees with the condition for applying the formula $T = 2\pi \sqrt{LC}$.

14.12. 1.04 times. **14.13.** $\delta T = \frac{8\rho \sqrt{\pi l C}}{d^2 \sqrt{\mu_0 \mu_r}} = 0.018$.
14.14. $t = \frac{T \log_e 100}{2\delta T} = 6.8 \times 10^{-3}$ s. **14.15.** $C = 0.7\ \mu$F.
14.16. $R = 4.1\ \Omega$. **14.17.** 300 Hz.
14.18. (1) $I = 4.6$ mA, (2) $U_1 = 73.4$ V, $U_2 = 146.6$ V.
14.19 (1) 74%, (2) 68%. **14.20.** (1) 72.5%, (2) 68.5%.
14.21. $C = 3.74\ \mu$F. **14.22.** $L = 0.055$ H.

14.23.

No.	Z	$\tan\varphi$
1	$\sqrt{R^2+\frac{1}{(\omega C)^2}}$	$\frac{1}{R\omega C}$
2	$\frac{R}{\sqrt{R^2\omega^2C^2+1}}$	$-R\omega C$
3	$\sqrt{R^2+(\omega L)^2}$	$\frac{\omega L}{R}$
4	$\frac{R\omega L}{\sqrt{R^2+(\omega L)^2}}$	$\frac{R}{\omega L}$
5	$\sqrt{R^2+\left(\omega L-\frac{1}{\omega C}\right)^2}$	$\frac{\omega L-\frac{1}{\omega C}}{R}$

14.24. (1) $Z=4{,}380\ \Omega$, (2) $Z=2{,}180\ \Omega$.
14.25. $I=1.34$ A, $U_C=121$ V, $U_R=134$ V, and $U_L=295$ V.
14.26. $R=12.3\ \Omega$. **14.27.** $R=40\ \Omega$, $L=0.074$ H.
14.28. $U_R=156$ V.

Chapter 5

OPTICS

15. Geometrical Optics and Photometry

15.1. Through 2α.
15.2. $a_2 = -15$ cm and $y' = 5$ mm. The image is real, inverse and diminished.
15.3. $a_2 = 0.12$ m, $y' = -8$ mm. The image is virtual, erect and diminished.
15.4. $a_2 = 7.5$ cm, $y' = -1.5$ cm. The image is virtual, erect and diminished.
15.5. $a_1 = -0.6$ m, $a_2 = -0.3$ m.
15.6. (1) $F = -10$ cm, (2) $D = -10$ diopters.
15.7. The total linear magnification of the system is 6.

15.8. $a_2 = \frac{R}{2}$; the image will be in the focus of the reflector; $y = 7.5$ cm.

15.9. Let us denote the longitudinal aberration AF by x and the transverse aberration FH by y. From the isosceles triangle OAM (see Fig. 62) we have $OA = \frac{R}{2\cos\alpha}$. But $x = AF = OA - OF = OA - \frac{R}{2}$, i. e., finally

$$x = \frac{R}{2}\left(\frac{1}{\cos\alpha} - 1\right) \quad (1)$$

If $\alpha = 0$, then $\cos\alpha = 1$ and $x = 0$.

Further, $y = FH = x\tan\angle HAF$. But $\angle HAF$ as an external angle of ΔAOM is equal to 2α, and hence $y = \frac{R}{2}\left(\frac{1}{\cos\alpha} - 1\right)\tan 2\alpha$. If $\alpha = 0$, then $\cos\alpha = 1$, $\tan 2\alpha = 0$ and $y = 0$.

15.10. $x = 1.8$ cm, $y = 1.5$ cm. **15.11.** $h = 8$ cm.
15.12. $d = 0.1$ m. **15.13.** $l = 5.8$ mm. **15.14.** $\tan i = n$.
15.15. (1) 41°8′, (2) 48°45′, (3) 61°10′.
15.16. At an angle of 41°15′ to the surface of the water.
15.17. 2.02×10^8 m/s.

15.18. We have $\frac{\sin i}{\sin r} = n_1$, where n_1 is the refractive index of glass. Total internal reflection from the surface separating the water and the glass will occur if the condition $\sin r = \frac{n_2}{n_1}$ is satisfied. Hence, $\sin i = n_1 \sin r = n_1 \frac{n_2}{n_1} = n_2 = 1.33$, i.e., $\sin i > 1$, and the conditions of the problem cannot be satisfied.

15.19. 0.114 m. **15.21.** $\varphi_r = 41°28'$ and $\varphi_v = 40°49'$.

15.22. The violet rays are subjected to total internal reflection, and the red rays emerge from the glass into the air.

15.23. 34°37′. **15.24.** 28°. **15.25.** 6°2′.
15.26. 10°8′. **15.27.** 77°22′. **15.28.** 4°47′.

15.29. $\sin\frac{\delta+\gamma}{2}=n\sin\frac{\gamma}{2}$. In this case the deflection of the ray from the initial direction will be minimum.

15.30. $\delta_r=30°37'$ and $\delta_v=33°27'$. **15.31.** $F=0.146$ m.

15.32. (1) 0.188 m, (2) 0.30 m, (3) 0.75 m, (4) —0.188 m, (5) —0.30 m, (6) —0.75 m.

15.33. (1) $\frac{F_1}{F_2}=1.4$, (2) in this liquid the first lens will be a diverging, and the second a converging one.

15.34. $D=2$ diopters. **15.35.** $a_2=0.3$ m, $y=4$ cm. **15.37.** 1 m.

15.38. (1) 0.48 m, (2) 2.65 m, (3) 0.864 m. **15.39.** $F=0.47$ m.

15.40. $F=-0.75$ m; the lens will be a diverging one.

15.41. $F=0.59$ m. **15.42.** $a_1=-90$ cm, $a_2=180$ cm.

15.43. $F_r-F_v=3$ cm. **15.44.** (1) 10 cm, (2) 5.7 cm.

15.46. (1) 12.5, (2) 7.5. **15.47.** $|R_1|=|R_2|=25$ mm. **15.48.** 5 mm.

15.49. $k=562$. **15.50.** $F=0.112$ m. **15.51.** 7°45'.

15.52. (1) The diameter of the image $d=2F\tan\frac{\alpha}{2}=4.6$ mm. (2) A beam of rays impinging onto the surface of a lens with the area $\frac{\pi D^2}{4}$ is concentrated in the Sun's image with an area $\frac{\pi d^2}{4}$. Hence $\frac{E'}{E}=\frac{4\pi D^2}{4\pi d^2}=\frac{D^2}{d^2}=383$.

15.53. (1) 1 m, (2) 0.71 m. **15.54.** $\Phi=8.34$ lm.

15.55. When a drawing whose dimensions are much greater than those of the plate is photographed completely, the image is obtained approximately in the principal focus of the lens. When parts of the drawing are photographed, a full-size image is obtained if the object is placed at a double focal length from the lens (the image on the plate is obtained at the same distance). In this case the area of the image increases $\left(\frac{2F}{F}\right)^2=4$ times. The illumination of the plate will decrease the same number of times. Therefore, the exposure should be increased four times.

15.56. 5.7 times. Thus, a standing man will become suntanned better than one lying down.

15.57. Two times.

15.58. The illumination in the corners of the room

$$E=\frac{I}{r^2}\cos\alpha \quad (1)$$

The distance from the lamp to a corner r, the quantity a (half the diagonal of the square floor), the side of the square floor b and the height of the lamp above the floor h are related by the obvious equation

$$a=r\sin\alpha=\frac{b}{\sqrt{2}}=h\tan\alpha \quad (2)$$

From formula (2), the illumination may be expressed as $E=\frac{I}{a^2}(\cos\alpha\sin^2\alpha)$. To find the maximum of E, let us take the derivative $\frac{dE}{d\alpha}$ and equate it to zero

$$\frac{dE}{d\alpha}=\frac{I}{a^2}(2\cos^2\alpha\sin\alpha-\sin^3\alpha)=0$$

whence $\tan^2\alpha=2$, and the sought height h will be

$$h=\frac{a}{\tan\alpha}=\frac{b}{\sqrt{2}\tan\alpha}=\frac{b}{2}=2.5 \text{ m}$$

15.60. When the desk lamp is burning. the illumination of the table edge is 1.2 times greater.

15.61. 2.25 times. **15.62.** $E\approx 8\times10^4$ lx.

15.63. (1) $M_1=1.6\times10^5$ lm/m², $L_1=5.1\times10^4$ nt,
(2) $M_2=4\times10^4$ lm/m², $L_2=1.27\times10^4$ nt.

15.64. (1) 1.2×10^7 nt, (2) 3×10^4 nt.

15.65. The illumination will be the same with both the transparent and the frosted bulb: $E_1=E_2=3.4$ lx.

15.66. $E=2\times10^3$ lx, $M=1.5\times10^3$ lm/m², $L=480$ nt.

15.67. $E=4.2\times10^4$ lx. **15.68.** $E=210$ lx.

15.69. (1) 1.61×10^{-3} W/lm, (2) approximately 2%.

16. Wave Optics

16.1. When one edge of the solar disk is photographed (the light source moves towards the observer)

$$\nu'=\frac{\nu c}{c-v} \quad (1)$$

and when the other edge is photographed (the light source moves away from the observer)

$$\nu''=\frac{\nu c}{c+v} \quad (2)$$

Since $\nu=\frac{c}{\lambda}$, from Eqs. (1) and (2) we find $\Delta\lambda=\frac{2v\lambda}{c}$. Hence $v=\frac{c\Delta\lambda}{2\lambda}=2\times10^3$ m/s.

16.2. $U=\frac{mc^2(\Delta\lambda)^2}{2\lambda^2Q}=2{,}500$ V.

16.3. The displacement of the spectral lines towards short wavelengths indicates that the star is moving towards the observer. The radial velocity of its motion (i.e., the velocity along a line connecting the star and the Earth) can be found from the equation $v=\frac{c\Delta\lambda}{\lambda}=103$ km/s.

16.4. 1.3 times. **16.5.** $y_1=1.8$ mm, $y_2=3.6$ mm, $y_3=5.4$ mm.

16.6. $\lambda=5\times10^{-7}$ m.

16.7. The glass plate causes the difference in paths between the interfering rays to change by $\Delta=nh-h=h(n-1)$, where h is the plate thickness and n is the refractive index of the plate material On the other hand, as a result of the introduction of the glass plate a displacement by k bands took place. Hence, the additional difference in paths due to the plate is $k\lambda$ Thus, $h(n-1)=k\lambda$, whence $h=\frac{k\lambda}{n-1}=6\times10^{-6}$ m.

16.8. $\Delta n\leqslant 5\times10^{-5}$. **16.9.** $h=0.13$ μ.

16.10. Let us denote the thicknesses of the film corresponding to adjacent bands by h_1 and h_2. Hence $\Delta h=h_2-h_1=\frac{\lambda}{2n}$. Let us denote the distance between adjacent bands by l (Fig. 108). It may be assumed that $\Delta h = l \tan\alpha$, where α is the wedge angle. Hence $\tan\alpha = \frac{k\lambda}{2nl} = 5.13\times10^{-5}$ and $\alpha = 11''$.

16.11. 1.9 mm. **16.12.** 5 bands per cm.

16.13. $k=5$, $k+1=6$, $\lambda = 5\times10^{-7}$ m. **16.14.** $\lambda = 5{,}890$ Å.

16.15. (1) $r_4 = \sqrt{4R\lambda_1} = 2.8\times10^{-3}$ m $= 2.8$ mm, (2) $r_3 = \sqrt{3R\lambda_2} = 3.1\times10^{-3}$ m $=$ $=3.1$ mm. It can easily be seen that the third red ring is farther than the fourth dark-blue ring. This is why Newton's rings can be observed in white light only if the air layer is thin, otherwise various colours will be superimposed on the same spot.

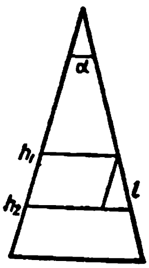


Fig. 108

16.16. $\lambda = 6{,}750$ Å. **16.17.** 3.6 mm. **16.18.** $k=275$.

16.19. When Newton's rings are observed in transmitted light, the condition of the maximum of light is determined by the formula

$$2hn = k\lambda \qquad (1)$$

The thickness of the layer h between the lens and the plate is related to the corresponding radius of the ring being observed as follows:

$$h = \frac{r_r^2}{2R} \qquad (2)$$

Upon inserting Eq. (2) into (1), we obtain $\frac{nr_r^2}{R} = k\lambda$, whence $n = \frac{k\lambda R}{r_r^2}$. Insertion of the numerical data gives $n = 1.33$.

16.20. 1.2 μ. **16.21.** 4.7×10^{-2} m. **16.22.** $n = 1.56$.

16.23. A shift L of the mirror over a distance of $\frac{\lambda}{2}$ corresponds to a change in the path difference of λ, i. e., a displacement of the interference pattern by one band. Thus, $L = k\frac{\lambda}{2}$ where k is the number of bands passed in the field of vision. Hence $\lambda = \frac{2L}{k} = 6.44\times10^{-7}$ m.

16.24. $n - 1 = \frac{k\lambda}{2l} = 3.8\times10^{-4}$, whence $n = 1.00038$.

16.25. In contrast to a Michelson interferometer, in this case the ray passes through the tube with chlorine only once (see Fig. 63). Therefore the path difference of the rays passing through the chlorine and through the vacuum is equal to $ln - l = l(n-1) = k\lambda$, where l is the length of the tube. Hence $n - 1 = \frac{k\lambda}{l} =$ $= 7.73\times10^{-4}$ and $n = 1.000773$.

16.26. $\lambda = 4.8\times10^{-7}$ m. **16.27.** $d = 1.15\times10^{-7}$ m.

16.28. (1) 5 zones, (2) bright.

16.29. The radius of the k-th zone $r_k = \sqrt{k\frac{ad\lambda}{a+d}}$, where a is the distance from the source of light to the wave surface and d is the distance from the wave surface to the point of observation. Upon inserting the numerical data, we find that $r_1 = 0.50$ mm, $r_2 = 0.71$ mm, $r_3 = 0.86$ mm, $r_4 = 1.0$ mm and $r_5 = 1.12$ mm.

16.30. $r_1 = 0.71$ mm, $r_2 = 1.0$ mm, $r_3 = 1.23$ mm, $r_4 = 1.42$ mm, $r_5 = 1.59$ mm.

16.31. 167 m.

16.32. Assume that the aperture in the diaphragm lets k Fresnel zones through. The radius of the k-th zone will therefore be the radius of the aperture, equal to $r_k=\sqrt{k\frac{ad\lambda}{a+d}}$. The minimum illumination of the centre of the rings observed on the screen corresponds to two zones. ($k=2$). Upon inserting the numerical data, we find that $r=10^{-3}$ m $=1$ mm.

16.33. At 0.8 m. **16.34.** $\varphi_1=17°8'$, $\varphi_2=36°5'$, $\varphi_3=62°$.

16.35. 5 cm. **16.36.** $\varphi=30°$. **16.37.** $d=2.8\times10^{-6}$ m, $N_0=3,570$ cm^{-1}.

16.38. $N_0=600$ mm^{-1}. **16.39.** $\lambda=4,099$ Å, $N_0=500$ mm^{-1}.

16.40. In our case $\sin\varphi=\frac{k_1\lambda_1}{d}=\frac{k_2\lambda_2}{d}$, or $k_1\lambda_1=k_2\lambda_2$. Hence $\frac{k_2}{k_1}=\frac{\lambda_1}{\lambda_2}=\frac{6,563}{4,102}=$ $=1.6$. Since k_1 and k_2 must be integers, then obviously the values $k_1=5$ and $k_2=8$ satisfy the condition that $\frac{k_2}{k_1}=1.6$. Thus

$$d=\frac{k_1\lambda_1}{\sin\varphi}=\frac{5\times6,563\times10^{-10}}{0.656}\text{ m}=5\times10^{-6}\text{ m}$$

16.41. $\lambda=6,600$ Å in the second-order spectrum.

16.42. $\lambda=4,470$ Å; the dark-blue line of the helium spectrum.

16.43. $\lambda=7.05\times10^{-7}$ m.

16.44. The maximum order of a spectrum obtained with the aid of this grating is 3.

16.45. $d=5\lambda$. **16.46.** Ten maxima, not counting the central one.

16.47. $d=3.9$ μ. **16.48.** $d=2.2\times10^{-3}$ cm.

16.49. $d=2.54\times10^{-2}$ mm. **16.50.** $\Delta\lambda=0.24$ Å.

16.51. We have

$$(a+b)\sin\varphi=k\lambda \tag{1}$$

Upon differentiating Eq. (1), we obtain $(a+b)\cos\varphi d\varphi=kd\lambda$ or $\frac{d\varphi}{d\lambda}=\frac{k}{(a+b)\cos\varphi}$. By inserting the numerical data, we find from Eq. (1) that $\sin\varphi=0.236$, whence $\varphi=13°38'$. Hence $\cos\varphi=0.972$ and $\frac{d\varphi}{d\lambda}=4.1\times10^5$ rad/m.

16.52. $d=5\times10^{-6}$ m. **16.53.** $D_1=8.1\times10^{-3}$ mm/Å

16.54. $D_1=0.031$ mm/Å, $x=0.65$ mm.

16.55. (1) $\lambda=4,750$ Å, (2) $N_0=460$ mm^{-1}, (3) $D=2.76\times10^4$ rad/cm.

16.56. $\lambda=5.1\times10^{-4}$ mm. **16.57.** $F=0.65$ m. **16.58.** 57°30'.

16.59. 54°44'. **16.60.** 37°. **16.61.** $n=1.73$.

16.62. (1) $n=1.63$, (2) $i=66°56'$.

16.63. $\lambda_o=3.55\times10^{-7}$ m, $\lambda_e=3.95\times10^{-7}$ m.

16.64. Let us denote the intensity of natural light by I_0. After passing through the polarizer, the ray has an intensity of $I_1=0.5\,I_0$. After the analyzer its intensity will be $I_2=I_1\cos^2\alpha=0.5\,I_0\cos^2\alpha$. According to the initial condition, $\frac{I_2}{I_0}=0.25$ and then $\cos^2\alpha=\frac{0.25}{0.50}=\frac{1}{2}$ and $\alpha=45°$.

16.65. 62°32'.

16.66. The reflection factor of incident light $k'=\frac{I_r}{I_0}$, where $I_r=I_\perp+I_\parallel$. Here

$$I_\perp=0.5I_0\frac{\sin^2(i-r)}{\sin^2(i+r)},\quad\text{and}\quad I_\parallel=0.5I_0\frac{\tan^2(i-r)}{\tan^2(i+r)}$$

In our case, when the light falls at the angle of complete polarization, $\tan i=n=1.54$ and, therefore, $i=57°$. Further, since $i+r=90°$, the refraction angle $r=90°-57°=33°$, and $i-r=24°$ Thus,

$$I_{\perp}=0.5I_0\frac{\sin^2 24°}{\sin^2 90°}=0.083I_0, \text{ and } I_{\parallel}=0.5I_0\frac{\tan^2 24°}{\tan^2 90°}=0$$

i.e., in reflected light, when the angle of incidence is equal to the angle of complete polarization, oscillations occur only in a plane perpendicular to the plane of incidence. In this case $k'=\frac{I_r}{I_0}=\frac{I_{\perp}+I_{\parallel}}{I_0}=0.083$. In other words, only 8.3 per cent of the energy of the incident natural rays is reflected from the glass. These will be rays with oscillations perpendicular to the plane of incidence. Hence, the energy of the oscillations perpendicular to the plane of incidence and which have passed into the second medium will be 41.7 per cent of the total energy of the rays falling upon the boundary, while the energy of the oscillations in the plane of incidence will be 50 per cent. The degree of polarization of the rays that have passed into the second medium will be

$$P=\frac{I_{\parallel}-I_{\perp}}{I_{\parallel}+I_{\perp}}=\frac{0.083}{0.917}=0.091=9.1\%$$

16.67. When a natural ray falls upon a glass plate at the angle of complete polarization, the refracted ray has an intensity of $I_1=0.917I_0$ (see the solution to the previous problem). In this refracted ray the oscillations perpendicular to the plane of incidence are responsible for $0.417I_0$, and those parallel to the plane of incidence—for $0.5I_0$. The intensity of the ray reflected from the second face of the plate is $I_2=0.083\times0.917I_0=0.076I_0$. Hence, the intensity of the ray emerging from the plate into the air will be $I_3=0.917I_0-0.076I_0=0.841I_0$. Here the rays with oscillations parallel to the plane of incidence are responsible for $0.5I_0$ and those with oscillations perpendicular to the plane of incidence—for $0.341I_0$. The degree of polarization will thus be

$$P=\frac{I_{\parallel}-I_{\perp}}{I_{\parallel}+I_{\perp}}=\frac{0.159}{0.841}=18.9\%$$

i.e., it will increase. This permits the use of a pile of plane-parallel plates ("Stoletov's pile") as a polarizer.

16.68. (1) $k'=\frac{I_r}{I_0}=5.06$ per cent, $P=83$ per cent, (2) 4.42 per cent

17. Elements of the Theory of Relativity

17.1. We have

$$l'=l_0\sqrt{1-\beta^2} \quad (1)$$

According to the initial condition, $\frac{l_0-l'}{l_0}=1-\frac{l'}{l_0}=0.25$. Hence $\frac{l'}{l_0}=0.75$, or

$$l'=0.75l_0 \quad (2)$$

Upon inserting Eq. (2) into (1), we obtain $\sqrt{1-\beta^2}=0.75$, or $1-\beta^2=(0.75)^2=0.5625$ and $\beta^2=0.4375$ Thus, $\beta=\frac{v}{c}=\sqrt{0.4375}=0.6615$, and finally $v=\beta c=0.662\times3\times10^8$ m/s $=198{,}000$ km/s

17.2. $v=2.6\times10^8$ m/s. **17.3.** $\frac{l_0-l'}{l_0}=68.8\%$ **17.4.** 7.1 times.

17.5. $\Delta\tau = 3.2$ s. 17.6. By 8.6×10^{-27} kg.

17.7. Figure 109 shows how the mass of an electron m and the ratio $\frac{e}{m}$ depend on the quantity $\frac{v}{c}$.

17.8. At $v = 2.6\times10^{8}$ m/s.

17.9. We have

$$E_k = m_0c^2\left(\frac{1}{\sqrt{1-\beta^2}} - 1\right) = c^2\left(\frac{m_0}{\sqrt{1-\beta^2}} - m_0\right) = c^2(m - m_0)$$

whence

$$\frac{E_k}{m_0} = \frac{c^2(m-m_0)}{m_0}$$

Let $\frac{m-m_0}{m_0} = k$. Hence $E_k = m_0c^2k$. According to the initial condition, $k = 0.05$.
(1) $E_k = 2.56\times10^{-2}$ MeV, (2) $E_k = 47$ MeV, (3) $E_k = 94$ MeV.

17.10. $U = 1.1\times10^{6}$ V. 17.11. $U = 510$ kV.

17.12. The total energy of the meson consists of its kinetic energy E_1 and its own energy E_2 (the energy of rest). We have

$$E_1 = m_0c^2\left(\frac{1}{\sqrt{1-\beta^2}} - 1\right) \qquad (1)$$

and

$$E_2 = m_0c^2 \qquad (2)$$

Hence, the total energy is

$$E = E_1 + E_2 = \frac{m_0c^2}{\sqrt{1-\beta^2}}$$

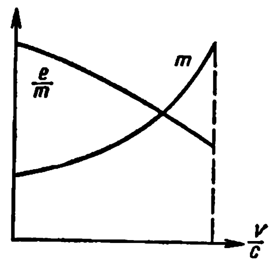


Fig. 109

According to the initial condition, $\frac{E}{E_2} = 10$, i. e., $\frac{1}{\sqrt{1-\beta^2}} = 10$. Hence $\beta = \frac{v}{c} = 0.995$ and $v = 2.985\times10^{8}$ m/s.

17.13. $\beta = 86.6\%$. 17.14. $\beta = 99.6\%$. 17.15. $\frac{l_0 - l'}{l_0} = 91.5\%$.

17.16. $\beta = 0.9$. 17.18. $E_k = 8.2\times10^{-14}$ J. 17.19. $\Delta m = 4.6\times10^{-17}$ kg.

17.20. $\Delta E = 931$ MeV. 17.21. $\Delta E = 8.2\times10^{-14}$ J $= 0.51$ MeV.

17.22. $\Delta m = 3.2\times10^{-9}$ kg/kmole. Thus, the reaction yields not 18 kg of water, but 3.2×10^{-9} kg less. This amount is beyond the sensitivity of the most accurate scales. The mass also changes in the same way during other chemical reactions. In nuclear reactions the change in mass is appreciable (see the next problem).

17.23. $\Delta m = 0.217$ kg/kmole. 17.24. In 7×10^{12} years.

18. Thermal Radiation

18.1. $T = 1000$°K. 18.2. $Q_e = 6.5\times10^{21}$ kW-h. 18.3. $Q_e = 0.46$ J.

18.4. $T = 1000$°K. 18.5. (1) $Q_e = 1.33\times10^{5}$ J, (2) $k = 0.3$.

18.6. $T = 2500$°K. 18.7. $S = 4\times10^{-5}$ m^2.

18.8. $q_e = 1.37\times10^{3}$ W/m^2 $= 8.21$ J/min·cm^2 $= 1.96$ cal/min·cm^2.

18.9. $N = 3.1\times10^{3}$ kW. 18.10. $q_e = 0.85$ cal/min·cm^2.

18.11. $Q_e = 7.35 \times 10^3$ J. **18.12.** $S = 6$ cm^2.

18.13. (1) $\lambda_{max} = 1\ \mu$—the infrared region, (2) $\lambda_{max} = 5 \times 10^{-5}$ cm—the visible light region, (3) $\lambda_{max} \cong 3\text{Å}$—the X-ray region.

18.14. (1) We find from the curve in Fig. 64 that the wavelength of the maximum energy density of the body is approximately 1.2 μ. From Wien's law we obtain $T = 2400°$K.

(2) The percentage of the radiant energy falling to the share of the visible spectrum is obviously determined by the part of the area limited by the curve $M_{e\lambda} = f(\lambda)$ which is cut off by the ordinates erected at the edges of the interval interesting us. The visible spectrum extends approximately from 0.4 to 0.75 μ. Upon plotting the curve of Fig. 64 on millimetre graph paper, we can find that about 3-5% of the total radiation falls to the share of the visible radiation at the given temperature.

18.15. 3.6 times. **18.16.** $\lambda = 9.3\ \mu$.

18.17. (1) 81 times, (2) from $\lambda_1 = 2.9$ to $\lambda_2 = 0.97\ \mu$, (3) 243 times.

18.18. $T_2 = \dfrac{C_1 T_1}{\Delta\lambda\, T_1 + C_1} = 290°$K.

18.19. It will grow 1.06 times. **18.20.** 0.84 W. **18.21.** By 0.2 μ.

18.22. (1) $\Delta m = \dfrac{\Delta Q_e}{c^2} = 1.4 \times 10^{17}$ kg, (2) $\tau = 7 \times 10^{12}$ years.

Chapter 6

ATOMIC AND NUCLEAR PHYSICS

19. Quantum Nature of Light and Wave Properties of Particles

19.1. (1) 3.2×10^{-36} kg, (2) 8.8×10^{-32} kg, (3) 1.8×10^{-30} kg.
19.2. $\varepsilon=1.15\times10^{-13}$ J, $m=1.38\times10^{-30}$ kg, $\bar{p}_{ph}=4.1\times10^{-22}$ kg·m/s.
19.3. (1) 6.2×10^{18} quanta, (2) 1.2×10^{19} quanta, (3) 1.1×10^{19} quanta, (4) 5.9×10^{18} quanta, (5) 4.6×10^{18} quanta, (6) 5.1×10^{18} quanta.
19.4. $v=9.2\times10^{6}$ m/s. **19.5.** $v=1{,}400$ m/s. **19.6.** 0.51 MeV.

19.7. $E=\frac{\bar{p}_{ph}c}{St}=150$ J/m²·s. **19.8.** $T=9800$°K.

19.9. $\lambda\geqslant4.1\times10^{-3}$ Å. **19.10.** $m=2.1\times10^{-32}$ kg.

19.11. We have $h\nu=W+\frac{mv^2}{2}$. For the photoeffect to appear, it is necessary that $h\nu>W$, i. e., $\nu>\frac{W}{h}$. But $\nu=\frac{c}{\lambda}$, and therefore the wavelength of the incident light should satisfy the inequality $\lambda<\frac{hc}{W}$. In Stoletov's experiments, $\lambda\leqslant2.95\times10^{-4}$ mm, whence $W=4.2$ eV.
19.12. 5.17×10^{-7} m, 5.4×10^{-7} m, 6.2×10^{-7} m, 6.6×10^{-7} m.
19.13. $\varepsilon=4.5$ eV.
19.14. (1) $W=4.5$ eV, (2) $v_{max}=9.1\times10^{5}$ m/s, (3) $E_{k\cdot max}=3.8\times10^{-19}$ J.
19.15. Since the photoeffect commences at $\nu_0=6\times10^{14}$ s^{-1}, the work of emission of an electron is

$$W=h\nu_0=\frac{6.62\times10^{-34}\times6\times10^{14}}{1.6\times10^{-19}}\text{ eV}=2.48\text{ eV}$$

Further, we have $h\nu=W+\frac{mv^2}{2}$ To retard escaping electrons, a retarding electric field is applied, and $eU_1=\frac{mv^2}{2}$ Thus, $h\nu=W+eU_1$, whence $\nu=\frac{W+eU_1}{h}$. Upon inserting the numerical data, we obtain $\nu=13.2\times10^{14}$ s^{-1}.
19.16. $U=1.75$ V. **19.17.** (1) 2,040 Å, (2) 2,340 Å.
19.18. $3\ 45\times10^{-25}$ kg·m/s. **19.19.** $h=6.6\times10^{-34}$ J·s.

19.20. (1) $U_x=\frac{h\nu-W}{e}+U_0=1.5$ V,

(2) $v=\sqrt{\frac{2}{m}(h\nu-W+eU_0)}=7.3\times10^{5}$ m/s.

19.21. At $\lambda\leqslant2{,}540$ Å.

19.22. (1) We have $p=\frac{F}{S}$, where p is the pressure of light, and F is the force exerted by this pressure on the surface S of a disk. But $F=\frac{M}{l}=\frac{k\alpha}{l}$, where M is the twisting moment, l the distance from the centre of the disk to the axis of rotation, and α the angle through which the disk turns. For the mirror light spot reflected onto the scale at a distance y from the mirror to deflect by x, the angle through which the mirror turns should satisfy the condition that $\tan 2\alpha=\frac{x}{y}$ or, if the angles are small, $\tan 2\alpha\approx 2\alpha=\frac{x}{y}$. Thus $\alpha=\frac{x}{2y}$. Then, finally, $p=\frac{kx}{2lyS}$ Insertion of the data gives $p=3.85\times10^{-6}$ N/m^2, (2) $E=7.7\times10^{-3}$ J/cm^2·s.

19.23. (1) $p=3.55\times10^{-7}$ N/m^2, (2) $P=2.1\times10^{-3}$ W.

19.24. (1) 1.2×10^{17} 1/cm^2·s, (2) (a) 1.42×10^{-6} N/m^2, (b) 2.13×10^{-6} N/m^2, (c) 2.84×10^{-6} N/m^2.

19.25. (1) $p=4.5\times10^{-6}$ N/m^2, (2) $m=7.8\times10^{-16}$ kg.

19.26. $p=1.04\times10^{-5}$ N/m^2.

19.27. (1) $p=7\times10^{-7}$ N/m^2, (2) $p=3.5\times10^{-7}$ N/m^2.

19.28. 2.9×10^{21} quanta.

19.29. (1) $\Delta\lambda=0.024$ Å and $\lambda=\lambda_0+\Delta\lambda=0.732$ Å, (2) $\Delta\lambda=0.048$ Å and $\lambda=0.756$ Å.

19.30. $\lambda_0=0.242$ Å.

19.31. (1) $\Delta\lambda=0.024$ Å, (2) $E_e=\frac{hc\,\Delta\lambda}{\lambda_0\lambda}=6.6\times10^3$ eV, (3) $\bar{p}_e=4.4\times\times10^{-23}$ kg·m/s.

19.32. $E=2.6\times10^5$ eV, $\bar{p}_{ph}=9.3\times10^{-12}$ kg·m/s.

19.33. $E=0.1$ MeV. **19.34.** (1) $\lambda=12.3$ Å, (2) $\lambda=1.23$ Å.

19.35. (1) $\lambda=0.29$ Å, (2) $\lambda=0.029$ Å.

19.36. (1) $\lambda=7.3$ Å, (2) $\lambda=1.44$ Å, (3) $\lambda=6.6\times10^{-27}$ cm, i. e., it is impossible to detect the wave properties of the ball.

19.37. 0.122 Å, 0.0087 Å. **19.38.** $m=1.67\times10^{-27}$ kg.

19.39.

$v\cdot10^{-8}$, m/s	2.0	2.2	2.4	2.6	2.8
$\lambda\cdot10^{12}$, m	2.70	2.25	1.82	1.39	0.925

19.40. $\lambda=0.1$ Å. **19.41.** $\lambda=1.8$ Å

20. Bohr's Atom. X-Rays

20.1. (1) $r_1=0.53\times10^{-10}$ m, $r_2=2.12\times10^{-10}$ m, $r_3=4.77\times10^{-10}$ m, (2) $v_1=2.19\times10^6$ m/s, $v_2=1.1\times10^6$ m/s, $v_3=7.3\times10^5$ m/s.

20.2. $E_k=\frac{me^4}{8\varepsilon_0^2h^2k^2}=13.6$ eV, $E_p=-2E_k=-27.2$ eV, $E_{tot}=E_k+E_p=-13.6$ eV.

20.3. $E_1=2.18\times10^{-18}$ J $=13.6$ eV, $E_2=5.44\times10^{-19}$ J $=3.40$ eV, $E_3=2.42\times10^{-19}$ J $=1.51$ eV, $E_4=0$.

20.4. (1) $T=1.43\times10^{-16}$ s, (2) $\omega=4.4\times10^{16}$ rad/s.

20.5. The wavelengths of the hydrogen spectral lines of all series are determined by the formula

$$\frac{1}{\lambda}=R\left(\frac{1}{k^2}-\frac{1}{n^2}\right) \qquad (1)$$

Various integral values of k and n give a number of spectral series:

k	n	Series
1	2, 3, 4, ...	Lyman in the ultraviolet region
2	3, 4, 5, ...	Balmer in the visible region
3	4, 5, 6, ...	Paschen } in the infrared region
4	5, 6, 7, ...	Brackett } in the infrared region
5	6, 7, 8, ...	Pfund } in the infrared region

Thus, the series in the visible region of the spectrum corresponds to $k=2$ and $n=3$, 4, 5, ... The wavelength of the spectral lines of this series will obviously be minimum when $n=\infty$. Hence from Eq. (1) we have $\frac{1}{\lambda_1}=\frac{R}{4}$, or $\lambda_1=\frac{4}{R}=3.65\times10^{-7}$ m (with an accuracy to the third significant figure). The maximum wavelength corresponds to $n=3$; i. e., $\lambda_2=6.56\times10^{-7}$ m. Thus the visible hydrogen spectrum lies within the wavelength range from $\lambda_1=3{,}650$ Å to $\lambda_2=6{,}560$ Å.

20.6. (1) $\lambda=1.21\times10^{-7}$ m, (2) $v=1.90\times10^6$ m/s.

20.7. The ionization potential U_i of an atom is determined by the equation $eU_i=W_i$, where W_i is the work of removing an electron from a normal orbit into infinity. For a hydrogen atom $W_i=h\nu=hcR\left(\frac{1}{k^2}-\frac{1}{n^2}\right)$. When $k=1$ and $n=\infty$, the work $W_i=hcR$ and the ionization potential $U_i=\frac{W_i}{e}=\frac{hcR}{e}=13.6$ V.

20.8. 10.2 V.

20.9. (1) All the lines of all the series of the hydrogen spectrum appear when the hydrogen atom is ionized. This will occur if the energy of the electrons is 13.6 eV (see Problem 20.7),

(2) $v_{min}=\sqrt{\frac{2eU_i}{m}}=2.2\times10^6$ m/s.

20.10. The energy needed to transfer an atom to the first excited state $E_1=10.2$ eV (see Problem 20.8). It is easy to see that the energy required to transfer an atom to the second excited state ($k=1$, $n=3$) is $E_2=12.1$ eV. Thus the hydrogen spectrum will have only one spectral line if the energy of the bombarding electrons is within $10.2\leqslant E\leqslant 12.1$ eV.

20.11. $E_{min}=12.1$ eV, $\lambda_1=1.21\times10^{-7}$ m, $\lambda_2=1.03\times10^{-7}$ m, $\lambda_3=6.56\times\times10^{-7}$ m.

20.12. 973 Å $\leqslant\lambda\leqslant$ 1,026 Å. 20.13. By 2.56 eV.

20.14. 973 Å $\leqslant\lambda\leqslant$ 1,026 Å. 20.15. From $n=3$ to $k=2$.

20.16. $\lambda=3.3$ Å 20.17. (1) $r_1=2.66\times10^{-11}$ m, (2) $v_1=4.37\times10^6$ m/s.

20.18. (1) $U=40.8$ V, (2) $U=91.8$ V. 20.19. (1) $U=54$ V, (2) $U=122$ V.

20.20. $\lambda=304$ Å. 20.21. $\lambda=135$ Å. 20.22. $\lambda=5{,}890$ Å.

20.23. At 2.1 V. 20.24. $\lambda=2{,}540$ Å.

20.25. The minimum angle corresponds to the first-order spectrum, i. e., $\lambda = 2d \sin\theta$, whence $\sin\theta = \frac{\lambda}{2d} = 0.033$, or $\theta = 1°54'$.

20.26. The volume of one kilomole of rock salt $V = \frac{\mu}{\rho}$. This volume contains $2N_A$ ions, where N_A is Avogadro's number. Hence the volume per ion is $V' = \frac{\mu}{2\rho N_A}$. Consequently, the distance d between ions (i. e., the lattice constant) can be found from the condition $V' = d^3$, in other words

$$d = \sqrt[3]{V'} = \sqrt[3]{\frac{\mu}{2\rho N_A}} = 2.81 \times 10^{-10} \text{ m} = 2.81 \text{ Å}$$

20.27. When the potential difference U applied to the electrodes of the X-ray tube is increased, a spectral line appears in the first-order spectrum whose wavelength λ satisfies the equation

$$eU = h\nu = h\frac{c}{\lambda} \qquad (1)$$

But according to Bragg's formula

$$\lambda = 2d \sin\theta \qquad (2)$$

We find from Eqs. (1) and (2) that

$$h = \frac{eU\lambda}{c} = \frac{eU\,2d\sin\theta}{c} \qquad (3)$$

Insertion of the numerical data into Eq. (3) gives

$$h = 6.6 \times 10^{-34} \text{ J·s}$$

20.28. $h = 6.6 \times 10^{-34}$ J·s.

20.29. (1) 0.413 Å, (2) 0.310 Å, (3) 0.248 Å.

20.30. 0.27 Å. **20.31.** 770 kV.

20.32. All the lines of the K-series (as well as those of other series) appear simultaneously as soon as an electron is removed from the K-orbit of the atom. For this purpose a potential difference U should be applied which satisfies the equation $eU = h\nu_1 = \frac{hc}{\lambda_1}$, where λ_1 is the wavelength corresponding to the transition of an infinitely removed electron to the K-orbit, i. e., the wavelength determining the boundary of the K-series. In our case, this boundary is equal (see Table XVII of the Appendix) to (1) 1.38 Å, (2) 0.484 Å, (3) 0.178 Å, (4) 0.158 Å. The sought potential difference can be found from the formula $U = \frac{hc}{e\lambda_1}$. Upon inserting the numerical data, we obtain (1) 9 kV, (2) 25.3 kV, (3) 69 kV, (4) 79 kV.

20.33. We have

$$\frac{1}{\lambda} = R(z-b)^2 \left(\frac{1}{k^2} - \frac{1}{n^2}\right) \qquad (1)$$

The maximum wavelength of the K-series corresponds to the line K_α, and it should be assumed in formula (1) that $b = 1$, $k = 1$, and $n = 2$. Upon solving formula (1) with respect to λ and inserting the numerical data, we obtain (1) 1.94 Å, (2) 1.55 Å, (3) 0.720 Å, (4) 0.574 Å, (5) 0.234 Å, (6) 0.228 Å, and (7) 0.205 Å. The wavelengths of the line K_α found experimentally are as follows: (1) 1.94 Å, (2) 1.54 Å, (3) 0.712 Å, (4) 0.563 Å, (5) 0.220 Å, (6) 0.214 Å, (7) 0.190 Å.

20.34. The transfer of an electron from the M-layer to the L-layer corresponds to $k=2$ and $n=3$. The atomic number of tungsten in the Mendeleyev Table $Z=74$. Upon inserting these numerical data into Moseley's formula, we find that $b=5.5$.

20.35. $Z=40$ (zirconium). **20.36.** $\frac{N_1}{N}=\frac{\mu D_e}{N_A e}=3.5\times10^{-10}$.

20.37. 1.6×10^{11} pairs. **20.38.** $I_s=2.7\times10^{-16}$ A.

20.39. $x=5\times10^{-4}$ m $=0.5$ mm. **20.40.** 3.7 times. **20.41.** $x=0.08$ mm.

20.42. (1)

Material	Water	Aluminium	Iron	Lead
μ, m^{-1}	6.7	16	44	77
$\mu_m\times10^3$, m^2/kg	6.7	6.2	5.6	6.8

(2) $\lambda=0.0124$ Å.

20.43. $n=\frac{\log_e 80}{\log_e 2}=6.35$.

21. Radioactivity

21.1 and **21.2.** The number of atoms of a radioactive substance which disintegrate during the time dt is determined by the formula

$$dN=-\lambda N\,dt \tag{1}$$

Obviously, this formula can be used for the final time interval Δt only when the number of available atoms N is constant during the time Δt, i. e., when Δt is much smaller than the half-life T. It can easily be seen (see the tables in the Appendix) that when solving Problem 21.1 the number of polonium atoms which disintegrate during twenty-four hours can be found from the formula

$$|\Delta N|=\lambda N\,\Delta t=\frac{\log_e 2}{T}N\Delta t \tag{2}$$

Upon inserting the numerical data of Problem 21.1, we obtain

$$\Delta N=\frac{0.693}{138}\times10^6\ \text{days}^{-1}=5{,}025\ \text{days}^{-1}$$

This approximate formula cannot be used for the solution of Problem 21.2, however, since the half-life of radon (see the tables) is only 3.82 days. The number of radon atoms which disintegrate during twenty-four hours can be found from the formula $N=N_1e^{-\lambda t}$. The sought number will thus be

$$\Delta N=N_1-N=N_1-N_1e^{-\lambda t}=N_1(1-e^{-\lambda t})$$

Upon inserting the numerical data of Problem 21.2, we find

$$\Delta N=10^6(1-0.833)=167{,}000\ \text{days}^{-1}$$

If we try to find ΔN from approximate formula (2), we get $\Delta N=192{,}000$ days^{-1}, i. e., an error of about 10% will be made.

Prove that the same answer with an accuracy to the third significant figure is obtained when Problem 21.1 is solved by formulas (1) and (2).

21.3. 3.7×10^{10} d/s = 1 curie. **21.4.** $m=6.5\times10^{-9}$ kg.

21.5. $m=2.2\times10^{-7}$ kg = 0.22 mg. **21.6.** $\lambda=2.1\times10^{-6}\ \text{s}^{-1}$.

21.7. (1) 7.9×10^{7} d/s·kg, (2) 5.7×10^{18} d/s·kg.

21.8. $m=3.5\times10^{-20}$ kg. **21.9.** 53. **21.10.** In 40 days.

21.11. The share of radioactivity in per cent introduced by each isotope into the total radioactivity of natural uranium can obviously be determined by the ratio of the number of disintegrations per second of each isotope to the total number of disintegrations per second of natural uranium. Let us denote the mass of natural uranium by M. The masses of the isotopes will thus be equal to $M_1=6\times10^{-5}M$, $M_2=7.1\times10^{-3}M$ and $M_3=99.28\times10^{-2}M$, respectively. The number of disintegrations per second produced by each isotope will be

$$\Delta N_1=\frac{\log_e 2}{T_1}N_1\Delta t=\frac{\log_e 2N_AM_1\Delta t}{T_1A_1}$$

$$\Delta N_2=\frac{\log_e 2N_AM_2\Delta t}{T_2A_2}$$

$$\Delta N_3=\frac{\log_e 2N_AM_3\Delta t}{T_3A_3}$$

where N_A = Avogadro's number, T_i = half-life of the isotope, A_i = atomic weight of the isotope. Hence the sought ratio for each isotope will be

$$x_i=\frac{\Delta N_i}{\Delta N_1+\Delta N_2+\Delta N_3}=\frac{\dfrac{M_i}{A_iT_i}}{\dfrac{M_1}{A_1T_1}+\dfrac{M_2}{A_2T_2}+\dfrac{M_3}{A_3T_3}}$$

Upon inserting the numerical data, it is easy to see that all the radioactivity of natural uranium is due to the isotope ${}_{92}U^{238}$, while that of ${}_{92}U^{235}$ and ${}_{92}U^{234}$ is vanishingly small.

21.12. (1) $v=1.52\times10^{7}$ m/s.

(2) The total energy evolved during the escape of the α-particle is equal to the sum of the kinetic energy of the α-particle E_1 and the kinetic energy E_2 of the residual nucleus. Thus,

$$E_x=E_1+E_2 \qquad (1)$$

Besides, account should be taken of the law of conservation of momentum. Since the momentum of the system before disintegration is zero, then after it $\bar{p}_1=\bar{p}_2$, whence

$$\bar{p}_1^2=\bar{p}_2^2 \qquad (2)$$

But $\bar{p}_1^2=(m_1v_1)^2=2m_1\dfrac{m_1v_1^2}{2}=2m_1E_1$ and, correspondingly, $\bar{p}_2^2=2m_2E_2$

From Eq. (2) we have $2m_1E_1=2m_2E_2$, whence $E_2=\dfrac{m_1E_1}{m_2}$.

Now from Eq. (1) we get $E_x=E_1+\dfrac{m_1E_1}{m_2}=E_1\dfrac{m_1+m_2}{m_2}$. Upon inserting the numerical data, we obtain $E_x=4.87$ MeV.

21.13. (1) $Q=120$ J, (2) $Q=1.6\times10^{4}$ J. **21.14.** $Q=5.2\times10^{12}$ J.

21.15. 2.8×10^{9} d/s. **21.16.** $N_A=6\times10^{26}\ \text{kmole}^{-1}$. **21.17.** $m=4.8\times10^{-9}$ kg.

21.18. (1) In 12.6 days. (2) The relation $\dfrac{N}{N'}=f(t)$ is shown in Fig. 110.

21.19. (1) The relation $\frac{N}{N'}=f(t)$ is shown in Fig. 111. (2) The half-life will be found as the abscissa of the point on the curve whose ordinate is 0.5. In our case from the curve $\frac{N}{N'}=f(t)$ with the scale indicated on the axes we get $T=3.8$ days.

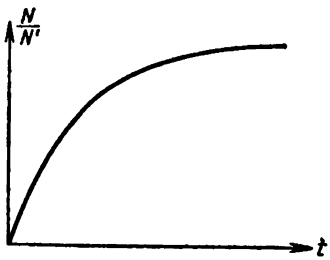


Fig. 110

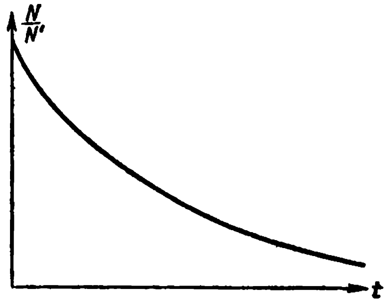


Fig. 111

21.20. $T \cong 4$ h. 21.21. In 10.4 days.

21.22. We have $N_t = N_u\left(1-e^{-\frac{0.693t}{T_u}}\right)$ or

$$\frac{M_t}{A_t}=\frac{M_u}{A_u}\left(1-e^{-\frac{0.693t}{T_u}}\right)$$

Hence $t=3\times10^9$ years.

21.23. 2.8×10^6 atoms. 21.24. From 7×10^3 kg of ore.

21.25. 1.1×10^{-7} cd. 21.26. 63.2%. 21.27. 1.67×10^8 d/s.

21.28. 5.25×10^{15} d/s·kg.

21.29. Before mixing, the specific activity of the preparation was equal to

$$a_1=\frac{\Delta N}{m_1\Delta t}=\frac{\lambda N}{m_1}=\frac{\log_e 2N_A m_1}{TA_1m_1}=\frac{\log_e 2N_A}{TA_1} \qquad (1)$$

After mixing

$$a_2=\frac{\Delta N}{(m_1+m_2)\,\Delta t}=\frac{\log_e 2N_A m_1}{TA_1(m_1+m_2)} \qquad (2)$$

It follows from Eqs. (1) and (2) that

$$\Delta a=\frac{\log_e 2N_A}{TA_1}\left(1-\frac{m_1}{m_1+m_2}\right)=\frac{\log_e 2N_A m_2}{TA_1(m_1+m_2)}=$$
$$=4.9\times10^{17}\text{ d/s·kg}=1.32\times10^7\text{ c/kg}$$

21.30. 11 mg. 21.31. ${}_{84}Po^{216}$. 21.32. ${}_{88}Ra^{226}$.

21.33. ${}_{92}U^{235}$. 21.34. ${}_{2}He^{4}$. 21.35. ${}_{55}Cs^{133}$.

21.36. (1) $v=1.92\times10^7$ m/s, (2) $E=7.83$ MeV (see the solution to Problem 21.12), (3) $n=2.26\times10^5$ ion pairs, (4) $I_s=1.33\times10^{-9}$ A.

22. Nuclear Reactions

22.1. (1) 12 protons and 12 neutrons, (2) 12 protons and 13 neutrons, (3) 12 protons and 14 neutrons.

22.2. We have $\Delta M = ZM_{_1H^1} + (M-Z) M_n - M_A$. In our case (see the tables in the Appendix) $\Delta M = 3\times 1.00814 + 4\times 1.00899 - 7.01823$ amu $= 0.04215$ amu. Since an energy of 931 MeV corresponds to the atomic mass unit (see Problem 17.20), the nuclear binding energy of $_3Li^7$ will finally be equal to $E = 0.04215\times\times 931$ MeV $= 39.3$ MeV. This energy must be spent to split the $_3Li^7$ nucleus into nucleons.

22.3. $E = 28.3$ MeV. **22.4.** 225 MeV.

22.5. (1) $E = 8.5$ MeV, (2) $E = 7.7$ MeV. The nucleus $_1H^3$ is more stable than $_2He^3$.

22.6. $E_0 = 7.97$ MeV. **22.7.** $E = 2.2$ MeV.

22.8. (1) 5.6 MeV, (2) 7.5 MeV, (3) 8.35 MeV, (4) 8.55 MeV, (5) 8.75 MeV, (6) 8.5 MeV, (7) 7.9 MeV, (8) 7.6 MeV.

22.9. We have $E = c^2 (\sum M_1 - \sum M_2)$. In our case the sum of the masses of the initial particles $\sum M_1 = 7.01823 + 1.00814 = 8.02637$. The sum of the masses of the formed particles $\sum M_2 = 4.00388 + 4.00388 = 8.00776$. Thus, the mass defect $\Delta M = 0.01861$ amu. Hence, the energy evolved during the reaction is $E = 0.01861\times 931$ MeV $= 17.3$ MeV.

22.10. 1.18 MeV. **22.11.** (1) 4.04 MeV, (2) 3.26 MeV.

22.12. (1) 18.3 MeV, (2) 22.4 MeV, (3) 4.02 MeV.

22.13. $M = 5.7\times 10^5$ kg. **22.15.** 15 MeV. **22.16.** 4,35 MeV.

22.17. $_7N^{14} + {_0n^1} \longrightarrow {_6C^{14}} + {_1H^1}$; $_6C^{14} \longrightarrow {_{-1}e^0} + {_7N^{14}}$.

22.18. 1.1×10^{23} d/s·kg. **22.19.** $T = 15$ h.

22.20. (1) $E_1 = 5.35\times 10^{22}$ MeV, (2) $E_2 = 3.6\times 10^{29}$ MeV.

Thus, $\frac{E_2}{E_1} \cong 7\times 10^6$, i. e., to effect this transformation, about seven million times more energy must be spent than is released during this reaction.

22.21. 6.017 amu.

22.22. As a result of this cycle, four hydrogen nuclei are transformed into one helium nucleus. The carbon, performing as a chemical catalyst, can be used anew. It is easy to find that this cycle releases energy equal to 4.3×10^{-12} J. On the other hand, we can find the radiation of the Sun per second, knowing the solar constant and the distance from the Earth to the Sun. It is $E = 3.8\times\times 10^{26}$ J. If the transformation of four hydrogen atoms produces energy equal to 4.3×10^{-12} J, it is obvious that $M = 5.9\times 10^{11}$ kg of hydrogen must be used per second for radiating energy equal to 3.8×10^{26} J. Since the mass of the Sun is 2×10^{30} kg, the amount of hydrogen in it is $2\times 10^{30}\times 0.35$ kg $= 7\times 10^{29}$ kg. This will be sufficient for 4×10^{10} years.

22.23. $m = 1.00899$ amu.

22.25. According to the definition,

$$k_1 = \frac{N_1}{N_2} \tag{1}$$

where N_1 is the number of atoms formed during a certain time, and N_2 is the number of particles which bombard the target during this time. On the other hand, since the activity of a preparation is determined by the number of disintegrations a second, it is obvious that

$$k_2 = \frac{\lambda N_1}{N_2} = \frac{\log_e 2N_1}{TN_2} \tag{2}$$

where T is the half-life of the radioactive isotope formed. Thus, it follows from Eqs. (1) and (2) that $k_2 = \frac{\log_e 2}{T} k_1$.

22.26. $k_1 = 2\times 10^{-3} = \frac{1}{500}$, i. e., only one proton of 500 will cause the reaction.

22.27. $k_1 = 1.2\times 10^{-3}$.

22.28. (2) The initial number of disintegrations per second of the source is $a_1 = \left(\frac{\Delta N}{\Delta t}\right)_1 = \lambda N_1$. The number of disintegrations per second after the time t has elapsed is $a_2 = \left(\frac{\Delta N}{\Delta t}\right)_2 = \lambda N_2$, where $N_2 = N_1 e^{-\lambda t}$. Since only one α-particle out of $n = 4{,}000$ induces a reaction, we can find the number of radon atoms introduced into the source:

$$N' = nN_1 = \frac{nN_2}{e^{-\lambda t}} = nN_2 e^{\lambda t}$$

The radon mass will thus be

$$M = \frac{AN'}{N_A} = \frac{A}{N_A} nN_2 e^{\lambda t} = \frac{Ane^{\lambda t} a_2}{N_A \lambda} \qquad (1)$$

In our case $A = 222$ kg/kg-atom; $n = 4\times 10^3$; $e^{\lambda t} = e^{\frac{\log_e 2t}{T}} = 2.45$; $a_2 = 1.2\times 10^6\ \text{s}^{-1}$; and $N_A = 6.02\times 10^{26}$ kg-atom^{-1}. Upon inserting these data into Eq. (1), we obtain $M = 2.1\times 10^{-9}$ kg $= 2.1$ μg.

22.29. $9.3\times 10^6\ \text{s}^{-1}$.

22.30. (1) $E = 6.9$ MeV, (2) $5\times 77\times 10^{-12}$ d/s.

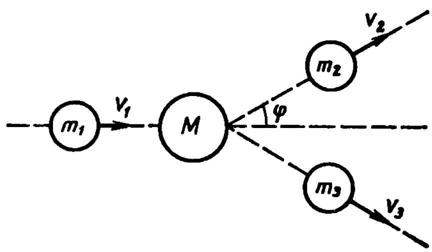


Fig. 112

22.31. Let us denote (Fig. 112) the mass numbers of a bombarding α-particle, proton and recoil nucleus (in our case an oxygen nucleus) by m_1, m_2 and m_3, and their kinetic energies by E_1, E_2 and E_3. If the nucleus M of nitrogen is immobile, the law of conservation of energy will be

$$E_1 + Q = E_2 + E_3 \qquad (1)$$

where Q is the energy of the nuclear reaction. The law of conservation of momentum in the vector form is

$$\bar{p}_1 = \bar{p}_2 + \bar{p}_3 \qquad (2)$$

From Eq. (2) for the numerical values of the momenta (see Fig. 112) we have

$$p_3^2 = \bar{p}_1^2 + \bar{p}_2^2 - 2\bar{p}_1\bar{p}_2 \cos\varphi \qquad (3)$$

Since

$$\bar{p}^2=(mv)^2=\frac{mv^2}{2}\,2m=E2m \tag{4}$$

equation (3) becomes

$$2m_3E_3=2m_1E_1+2m_2E_2-2\cos\varphi\sqrt{2m_1E_12m_2E_2}$$

or

$$E_3=\frac{m_1}{m_3}E_1+\frac{m_2}{m_3}E_2-\frac{2\cos\varphi}{m_3}\sqrt{m_1m_2E_1E_2} \tag{5}$$

From Eq. (1) we have $E_3=E_1+Q-E_2$. Upon substituting this expression for E_3 in Eq. (5), we obtain the final formula relating the kinetic energy of the bombarding particles to that of the particles obtained:

$$E_1\left(\frac{m_3-m_1}{m_3}\right)+Q=E_2\left(\frac{m_2+m_3}{m_3}\right)-\frac{2\cos\varphi}{m_3}\sqrt{m_1m_2E_1E_2} \tag{6}$$

Here $Q=-1.18$ MeV. Upon solving Eq. (6) with respect to $\cos\varphi$ and inserting the numerical data, we find that

$$\cos\varphi=\frac{m_2+m_3}{2}\sqrt{\frac{E_2}{m_1m_2E_1}}-\frac{m_3-m_1}{2}\sqrt{\frac{E_1}{m_1m_2E_2}}-\frac{m_3Q}{2\sqrt{m_1m_2E_1E_2}}=$$
$$=0.59 \text{ and } \varphi\cong 54^\circ$$

22.32. (1) $E_k=8.75$ MeV, (2) $\varphi\cong 87^\circ$.

22.33. (2) $Q=-0.78$ MeV: the reaction proceeds with the absorption of energy, (3) $E_x=|Q|\frac{M+m}{M}=1.04$ MeV, where M is the mass of a nucleus at rest, and m is the mass of a bombarding particle.

22.34. $E_x=1.52$ MeV. **22.35.** $E_x=1.89$ MeV.

22.36. (2) $Q=-0.30$ MeV, (3) $E_x=0.35$ MeV, (4) $E=E_x+Q=0.05$ MeV.

22.37. (1) $Q=2.8$ MeV, (2) $v=9.3\times10^6$ m/s, $E_\alpha=1.8$ MeV.

22.38. $E=1$ MeV. **22.39.** $h\nu=2.2$ MeV. **22.40.** $h\nu=16.6$ MeV

22.41. 2.3×10^4 kW-h. **22.42.** $M=31$ g.

22.43. (2) $Q=17.6$ MeV, (3) $E=11.8\times10^4$ kW-h.

23. Elementary Particles. Particle Accelerators

23.1. (1) $N=2.2\times10^{16}$, (2) $N=1.1\times10^{16}$

23.2. $M=12$ amu (graphite). **23.3.** 92%.

23.4. (1) about 100%, (2) 1.9%, i. e., the neutrons are retarded in the layer of lead much slower than in the material containing hydrogen (paraffin, for example).

23.5. The direction of the velocity v of the neutron colliding with an immobile proton is the bisectrix of the right angle at which the particles diverge. The velocities of these particles are the same and equal to $v'=\frac{v\sqrt{2}}{2}$. Hence, the energy will be equally distributed between the neutron and the proton.

23.6. Upon each collision the kinetic energy of the neutron is halved (see the previous problem). Therefore, after n collisions the energy of the neutron will be $E=\left(\frac{1}{2}\right)^n E_0$. Hence, $n\log_{10}2=\log_{10}\frac{E_0}{E}=\log_{10}(2\times10^7)$ and $n=\frac{\log_{10}(2\times10^7)}{\log_{10}2}=24$ collisions.

23.7. $q=2e=3.2\times10^{-19}$ C. **23.8.** $\frac{q}{m}=4.8\times10^{7}$ C/kg.

23.9. (1) $m=1.23\times10^{-30}$ kg, $v=2.02\times10^{8}$ m/s, $E=1.8\times10^{5}$ eV, $\frac{e}{m}=$ $=1.3\times10^{11}$ C/kg, (2) $v=2.52\times10^{8}$ m/s.

23.10. According to the initial condition, $\frac{E}{E_0}=\frac{1}{\sqrt{1-\beta^2}}=30$, whence $v=$ $=2.998\times10^{8}$ m/s. The lifetime of a moving meson as registered by a laboratory clock is $\tau'=\frac{\tau_0}{\sqrt{1-\beta^2}}=30\,\tau_0$. In this time the meson will cover the distance $l=v\tau'=v\,30\tau_0\cong18{,}000\text{ m}=18\text{ km}$.

23.11. Eight times.

23.12. (1) The energy of each photon $E_0=0.51$ MeV, (2) $\lambda=0.024$ Å.

23.13. If a γ-quantum with the energy $h\nu$ is converted into a couple of particles, then according to the law of conservation of energy

$$h\nu=2m_0c^2+E_1+E_2$$

where m_0c^2 is the rest energy of each particle, and E_1 and E_2 the kinetic energies of the particles at the moment of formation. In our case $m_0c^2=0.51$ MeV, and therefore $2m_0c^2=1.02$ MeV. Hence $E_1+E_2=(2.62-1.02)\text{ MeV}=1.60$ MeV.

23.14. The Lorentz force $Bqv=\frac{mv^2}{R}$, whence $B=\frac{mv}{qR}$. According to the formulas of the theory of relativity, the impulse of a particle $\bar{p}=mv$ is related to its kinetic energy E_k by the expression $\bar{p}=\frac{1}{c}\sqrt{E_k(E_k+2m_0c^2)}$, where m_0 is the rest mass of the particle. Hence

$$B=\frac{1}{cqR}\sqrt{E_k\,(E_k+2m_0c^2)} \qquad (1)$$

It can easily be shown (see the solution to the previous problem) that the kinetic energy of each particle $E_k=2.34$ MeV. Upon inserting the numerical data into Eq. (1), we obtain $B=0.31$ T.

23.15. $E=67.5$ MeV. **23.16.** 940 MeV.

23.17. (1) $m=273\,m_0$, where m_0 is the rest mass of an electron, (2) $v=$ $=2.48\times10^{8}$ m/s.

23.18. (1) $\nu=\frac{Bq}{2\pi m}$, (2) (a) $\nu=9.7\times10^{6}\text{ Hz}=9.7$ MHz, (b) $v=19.4$ MHz, (c) $\nu=9.7$ MHz.

23.19. (1) $E=2\pi^2 m\nu^2R^2$, (2) (a) $E=13.8$ MeV, (b) $E=6.9$ MeV, (c) $E=$ $=27.6$ MeV.

23.20. (1) $B=0.9$ T, (2) $E=4.8$ MeV.

23.21. (1) $B=1.8$ T, $E=9.6$ MeV, (2) $B=1.8$ T, $E=19.2$ MeV.

23.22. One gramme of radium emits 3.7×10^{10} α-particles a second. A current of 15 μA corresponds to a flux of 4.7×10^{13} α-particles per second. Thus the given cyclotron is 1,000 times more efficient than one gramme of radium.

23.23. $U=\frac{R^2B^2q}{2m}=1.2\times10^{7}$ V. **23.24.** $R=0.36$ m.

23.25. (1) $B=1.3$ Wb/m² for deuterons and α-particles, and $B=0.65$ Wb/m² for protons.

(2) $v=3.13\times10^{7}$ m/s for deuterons, protons and α-particles. The energy of particles flying out from the cyclotron will be different: $E=10.2$ MeV for deuterons, $E=5.1$ MeV for protons and $E=20.4$ MeV for α-particles.

(3) Upon each complete revolution a charged particle passes through the space between the dees twice and therefore receives an additional impulse two times. For this reason a charged particle after N revolutions will acquire an energy equivalent to the accelerating potential $U' = 2NU$, where U is the potential difference applied between the dees. Hence, $N = \frac{U'}{2U}$. For deuterons and α-particles $N = 68$, and for protons $N = 34$.

23.26. $E = 188$ MeV.

23.27. (1) $\frac{M}{M_0} = 1.1$, (2) $\beta = \frac{v}{c} = 0.44$ and $v = 1.32 \times 10^8$ m/s.

23.28. (1) $B = \frac{2\pi m_0 \nu_0}{q} = \frac{2\pi m \nu}{q} = 1.62$ T. (2) Since

$$\frac{\nu_0}{\nu} = \frac{m}{m_0} = \frac{1}{\sqrt{1-\beta^2}}$$

then

$$E_k = m_0 c^2 \left(\frac{1}{\sqrt{1-\beta^2}} - 1 \right) = \frac{m_0 c^2 (\nu_0 - \nu)}{\nu} = 300 \text{ MeV}$$

23.29. (1) $\frac{T}{T_0} = 1.7$, (2) $\frac{T}{T_0} = 1.9$.

APPENDIX

INDUCTION *B* VERSUS INTENSITY *H* OF A MAGNETIC FIELD FOR A CERTAIN GRADE OF IRON

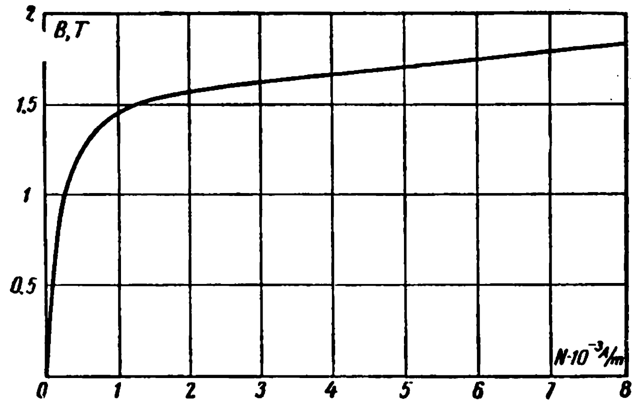


Fig. 113

RELATIONSHIP BETWEEN RATIONALIZED AND NON-RATIONALIZED EQUATIONS OF AN ELECTROMAGNETIC FIELD

Rationalized equations of an electromagnetic field can be obtained from the non-rationalized ones by means of the following transformations:

1. The relative permittivity (dielectric constant) ε_r in the non-rationalized equations is replaced by the quantity $4\pi\varepsilon=4\pi\varepsilon_0\varepsilon_r$, where ε_0 is the absolute permittivity in a vacuum and ε_r the relative permittivity of the medium with respect to a vacuum, i. e., the usual tabulated value of ε_r.

2. The relative permeability μ_r in the non-rationalized equations is replaced by

$$\frac{\mu}{4\pi}=\frac{\mu_0\mu_r}{4\pi}$$

where μ_0 is the permeability in a vacuum and μ_r the relative permeability of the medium with respect to a vacuum, i. e., the usual tabulated value of μ_r.

3. The electric displacement (induction) $D = \varepsilon E$ in the non-rationalized equations is replaced by the quantity

$$4\pi D = 4\pi\varepsilon_0\varepsilon_r E$$

4. The intensity of a magnetic field $H = \frac{B}{\mu_r}$ in the non-rationalized equations is replaced by the quantity

$$4\pi H = 4\pi\frac{B}{\mu_0\mu_r}$$

All the equations in which the quantities ε_r, μ_r, D and H are absent have the same form both in the non-rationalized and in the rationalized systems.

The most important equations of Sections 9 and 11 of Chapter 3 in the non-rationalized and rationalized form are compared in the following table.

	Non-rationalized form (Gaussian system)	Rationalized form (SI)
Coulomb law	$F=\frac{Q_1Q_2}{\varepsilon_r r^2}$	$F=\frac{Q_1Q_2}{4\pi\varepsilon_0\varepsilon_r r^2}$
Intensity of an electric field	$E=\frac{F}{Q}$	$E=\frac{F}{Q}$
Intensity of the field of a point charge	$E=\frac{Q}{\varepsilon_r r^2}$	$E=\frac{Q}{4\pi\varepsilon_0\varepsilon_r r^2}$
Gauss theorem	$N_E=\frac{4\pi\sum Q}{\varepsilon_r}$ $N_D=4\pi\sum Q$	$N_E=\frac{\sum Q}{\varepsilon_0\varepsilon_r}$ $N_D=\sum Q$
Intensity of the field produced by a charged filament	$E=\frac{2\tau}{\varepsilon_r r}$	$E=\frac{\tau}{2\pi\varepsilon_0\varepsilon_r r}$
Intensity of the field produced by a charged plane	$E=\frac{2\pi\sigma}{\varepsilon_r}$	$E=\frac{\sigma}{2\varepsilon_0\varepsilon_r}$
Field of a plane capacitor	$E=\frac{4\pi\sigma}{\varepsilon_r}$	$E=\frac{\sigma}{\varepsilon_0\varepsilon_r}$
Difference of potentials	$U=\frac{W}{Q}$	$U=\frac{W}{Q}$
Potential of the field of a point charge	$U=\frac{Q}{\varepsilon_r r}$	$U=\frac{Q}{4\pi\varepsilon_r\varepsilon_r r}$
Dependence between field intensity and the potential	$E=-\frac{dU}{dr}$	$E=-\frac{dU}{dr}$
Ditto for a homogeneous field	$E=-\frac{U}{d}$	$E=-\frac{U}{d}$
Relation between the capacitance, charge and potential of a conductor	$Q=CU$	$Q=CU$
Capacitance of a plane capacitor	$C=\frac{\varepsilon_r A}{4\pi d}$	$C=\frac{\varepsilon_0\varepsilon_r A}{d}$
Capacitance of a spherical capacitor	$C=\frac{\varepsilon_r rR}{R-r}$	$C=\frac{4\pi\varepsilon_0\varepsilon_r rR}{R-r}$
Capacitance of a sphere	$C=\varepsilon_r r$	$C=4\pi\varepsilon_0\varepsilon_r r$
Energy of a charged conductor	$W_e=\frac{1}{2}QU=$ $=\frac{1}{2}CU^2=$ $=\frac{1}{2}\frac{Q^2}{C}$	$W_e=\frac{1}{2}QU=$ $=\frac{1}{2}CU^2=$ $=\frac{1}{2}\frac{Q^2}{C}$

(Continued)

	Non-rationalized form (Gaussian system)	Rationalized form (SI)
Energy of the field of a plane capacitor	$W_e=\frac{\varepsilon_r AU^2}{8\pi d}=$ $=\frac{\varepsilon_r E^2 Ad}{8\pi}=$ $=\frac{2\pi\sigma^2}{\varepsilon_r}Ad$	$W=\frac{\varepsilon_0\varepsilon_r AU^2}{2d}=$ $=\frac{\varepsilon_0\varepsilon_r E^2 Ad}{2}=$ $=\frac{\sigma^2 Ad}{2\varepsilon_0\varepsilon_r}$
Volume energy density of an electric field	$W_0=\frac{\varepsilon_r E^2}{8\pi}$	$W_0=\frac{\varepsilon_0\varepsilon_r E^2}{2}$
Force of attraction of plates in a plane capacitor	$F=\frac{\varepsilon_r E^2}{8\pi}A=$ $=\frac{\varepsilon_r AU^2}{8\pi d^2}=$ $=\frac{2\pi\sigma^2 A}{\varepsilon_r}$	$F=\frac{\varepsilon_0\varepsilon_r E^2 A}{2}=$ $=\frac{\varepsilon_0\varepsilon_r AU^2}{2d^2}=$ $=\frac{\sigma^2 A}{2\varepsilon_0\varepsilon_r}$
Ampere's law	$dH=\frac{1}{c}\frac{I\sin\alpha dl}{r^2}$	$dH=\frac{I\sin\alpha dl}{4\pi r^2}$
Intensity of a magnetic field in the centre of a circular current	$H=\frac{1}{c}\frac{2\pi I}{R}$	$H=\frac{I}{2R}$
Intensity of the magnetic field of a direct current	$H=\frac{1}{c}\frac{2I}{a}$	$H=\frac{I}{2\pi a}$
Intensity of the magnetic field inside a solenoid	$H=\frac{1}{c}4\pi In$	$H=In$
Relationship between the intensity of a magnetic field and magnetic induction	$B=\mu_r H$	$B=\frac{\mu_0\mu_r}{4\pi}4\pi H=$ $=\mu_0\mu_r H$
Density of the energy of a magnetic field	$W_0=\frac{HB}{8\pi}$	$W_0=\frac{HB}{2}$
Ampere force	$dF=\frac{1}{c}BI\sin\alpha dl$	$dF=BI\sin\alpha dl$
Lorentz force	$F=\frac{1}{c}BQv\sin\alpha$	$F=BQv\sin\alpha$
Force of interaction of parallel currents	$F=\frac{1}{c^2}\frac{2\mu_r I_1 I_2 l}{d}$	$F=\frac{\mu_0\mu_r I_1 I_2 l}{2\pi d}$
Inductance of a solenoid	$L=4\pi\mu_r n^2 lA$	$L=\mu_0\mu_r n^2 lA$

By making the transformations indicated above, supplement this table with the formulas from Sections 9 and 11 of Chapter 3 which have not been included in it. It can easily be seen that all the equations in Sec. 10 of Chapter 3 have the same form in the rationalized and non-rationalized systems.

TABLE I. BASIC PHYSICAL QUANTITIES

Quantity	Numerical value
Gravity constant γ	6.67×10^{-11} m³/kg·s²
Number of molecules in 1 kmole (Avogadro's number) N_A	6.025×10^{26} kmole⁻¹
Volume of 1 kmole of an ideal gas under standard conditions V_0	22.4 m³
Universal gas constant R	8.31×10^{3} J/kmole·deg
Boltzmann's constant k	1.38×10^{-23} J/deg
Faraday's number F	9.65×10^{7} C/kg-eq
Stefan-Boltzmann's constant σ	5.67×10^{-8} W/m²·deg⁴
Planck's constant h	6.625×10^{-34} J·s
Electron charge e	1.602×10^{-19} C
Rest mass of an electron m_e	9.11×10^{-31} kg$=5.49\times10^{-4}$ amu
Rest mass of a proton m_p	1.672×10^{-27} kg$=1.00759$ amu
Rest mass of a neutron m_n	1.675×10^{-27} kg$=1.00899$ amu
Velocity of light propagation in a vacuum	3.00×10^{8} m/s

TABLE II ASTRONOMIC QUANTITIES

Mean radius of the Earth	6.37×10^{6} m
Mean density of the Earth	5,500 kg/m³
Mass of the Earth	5.96×10^{24} kg
Radius of the Sun	6.95×10^{8} m
Mass of the Sun	1.97×10^{30} kg
Radius of the Moon	1.74×10^{6} m
Mass of the Moon	7.3×10^{22} kg
Mean distance between the centres of the Moon and the Earth	3.84×10^{8} m
Mean distance between the centres of the Earth and the Sun	1.5×10^{11} m
Period of revolution of the Moon about the Earth	27 days 7h 43 min
Mean density of the Sun	1,400 kg/m³

TABLE III. DATA ON THE PLANETS OF THE SOLAR SYSTEM

	Mercury	Venus	Earth	Mars	Jupiter	Saturn	Uranus	Neptune	Pluto
Mean distance from the Sun, million km	57.9	108.0	149.5	227.8	777.8	1,426.1	2,869.1	4,495.6	5,229
Period of revolution around the Sun, terrestrial year	0.24	0.62	1.0	1.88	11.86	29.46	84.02	164.8	249.7
Equatorial diameter, km	4,840	12,400	12,742	6,780	139,760	115,100	51,000	50,000	—
Volume with respect to that of Earth	0.055	0.92	1.0	0.150	1,345	767	73.5	59.5	—
Mass with respect to that of Earth	0.054	0.81	1.0	0.107	318.4	95.2	14.58	17.26	—
Gravity acceleration with respect to that on Earth's surface ($g = 980.7$ m/s^2)	0.38	0.85	1.0	0.38	2.64	1.17	0.92	1.14	—

TABLE IV. DIAMETERS OF ATOMS AND MOLECULES

Helium (He)	2×10^{-10} m
Hydrogen (H_2)	2.3×10^{-10} m
Oxygen (O_2)	3×10^{-10} m
Nitrogen (N_2)	3×10^{-10} m

TABLE V. CRITICAL VALUES OF T_{cr} AND p_{cr}

Substance	T_{cr}, °K	p_{cr}, atm	$p_{cr}\cdot10^{-6}$, N/m²
Steam (water vapour)	647	217	22.0
Carbon dioxide	304	73	7.4
Oxygen	154	50	5.07
Argon	151	48	4.87
Nitrogen	126	33.6	3.4
Hydrogen	33	12.8	1.3
Helium	5.2	2.25	0.23

TABLE VI PRESSURE OF WATER VAPOUR SATURATING A SPACE AT VARIOUS TEMPERATURES

t, °C	p_s, mm Hg	t, °C	p_s, mm Hg	t, °C	p_s, mm Hg
−5	3.01	8	8.05	30	31.8
0	4.58	9	8.61	40	55.3
1	4.93	10	9.21	50	92.5
2	5.29	12	10.5	60	149
3	5.69	14	12.0	70	234
4	6.10	16	13.6	80	355
5	6.54	18	15.5	90	526
6	7.01	20	17.5	100	760
7	7.71	25	23.8	150	4.8 atm
				200	15.3 atm

TABLE VII SPECIFIC HEAT OF VAPORIZATION OF WATER AT VARIOUS TEMPERATURES

t, °C	0	50	100	200
r, cal/g	595	568	539	464
$r\cdot10^{-5}$, J/kg	24.9	23.8	22.6	19.4

TABLE VIII. **PROPERTIES OF SOME LIQUIDS**

Liquid	Density, kg/m³	Specific heat at 20° C		Coefficient of surface tension at 20° C, N/m
		J/kg·deg	cal/g·deg	
Alcohol	790	2,510	0.6	0.02
Benzene	880	1,720	0.41	0.03
Castor oil	900	1,800	0.43	0.035
Glycerine	1,200	2,430	0.58	0.064
Kerosene	800	2,140	0.051	0.03
Mercury	13,600	138	0.033	0.5
Water	1,000	4,190	1.0	0.073

TABLE IX. **PROPERTIES OF SOME SOLIDS**

Material	Density, kg/m³	Melting point, °C	Specific heat		Specific heat of fusion, J/kg	Coefficient of linear thermal expansion, deg⁻¹
			J/kg·deg	kcal/kg·deg		
Aluminium	2,600	659	896	0.214	3.22×10^5	2.3×10^{-5}
Brass	8,400	900	386	0.092	—	1.9×10^{-5}
Copper	8,600	1,100	395	0.094	1.76×10^5	1.6×10^{-5}
Cork	200	—	2,050	0.49	—	—
Ice	900	0	2,100	0.5	3.35×10^5	—
Iron	7,900	1,530	500	0.119	2.72×10^5	1.2×10^{-5}
Lead	11,300	327	126	0.030	2.26×10^4	2.9×10^{-5}
Platinum	21,400	1,770	117	0.028	1.13×10^5	0.89×10^{-5}
Silver	10,500	960	234	0.056	8.8×10^4	1.9×10^{-5}
Steel	7,700	1,300	460	0.11	—	1.06×10^{-5}
Tin	7,200	232	230	0.055	5.86×10^4	2.7×10^{-5}
Zinc	7,000	420	391	0.093	1.17×10^5	2.9×10^{-5}

TABLE X **ELASTIC PROPERTIES OF SOME SOLIDS**

Material	Ultimate strength	Young's modulus
	N/m²	N/m²
Aluminium	1.1×10^8	6.9×10^{10}
Copper	2.45×10^8	11.8×10^{10}
Iron	2.94×10^8	19.6×10^{10}
Lead	0.2×10^8	1.57×10^{10}
Silver	2.9×10^8	7.4×10^{10}
Steel	7.85×10^8	21.6×10^{10}

TABLE XI. **THERMAL CONDUCTIVITY OF SOME SOLIDS**
(λ, W/m·deg)

Aluminium	210
Copper	390
Cork	0.050
Dry sand	0.325
Ebonite	0.174
Felt	0.046
Fused quartz	1.37
Iron	58.7
Silver	460

TABLE XII. DIELECTRIC CONSTANT (RELATIVE PERMITTIVITY) OF DIELECTRICS

Ebonite	2.6
Glass	6
Kerosene	2
Mica	6
Oil	5
Paraffine	2
Paraffined paper	2
Porcelain	6
Water	81
Wax	7.8

TABLE XIII. RESISTIVITY OF CONDUCTORS ($\Omega\cdot$m at 0° C)

Aluminium	2.53×10^{-8}
Copper	1.7×10^{-8}
Graphite	3.9×10^{-7}
Iron	8.7×10^{-8}
Lead	2.2×10^{-7}
Mercury	9.4×10^{-7}
Nichrome	1.0×10^{-6}
Steel	1.0×10^{-7}

TABLE XIV MOBILITY OF IONS IN ELECTROLYTES (m^2/V·s)

NO_3^-	6.4×10^{-8}
H^+	3.26×10^{-7}
K^+	6.7×10^{-8}
Cl^-	6.8×10^{-8}
Ag^+	5.6×10^{-8}

TABLE XV. WORK OF EXIT OF ELECTRONS FROM METALS (eV)

W	4.5
W+Cs	1.6
W+Th	2.63
Pt+Cs	1.40
Pt	5.3
Ag	4.74
Li	2.4
Na	2.3
K	2.0
Cs	1.9

TABLE XVI. REFRACTIVE INDICES

Carbon bisulphide	1.63
Diamond	2.42
Glass	1.5-1.9
Ice	1.31
Turpentine	1.48
Water	1.33

TABLE XVII. BOUNDARY OF *K*-SERIES OF X-RAYS FOR VARIOUS MATERIALS OF THE ANTICATHODE (Å)

Copper	1.38
Gold	0.153
Platinum	0.158
Silver	0.484
Tungsten	0.178

TABLE XVIII. SPECTRAL LINES OF MERCURY ARC (Å)

2537	4047	5461	6128
3650	4358	5770	6908
3655	5235	5791	7082

TABLE XIX. **MASSES OF SOME ISOTOPES**
(amu)

Isotope	Mass	Isotope	Mass	Isotope	Mass
$_1H^1$	1.00814	$_4Be^9$	9.01505	$_{14}Si^{30}$	29.98325
$_1H^2$	2.01474	$_5B^{10}$	10.01612	$_{20}Ca^{40}$	39.97542
$_1H^3$	3.01700	$_6C^{12}$	12.00380	$_{27}Co^{56}$	55.95769
$_2He^3$	3.01699	$_7N^{13}$	13.00987	$_{29}Cu^{63}$	62.94962
$_2He^4$	4.00388	$_7N^{14}$	14.00752	$_{48}Cd^{113}$	112.94206
$_3Li^6$	6.01703	$_8O^{17}$	17.00453	$_{80}Hg^{200}$	200.02800
$_3Li^7$	7.01823	$_{12}Mg^{23}$	23.00145	$_{92}U^{235}$	235.11750
$_4Be^7$	7.01916	$_{12}Mg^{24}$	23.99267	$_{92}U^{238}$	238.12376
$_4Be^8$	8.00785	$_{13}Al^{27}$	26.99010		

TABLE XX. **HALF-LIVES OF SOME RADIOACTIVE ELEMENTS**

$_{20}Ca^{45}$	164 days
$_{38}Sr^{90}$	28 years
$_{84}Po^{210}$	138 days
$_{86}Rn^{222}$	3.82 days
$_{88}Ra^{226}$	1,590 years
$_{92}U^{235}$	$7.1 \cdot 10^8$ years
$_{92}U^{238}$	$4.5 \cdot 10^9$ years

TABLE XXI COMMON LOGARITHMS

	0	1	2	3	4	5	6	7	8	9	1	2	3	4	5	6	7	8	9
10	0000	0043	0086	0128	0170	0212	0253	0294	0334	0374	4	8	12	17	21	25	29	33	37
11	0414	0453	0492	0531	0569	0607	0645	0682	0719	0755	4	8	11	15	19	23	26	30	34
12	0792	0828	0864	0899	0934	0969	1004	1038	1072	1106	3	7	10	14	17	21	24	28	31
13	1139	1173	1206	1239	1271	1303	1335	1367	1399	1430	3	6	10	13	16	19	23	26	29
14	1461	1492	1523	1553	1584	1614	1644	1673	1703	1732	3	6	9	12	15	18	21	24	27
15	1761	1790	1818	1847	1875	1903	1931	1959	1987	2014	3	6	8	11	14	17	20	22	25
16	2041	2068	2095	2122	2148	2175	2201	2227	2253	2279	3	5	8	11	13	16	18	21	24
17	2304	2330	2355	2380	2405	2430	2455	2480	2504	2529	2	5	7	10	12	15	17	20	22
18	2553	2577	2601	2625	2648	2672	2695	2718	2742	2765	2	5	7	9	12	14	16	19	21
19	2788	2810	2833	2856	2878	2900	2923	2945	2967	2989	2	4	7	9	11	13	16	18	20
20	3010	3032	3054	3075	3096	3118	3139	3160	3181	3201	2	4	6	8	11	13	15	17	19
21	3222	3243	3263	3284	3304	3324	3345	3365	3385	3404	2	4	6	8	10	12	14	16	18
22	3424	3444	3464	3483	3502	3522	3541	3560	3579	3598	2	4	6	8	10	12	14	15	17
23	3617	3636	3655	3674	3692	3711	3729	3747	3766	3784	2	4	6	7	9	11	13	15	17
24	3802	3820	3838	3856	3874	3892	3909	3927	3945	3962	2	4	5	7	9	11	12	14	16
25	3979	3997	4014	4031	4048	4065	4082	4099	4116	4133	2	3	5	7	9	10	12	14	15
26	4150	4166	4183	4200	4216	4232	4249	4265	4281	4298	2	3	5	7	8	10	11	13	15
27	4314	4330	4346	4362	4378	4393	4409	4425	4440	4456	2	3	5	6	8	9	11	13	14
28	4472	4487	4502	4518	4533	4548	4564	4579	4594	4609	2	3	5	6	8	9	11	12	14
29	4624	4639	4654	4669	4683	4698	4713	4728	4742	4757	1	3	4	6	7	9	10	12	13
30	4771	4786	4800	4814	4829	4843	4857	4871	4886	4900	1	3	4	6	7	9	10	11	13
31	4914	4928	4942	4955	4969	4983	4997	5011	5024	5038	1	3	4	6	7	8	10	11	12
32	5051	5065	5079	5092	5105	5119	5132	5145	5159	5172	1	3	4	5	7	8	9	11	12
33	5185	5198	5211	5224	5237	5250	5263	5276	5289	5302	1	3	4	5	6	8	9	10	12
34	5315	5328	5340	5353	5366	5378	5391	5403	5416	5428	1	2	4	5	6	8	9	10	11

Table XXI (continued)

	0	1	2	3	4	5	6	7	8	9	1	2	3	4	5	6	7	8	9
35	5441	5453	5465	5478	5490	5502	5514	5527	5539	5551	1	2	4	5	6	7	9	10	11
36	5563	5575	5587	5599	5611	5623	5635	5647	5658	5670	1	2	4	5	6	7	8	10	11
37	5682	5694	5705	5717	5729	5740	5752	5763	5775	5786	1	2	3	5	6	7	8	9	10
38	5798	5809	5821	5832	5843	5855	5866	5877	5888	5899	1	2	3	5	6	7	8	9	10
39	5911	5922	5933	5944	5955	5966	5977	5988	5999	6010	1	2	3	4	5	7	8	9	10
40	6021	6031	6042	6053	6064	6075	6085	6096	6107	6117	1	2	3	4	5	6	8	9	10
41	6128	6138	6149	6160	6170	6180	6191	6201	6212	6222	1	2	3	4	5	6	7	8	9
42	6232	6243	6253	6263	6274	6284	6294	6304	6314	6325	1	2	3	4	5	6	7	8	9
43	6335	6345	6355	6365	6375	6385	6395	6405	6415	6425	1	5	3	4	5	6	7	8	9
44	6435	6444	6454	6464	6474	6484	6493	6503	6513	6522	1	2	3	4	5	6	7	8	9
45	6532	6542	6551	6561	6571	6580	6590	6599	6609	6618	1	2	3	4	5	6	7	8	9
46	6628	6637	6646	6656	6665	6675	6684	6693	6702	6712	1	2	3	4	5	6	7	7	8
47	6721	6730	6739	6749	6758	6767	6776	6785	6794	6803	1	2	3	4	5	5	6	7	8
48	6812	6821	6830	6839	6848	6857	6866	6875	6884	6893	1	2	3	4	4	5	6	7	8
49	6902	6911	6920	6928	6937	6947	6955	6964	6972	6981	1	2	3	4	4	5	6	7	8
50	6990	6998	7007	7016	7024	7033	7042	7050	7059	7067	1	2	3	3	4	5	6	7	8
51	7076	7084	7093	7101	7110	7118	7126	7135	7143	7152	1	2	2	3	4	5	6	7	8
52	7160	7168	7177	7185	7193	7202	7210	7218	7226	7235	1	2	2	3	4	5	6	7	7
53	7243	7251	7259	7267	7275	7284	7292	7300	7308	7316	1	2	2	3	4	5	6	6	7
54	7324	7332	7340	7348	7356	7364	7372	7380	7388	7396	1	2	2	3	4	5	6	6	7
55	7404	7412	7419	7427	7435	7443	7451	7459	7467	7474	1	2	2	3	4	5	5	6	7
56	7482	7490	7497	7505	7513	7520	7528	7536	7543	7551	1	2	2	3	4	5	5	6	7
57	7559	7566	7574	7582	7589	7597	7604	7612	7619	7627	1	2	2	3	4	5	5	6	7
58	7634	7642	7649	7657	7664	7672	7679	7686	7694	7701	1	1	2	3	4	4	5	6	7
59	7709	7716	7723	7731	7738	7745	7752	7760	7767	7774	1	1	2	3	4	4	5	6	7

60	7782	7789	7796	7803	7810	7818	7825	7832	7839	7846	1	1	2	3	4	4	5	6	6
61	7853	7860	7868	7875	7882	7889	7896	7903	7910	7917	1	1	2	3	4	4	5	6	6
62	7924	7931	7938	7945	7952	7959	7966	7973	7980	7987	1	1	2	3	3	4	5	6	6
63	7993	8000	8007	8014	8021	8028	8035	8041	8048	8055	1	1	2	3	3	4	5	5	6
64	8062	8069	8075	8082	8089	8096	8102	8109	8116	8122	1	1	2	3	3	4	5	5	6
65	8129	8136	8142	8149	8156	8162	8169	8176	8182	8189	1	1	2	3	3	4	5	5	6
66	8195	8202	8209	8215	8222	8228	8235	8241	8248	8254	1	1	2	3	3	4	5	5	6
67	8261	8267	8274	8280	8287	8293	8299	8306	8212	8319	1	1	2	3	3	4	5	5	6
68	8325	8331	8338	8344	8351	8357	8363	8370	8376	8382	1	1	2	3	3	4	5	5	6
69	8388	8395	8401	8407	8414	8420	8426	8432	8439	8445	1	1	2	2	3	4	4	5	6
70	8451	8457	8463	8470	8476	8482	8488	8494	8500	8506	1	1	2	2	3	4	4	5	6
71	8513	8519	8525	8531	8537	8543	8549	8555	8561	8567	1	1	2	2	3	4	4	5	5
72	8573	8579	8585	8591	8597	8603	8609	8615	8621	8627	1	1	2	2	3	4	4	5	5
73	8633	8639	8645	8651	8657	8663	8669	8675	8681	8686	1	1	2	2	3	4	4	5	5
74	8692	8698	8704	8710	8716	8722	8727	8733	8739	8745	1	1	2	2	3	4	4	5	5
75	8751	8756	8762	8768	8774	8779	8785	8791	8797	8802	1	1	2	2	3	3	4	5	5
76	8808	8814	8820	8825	8831	8837	8842	8848	8854	8859	1	1	2	2	3	3	4	5	5
77	8865	8871	8876	8882	8887	8893	8899	8904	8910	8915	1	1	2	2	3	3	4	4	5
78	8921	8927	8932	8938	8943	8949	8954	8960	8965	8971	1	1	2	2	3	3	4	4	5
79	8976	8982	8987	8993	8998	9004	9009	9015	9020	9025	1	1	2	2	3	3	4	4	5
80	9031	9036	9042	9047	9053	9058	9063	9069	9074	9079	1	1	2	2	3	3	4	4	5
81	9085	9090	9096	9101	9106	9112	9117	9122	9128	9133	1	1	2	2	3	3	4	4	5
82	9138	9143	9149	9154	9159	9165	9170	9175	9180	9186	1	1	2	2	3	3	4	4	5
83	9191	9196	9201	9206	9212	9217	9222	9227	9232	9238	1	1	2	2	3	3	4	4	5
84	9243	9248	9253	9258	9263	9269	9274	9279	9284	9289	1	1	2	2	3	3	4	4	5

	0	1	2	3	4	5	6	7	8	9	1	2	3	4	5	6	7	8	9
85	9294	9299	9304	9309	9315	9320	9325	9330	9335	9340	1	1	2	2	3	3	4	4	5
86	9345	9350	9355	9360	9365	9370	9375	9380	9385	9390	1	1	2	2	3	3	4	4	5
87	9395	9400	9405	9410	9415	9420	9425	9430	9435	9440	0	1	1	2	2	3	3	4	4
88	9445	9450	9455	9460	9465	9469	9474	9479	9484	9489	0	1	1	2	2	3	3	4	4
89	9494	9499	9504	9509	9513	9518	9523	9528	9533	9538	0	1	1	2	2	3	3	4	4
90	9542	9547	9552	9557	9562	9566	9571	9576	9581	9586	0	1	1	2	2	3	3	4	4
91	9590	9595	9600	9605	9609	9614	9619	9624	9628	9638	0	1	1	2	2	3	3	4	4
92	9638	9643	9647	9652	9657	9661	9666	9671	9675	9680	0	1	1	2	2	3	3	4	4
93	9685	9689	9694	9699	9703	9708	9713	9717	9722	9727	0	1	1	2	2	3	3	4	4
94	9731	9736	9741	9745	9750	9754	9759	9763	9768	9773	0	1	1	2	2	3	3	4	4
95	9777	9782	9786	9791	9795	9800	9805	9809	9814	9818	0	1	1	2	2	3	3	4	4
96	9823	9827	9832	9836	9841	9845	9850	9854	9859	9863	0	1	1	2	2	3	3	4	4
97	9868	9872	9877	9881	9886	9890	9894	9899	9903	9908	0	1	1	2	2	3	3	4	4
98	9912	9917	9921	9926	9930	9934	9939	9943	9948	9952	0	1	1	2	2	3	3	4	4
99	9956	9961	9965	9969	9974	9978	9983	9987	9991	9996	0	1	1	2	2	3	3	3	4

TABLE XXII. SINES

Degrees	0′	6′	12′	18′	24′	30′	36′	42′	48′	54′	60′	
0	0.0000	0.0017	0.0035	0.0052	0.0070	0.0087	0.0105	0.0122	0.0140	0.0157	0.0175	89
1	0.0175	0.0192	0.0209	0.0227	0.0244	0.0262	0.0279	0.0297	0.0314	0.0332	0.0349	88
2	0.0349	0.0366	0.0384	0.0401	0.0419	0.0436	0.0454	0.0471	0.0488	0.0506	0.0523	87
3	0.0523	0.0541	0.0558	0.0576	0.0593	0.0610	0.0628	0.0645	0.0663	0.0680	0.0698	86
4	0.0698	0.0715	0.0732	0.0750	0.0767	0.0785	0.0802	0.0819	0.0837	0.0854	0.0872	85
5	0.0872	0.0889	0.0906	0.0924	0.0941	0.0958	0.0976	0.0993	0.1011	0.1028	0.1045	84
6	0.1045	0.1063	0.1080	0.1097	0.1115	0.1132	0.1149	0.1167	0.1184	0.1201	0.1219	83
7	0.1219	0.1236	0.1253	0.1271	0.1288	0.1305	0.1323	0.1340	0.1357	0.1374	0.1392	82
8	0.1392	0.1409	0.1426	0.1444	0.1461	0.1478	0.1495	0.1513	0.1530	0.1547	0.1564	81
9	0.1564	0.1582	0.1599	0.1616	0.1633	0.1650	0.1668	0.1685	0.1702	0.1719	0.1736	80
10	0.1736	0.1754	0.1771	0.1788	0.1805	0.1822	0.1840	0.1857	0.1874	0.1891	0.1908	79
11	0.1908	0.1925	0.1942	0.1959	0.1977	0.1994	0.2011	0.2028	0.2045	0.2062	0.2079	78
12	0.2079	0.2096	0.2113	0.2130	0.2147	0.2164	0.2181	0.2198	0.2215	0.2233	0.2250	77
13	0.2250	0.2267	0.2284	0.2300	0.2317	0.2334	0.2351	0.2368	0.2385	0.2402	0.2419	76
14	0.2419	0.2436	0.2453	0.2470	0.2487	0.2504	0.2521	0.2538	0.2554	0.2571	0.2588	75
15	0.2588	0.2605	0.2622	0.2639	0.2656	0.2672	0.2689	0.2706	0.2723	0.2740	0.2756	74
16	0.2756	0.2773	0.2790	0.2807	0.2823	0.2840	0.2857	0.2874	0.2890	0.2907	0.2924	73
17	0.2924	0.2940	0.2957	0.2974	0.2990	0.3007	0.3024	0.3040	0.3057	0.3074	0.3090	72
18	0.3090	0.3107	0.3123	0.3140	0.3156	0.3173	0.3190	0.3206	0.3223	0.3239	0.3256	71
19	0.3256	0.3272	0.3289	0.3305	0.3322	0.3338	0.3355	0.3371	0.3387	0.3404	0.3420	70
	60′	54′	48′	42′	36′	30′	24′	18′	12′	6′	0′	Degrees

COSINES

SINES *Table XXII (continued)*

Degrees	0′	6′	12′	18′	24′	30′	36′	42′	48′	54′	60′	
20	0.3420	0.3437	0.3453	0.3469	0.3486	0.3502	0.3518	0.3535	0.3551	0.3567	0.3584	69
21	0.3584	0.3600	0.3616	0.3633	0.3649	0.3665	0.3681	0.3697	0.3714	0.3730	0.3746	68
22	0.3746	0.3762	0.3778	0.3795	0.3811	0.3827	0.3843	0.3859	0.3875	0.3891	0.3907	67
23	0.3907	0.3923	0.3939	0.3955	0.3971	0.3987	0.4003	0.4019	0.4035	0.4051	0.4067	66
24	0.4067	0.4083	0.4099	0.4115	0.4131	0.4147	0.4163	0.4179	0.4195	0.4210	0.4226	**65**
25	0.4226	0.4242	0.4258	0.4274	0.4289	0.4305	0.4321	0.4337	0.4352	0.4368	0.4384	64
26	0.4384	0.4399	0.4415	0.4431	0.4446	0.4462	0.4478	0.4493	0.4509	0.4524	0.4540	63
27	0.4540	0.4555	0.4571	0.4586	0.4602	0.4617	0.4633	0.4648	0.4664	0.4679	0.4695	62
28	0.4695	0.4710	0.4726	0.4741	0.4756	0.4772	0.4787	0.4802	0.4818	0.4833	0.4848	61
29	0.4848	0.4863	0.4879	0.4894	0.4909	0.4924	0.4939	0.4955	0.4970	0.4985	0.5000	**60**
30	0.5000	0.5015	0.5030	0.5045	0.5060	0.5075	0.5090	0.5105	0.5120	0.5135	0.5150	59
31	0.5150	0.5165	0.5180	0.5195	0.5210	0.5225	0.5240	0.5255	0.5270	0.5284	0.5299	58
32	0.5299	0.5314	0.5329	0.5344	0.5358	0.5373	0.5388	0.5402	0.5417	0.5432	0.5446	57
33	0.5446	0.5461	0.5476	0.5490	0.5505	0.5519	0.5534	0.5548	0.5563	0.5877	0.5592	56
34	0.5592	0.5606	0.5621	0.5635	0.5650	0.5664	0.5678	0.5693	0.5707	0.5721	0.5736	**55**
35	0.5736	0.5750	0.5764	0.5779	0.5793	0.5807	0.5821	0.5835	0.5850	0.5864	0.5878	54
36	0.5878	0.5892	0.5906	0.5920	0.5934	0.5948	0.5962	0.5976	0.5990	0.6004	0.6018	53
37	0.6018	0.6032	0.6046	0.6060	0.6074	0.6088	0.6101	0.6115	0.6129	0.6143	0.6157	52
38	0.6157	0.6170	0.6184	0.6198	0.6211	0.6225	0.6239	0.6252	0.6266	0.6280	0.6293	51
39	0.6293	0.6307	0.6320	0.6334	0.6347	0.6361	0.6374	0.6388	0.6401	0.6414	0.6428	**50**
40	0.6428	0.6441	0.6455	0.6468	0.6481	0.6494	0.6508	0.6521	0.6534	0.6547	0.6561	49
41	0.6561	0.6574	0.6587	0.6600	0.6613	0.6626	0.6639	0.6652	0.6665	0.6678	0.6691	48
42	0.6691	0.6704	0.6717	0.6730	0.6743	0.6756	0.6769	0.6782	0.6794	0.6807	0.6820	47
43	0.6820	0.6833	0.6845	0.6858	0.6871	0.6884	0.6896	0.6909	0.6921	0.6934	0.6947	46
44	0.6947	0.6959	0.6972	0.6984	0.6997	0.7009	0.7022	0.7034	0.7046	0.7059	0.7071	**45**

	60′	54′	48′	42′	36′	30′	24′	18′	12′	6′	0′	Degrees
45	0.7071	0.7083	0.7096	0.7108	0.7120	0.7133	0.7145	0.7157	0.7169	0.7181	0.7193	44
46	0.7193	0.7206	0.7218	0.7230	0.7242	0.7254	0.7266	0.7278	0.7290	0.7302	0.7314	43
47	0.7314	0.7325	0.7337	0.7349	0.7361	0.7373	0.7385	0.7396	0.7408	0.7420	0.7431	42
48	0.7431	0.7443	0.7455	0.7466	0.7478	0.7490	0.7501	0.7513	0.7524	0.7536	0.7547	41
49	0.7547	0.7559	0.7570	0.7581	0.7593	0.7604	0.7615	0.7627	0.7638	0.7649	0.7660	40
50	0.7660	0.7672	0.7683	0.7694	0.7705	0.7716	0.7727	0.7738	0.7749	0.7760	0.7771	39
51	0.7771	0.7782	0.7793	0.7804	0.7815	0.7826	0.7837	0.7848	0.7859	0.7869	0.7880	38
52	0.7880	0.7891	0.7902	0.7912	0.7923	0.7934	0.7944	0.7955	0.7965	0.7976	0.7986	37
53	0.7986	0.7997	0.8007	0.8018	0.8028	0.8039	0.8049	0.8059	0.8070	0.8080	0.8090	36
54	0.8090	0.8100	0.8111	0.8121	0.8131	0.8141	0.8151	0.8161	0.8171	0.8181	0.8192	35
55	0.8192	0.8202	0.8211	0.8221	0.8231	0.8241	0.8251	0.8261	0.8271	0.8281	0.8290	34
56	0.8290	0.8300	0.8310	0.8320	0.8329	0.8339	0.8348	0.8358	0.8368	0.8377	0.8387	33
57	0.8387	0.8396	0.8406	0.8415	0.8425	0.8434	0.8443	0.8453	0.8462	0.8471	0.8480	32
58	0.8480	0.8490	0.8499	0.8508	0.8517	0.8526	0.8536	0.8545	0.8554	0.8563	0.8572	31
59	0.8572	0.8581	0.8590	0.8599	0.8607	0.8616	0.8625	0.8634	0.8643	0.8652	0.8660	30
60	0.8660	0.8669	0.8678	0.8686	0.8695	0.8704	0.8712	0.8721	0.8729	0.8738	0.8746	29
61	0.8746	0.8755	0.8763	0.8771	0.8780	0.8788	0.8796	0.8805	0.8813	0.8821	0.8829	28
62	0.8829	0.8838	0.8846	0.8854	0.8862	0.8870	0.8878	0.8886	0.8894	0.8902	0.8910	27
63	0.8910	0.8918	0.8926	0.8934	0.8942	0.8949	0.8957	0.8965	0.8973	0.8980	0.8988	26
64	0.8988	0.8996	0.9003	0.9011	0.9018	0.9026	0.9033	0.9041	0.9048	0.9056	0.9063	25
65	0.9063	0.9070	0.9078	0.9085	0.9092	0.9100	0.9107	0.9114	0.9121	0.9128	0.9135	24
66	0.9135	0.9143	0.9150	0.9157	0.9164	0.9171	0.9178	0.9184	0.9191	0.9198	0.9205	23
67	0.9205	0.9212	0.9219	0.9225	0.9232	0.9239	0.9245	0.9252	0.9259	0.9265	0.9272	22
68	0.9272	0.9278	0.9285	0.9291	0.9298	0.9304	0.9311	0.9317	0.9323	0.9330	0.9336	21
69	0.9336	0.9342	0.9348	0.9354	0.9361	0.9367	0.9373	0.9379	0.9385	0.9391	0.9397	20

COSINES

SINES *Table XXII (concluded)*

Degrees	0′	6′	12′	18′	24′	30′	36′	42′	48′	54′	60′	
70	0.9397	0.9403	0.9409	0.9415	0.9421	0.9426	0.9433	0.9438	0.9444	0.9449	0.9455	19
71	0.9455	0.9461	0.9466	0.9472	0.9478	0.9483	0.9489	0.9494	0.9500	0.9505	0.9511	18
72	0.9511	0.9516	0.9521	0.9527	0.9532	0.9537	0.9542	0.9548	0.9553	0.9558	0.9563	17
73	0.9563	0.9568	0.9573	0.9578	0.9583	0.9588	0.9593	0.9598	0.9603	0.9608	0.9613	16
74	0.9613	0.9617	0.9622	0.9627	0.9632	0.9636	0.9641	0.9646	0.9650	0.9655	0.9659	15
75	0.9659	0.9664	0.9668	0.9673	0.9677	0.9681	0.9686	0.9690	0.9694	0.9699	0.9703	14
76	0.9703	0.9707	0.9711	0.9715	0.9720	0.9724	0.9728	0.9732	0.9736	0.9740	0.9744	13
77	0.9744	0.9748	0.9751	0.9755	0.9759	0.9763	0.9767	0.9770	0.9774	0.9778	0.9781	12
78	0.9781	0.9785	0.9789	0.9792	0.9796	0.9799	0.9803	0.9806	0.9810	0.9813	0.9816	11
79	0.9816	0.9820	0.9823	0.9826	0.9829	0.9833	0.9836	0.9839	0.9842	0.9845	0.9848	10
80	0.9848	0.9851	0.9854	0.9857	0.9860	0.9863	0.9866	0.9869	0.9871	0.9874	0.9877	9
81	0.9877	0.9880	0.9882	0.9885	0.9888	0.9890	0.9893	0.9895	0.9898	0.9900	0.9903	8
82	0.9903	0.9905	0.9907	0.9910	0.9912	0.9914	0.9917	0.9919	0.9921	0.9923	0.9925	7
83	0.9925	0.9928	0.9930	0.9932	0.9934	0.9936	0.9938	0.9940	0.9942	0.9943	0.9945	6
84	0.9945	0.9947	0.9949	0.9951	0.9952	0.9954	0.9956	0.9957	0.9959	0.9960	0.9962	5
85	0.9962	0.9963	0.9965	0.9966	0.9968	0.9969	0.9971	0.9972	0.9973	0.9974	0.9976	4
86	0.9976	0.9977	0.9978	0.9979	0.9980	0.9981	0.9982	0.9983	0.9984	0.9985	0.9986	3
87	0.9986	0.9987	0.9988	0.9989	0.9990	0.9990	0.9991	0.9992	0.9993	0.9993	0.9994	2
88	0.9994	0.9995	0.9995	0.9996	0.9996	0.9997	0.9997	0.9997	0.9998	0.9998	0.9998	1
89	0.9998	0.9999	0.9999	0.9999	0.9999	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	0
	60′	54′	48′	42′	36′	30′	24′	18′	12′	6′	0′	Degrees

COSINES

TABLE XXIII. TANGENTS

Degrees	0′	10′	20′	30′	40′	50′	60′	1′	2′	3′	4′	5′	
0	0.0000	0.0029	0.0058	0.0087	0.0116	0.0145	0.0175	3	6	9	12	14	89
1	0.0175	0.0204	0.0233	0.0262	0.0291	0.0320	0.0349	3	6	9	12	15	88
2	0.0349	0.0378	0.0407	0.0437	0.0466	0.0495	0.0524	3	6	9	12	15	87
3	0.0524	0.0553	0.0582	0.0612	0.0641	0.0670	0.0699	3	6	9	12	15	86
4	0.0699	0.0729	0.0758	0.0787	0.0816	0.0846	0.0875	3	6	9	12	15	85
5	0.0875	0.0904	0.0934	0.0963	0.0992	0.1022	0.1051	3	6	9	12	15	84
6	0.1051	0.1080	0.1110	0.1139	0.1169	0.1198	0.1228	3	6	9	12	15	83
7	0.1228	0.1257	0.1287	0.1317	0.1346	0.1376	0.1405	3	6	9	12	15	82
8	0.1405	0.1435	0.1465	0.1495	0.1524	0.1554	0.1584	3	6	9	12	15	81
9	0.1584	0.1614	0.1644	0.1673	0.1703	0.1733	0.1763	3	6	9	12	15	80
10	0.1763	0.1793	0.1823	0.1853	0.1883	0.1914	0.1944	3	6	9	12	15	79
11	0.1944	0.1974	0.2004	0.2035	0.2065	0.2095	0.2126	3	6	9	12	15	78
12	0.2126	0.2156	0.2186	0.2217	0.2247	0.2278	0.2309	3	6	9	12	15	77
13	0.2309	0.2339	0.2370	0.2401	0.2432	0.2462	0.2493	3	6	9	12	15	76
14	0.2493	0.2524	0.2555	0.2586	0.2617	0.2648	0.2679	3	6	9	12	16	75
15	0.2679	0.2711	0.2742	0.2773	0.2805	0.2836	0.2867	3	6	9	13	16	74
16	0.2867	0.2899	0.2931	0.2962	0.2994	0.3026	0.3057	3	6	9	13	16	73
17	0.3057	0.3089	0.3121	0.3153	0.3185	0.3217	0.3249	3	6	10	13	16	72
18	0.3249	0.3281	0.3314	0.3346	0.3378	0.3411	0.3443	3	6	10	13	16	71
19	0.3443	0.3476	0.3508	0.3541	0.3574	0.3607	0.3640	3	6	10	13	17	70
	60′	50′	40′	30′	20′	10′	0′						Degrees

COTANGENTS

Degrees	0′	10′	20′	30′	40′	50′	60′	1′	2′	3′	4′	5′	
20	0.3640	0.3673	0.3706	0.3739	0.3772	0.3805	0.3839	3	7	10	13	17	69
21	0.3839	0.3872	0.3906	0.3939	0.3973	0.4006	0.4040	3	7	10	13	17	68
22	0.4040	0.4074	0.4108	0.4142	0.4176	0.4210	0.4245	3	7	10	14	17	67
23	0.4245	0.4279	0.4314	0.4348	0.4383	0.4417	0.4452	3	7	10	14	17	66
24	0.4452	0.4487	0.4522	0.4557	0.4592	0.4628	0.4663	4	7	10	14	18	65
25	0.4663	0.4899	0.4734	0.4770	0.4806	0.4841	0.4877	4	7	11	14	18	64
26	0.4877	0.4913	0.4950	0.4986	0.5022	0.5059	0.5095	4	7	11	15	18	63
27	0.5095	0.5132	0.5169	0.5206	0.5243	0.5280	0.5317	4	7	11	15	18	62
28	0.5317	0.5354	0.5392	0.5430	0.5467	0.5505	0.5543	4	8	11	15	19	61
29	0.5543	0.5581	0.5619	0.5658	0.5696	0.5735	0.5774	4	8	12	15	19	60
30	0.5774	0.5812	0.5851	0.5890	0.5930	0.5969	0.6009	4	8	12	16	20	59
31	0.6009	0.6048	0.6088	0.6128	0.6168	0.6208	0.6249	4	8	12	16	20	58
32	0.6249	0.6289	0.6330	0.6371	0.6412	0.6453	0.6494	4	8	12	16	20	57
33	0.6494	0.6536	0.6577	0.6619	0.6661	0.6703	0.6745	4	8	13	17	21	56
34	0.6745	0.6787	0.6830	0.6873	0.6916	0.6959	0.7002	4	9	13	17	21	55
35	0.7002	0.7046	0.7089	0.7133	0.7177	0.7221	0.7265	4	9	13	18	22	54
36	0.7265	0.7310	0.7355	0.7400	0.7445	0.7490	0.7536	5	9	14	18	23	53
37	0.7536	0.7581	0.7627	0.7673	0.7720	0.7766	0.7813	5	9	14	18	23	52
38	0.7813	0.7860	0.7907	0.7954	0.8002	0.8050	0.8098	5	10	14	19	24	51
39	0.8098	0.8146	0.8195	0.8243	0.8292	0.8342	0.8391	5	10	15	20	24	50
40	0.8391	0.8441	0.8491	0.8541	0.8591	0.8642	0.8693	5	10	15	20	25	49
41	0.8693	0.8744	0.8796	0.8847	0.8899	0.8952	0.9004	5	10	16	21	26	48
42	0.9004	0.9057	0.9110	0.9163	0.9217	0.9271	0.9325	5	11	16	21	27	47
43	0.9325	0.9380	0.9435	0.9490	0.9545	0.9601	0.9657	6	11	17	22	28	46
44	0.9657	0.9713	0.9770	0.9827	0.9884	0.9942	1.0000	6	11	17	32	29	45

45	1.0000	1.0060	1.0117	0.0176	1.0237	1.0295	1.0355	6	12	18	25	30	44
46	1.0355	1.0417	1.0477	1.0528	1.0600	1.0661	1.0724	6	12	18	25	31	43
47	1.0724	1.0786	1.0850	1.0913	1.0976	1.1041	1.1106	6	13	19	25	32	42
48	1.1106	1.1171	1.1237	1.1303	1.1369	1.1436	1.1504	7	13	20	26	33	41
49	1.1504	1.1571	1.1640	1.1708	1.1778	1.1847	1.1918	7	14	21	27	34	**40**
50	1.1918	1.1989	1.2059	1.2131	1.2203	1.2275	1.2349	7	14	22	29	36	39
51	1.2349	1.2423	1.2497	1.2572	1.2647	1.2723	1.2799	8	15	23	30	38	38
52	1.2799	1.2877	1.2954	1.3032	1.3110	1.3191	1.3270	8	16	23	31	39	37
53	1.3270	1.3352	1.3432	1.3514	1.3597	1.3680	1.3764	8	16	25	33	41	36
54	1.3764	1.3848	1.3933	1.4019	1.4105	1.4193	1.4281	9	17	26	34	43	**35**
55	1.4281	1.4371	1.4460	1.4550	1.4641	1.4733	1.4826	9	18	27	36	45	34
56	1.4826	1.4920	1.5012	1.5108	1.5204	1.5301	1.5399	10	19	29	38	48	33
57	1.5399	1.5498	1.5597	1.5697	1.5797	1.5900	1.6003	10	20	30	40	50	32
58	1.6003	1.6107	1.6213	1.6319	1.6426	1.6533	1.6643	11	21	32	43	53	31
59	1.6643	1.6754	1.6865	1.6977	1.7090	1.7205	1.7311	11	23	34	45	56	**30**
60	1.7321	1.7439	1.7556	1.7675	1.7795	1.7917	1.8040	12	24	36	48	60	29
61	1.8040	1.8166	1.8291	1.8418	1.8546	1.8676	1.8807	13	26	38	51	64	28
62	1.8807	1.8942	1.9074	1.9210	1.9347	1.9485	1.9626	14	27	41	55	68	27
63	1.9626	1.9769	1.9912	2.0057	2.0203	2.0352	2.0503	15	29	44	58	73	26
64	2.0503	2.0657	2.0809	2.0965	2.1123	2.1282	2.1445	16	31	47	63	77	**25**
65	2.1445	2.1611	2.1776	2.1943	2.2113	2.2285	2.2460	17	34	51	68	85	**24**
66	2.2460	2.2640	2.2818	2.2998	2.3183	2.3369	2.3559	18	37	55	74	92	23
67	2.3559	2.3752	2.3946	2.4142	2.2421	2.4544	2.4751	20	40	60	79	99	22
68	2.4751	2.4963	2.5172	2.5386	2.5604	2.5825	2.6051	22	43	65	87	108	21
69	2.6051	2.6282	2.6511	2.6746	2.6984	2.7226	2.7475	24	47	71	95	118	**20**
	60′	50′	40′	30′	20′	10′	0′						**Degrees**

COTANGENTS

TANGENTS

Table XXIII (concluded)

Degrees	0′	10′	20′	30′	40′	50′	60′	1′	2′	3′	4′	5′	
70	2.7475	2.7729	2.7981	2.8239	2.8501	2.8768	2.9042	26	25	78	104	130	19
71	2.9042	2.9323	2.9602	2.9887	3.0176	3.0473	3.0777	29	58	87	115	144	18
72	3.0777	3.1080	3.1398	3.1716	3.2039	3.2369	3.2709	32	64	96	129	161	17
73	3.2709	3.3058	3.3404	3.3759	3.4121	3.4492	3.4874	36	72	108	144	180	16
74	3.4874	3.5267	3.5658	3.6059	3.6467	3.6888	3.7321	41	82	122	162	203	**15**
75	3.7321	3.7769	3.8219	3.8667	3.9133	3.9614	4.0108	46	94	139	186	232	14
76	4.0108	4.0622	4.1129	4.1653	4.2190	4.2742	4.3315	53	107	160	214	267	13
77	4.3315	4.391	4.4497	4.5107	4.5731	4.6376	4.7046	62	124	186	248	310	12
78	4.7046	4.7745	4.8434	4.9152	4.9886	5.0650	5.1446	73	146	219	292	365	11
79	5.1446	5.2279	5.3099	5.3955	5.4836	5.5753	5.6713	87	175	262	350	437	**10**
80	5.671	5.769	5.871	5.976	6.084	6.197	6.314						9
81	6.314	6.435	6.561	6.691	6.827	6.968	7.115						8
82	7.115	7.269	7.429	7.596	7.770	7.953	8.144						7
83	8.144	8.345	8.556	8.777	9.010	9.255	9.514						6
84	9.514	9.788	10.078	10.385	10.712	11.059	11.430						**5**